Single-Mode Displacement Sensor

Barbara Terhal

JARA Institute for Quantum Information

RWTH Aachen University

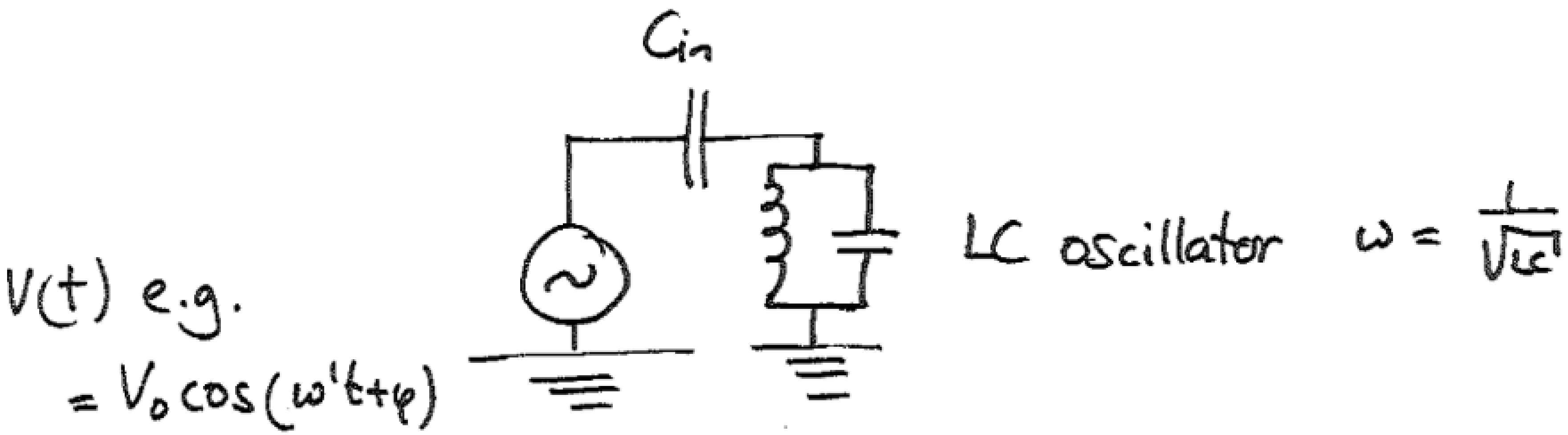
- B.M. Terhal and D. Weigand
Encoding a Qubit into a Cavity Mode in Circuit-QED using Phase Estimation, Phys. Rev. A 93, 012315 (2016)
- B.M. Terhal and K. Duivenvoorden,
Single-Mode Displacement Sensor, arXiv.org: 1603.02242

Displacement Sensor

Assume weak time-dependent unknown force $F(t)$ on oscillator so Hamiltonian $H(t) = \hbar\omega(a^\dagger a + 1/2) - \hat{q}F(t)$.

For example LC oscillator

$$H(t) = \hbar\omega(a^\dagger a + 1/2) + g\,V(t)(a + a^\dagger),\ \hat{q} = q = \frac{1}{\sqrt{2}}(a + a^\dagger)$$



What are the limits in determining the displacement caused by V(t)?

$\hat{p} = \frac{i}{\sqrt{2}}(a^\dagger - a)$

Displacement Sensor

Force induces a certain displacement $e^{-i\,u\hat{p}+i\,v\,\hat{q}}$, assume u and v small.

Displacement acts on an oscillator sensor state $|\psi_{sensor}\rangle$.

Goal: estimate u and v as $\tilde{u}$ and $\tilde{v}$ by clever measurement.

Single shot, single mode

Can we get accuracy down to zero (for increasing photon number) for both these estimates or

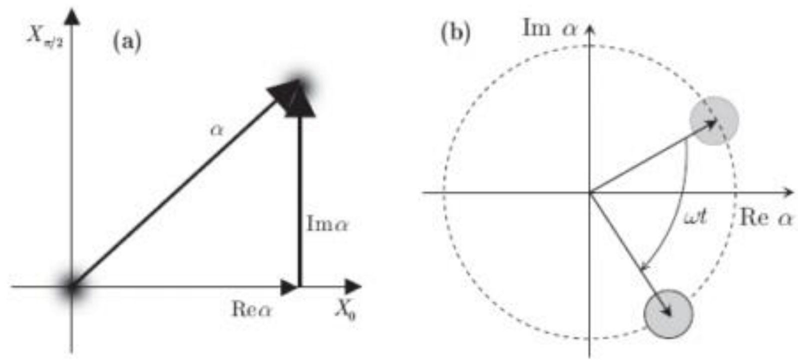
is this forbidden by Heisenberg uncertainty?

Displacement Intermezzo

Let $D(\alpha) = \exp(\alpha a^{\dagger} - \alpha^{*} a^{\dagger})$ be a unitary displacement

$|\alpha\rangle = D(\alpha)|0\rangle$ with vacuum state $|0\rangle$.

$$D(\alpha)D(\beta) = e^{i\,Im(\alpha\beta^{*})}D(\alpha+\beta)$$



Horizontal axis: $\left\langle (a + a^{\dagger})/2 \right\rangle \propto \langle q \rangle (= Re(\alpha))$
Vertical axis: $\left\langle -i(a - a^{\dagger})/2 \right\rangle \propto \langle p \rangle\ (= Im(\alpha))$

Displacement is translation in phase space

$R(\theta) = e^{-i\theta a^{\dagger} a}$ is a rotation, $R(\theta)|\alpha\rangle = \left|\alpha e^{-i\theta}\right\rangle$

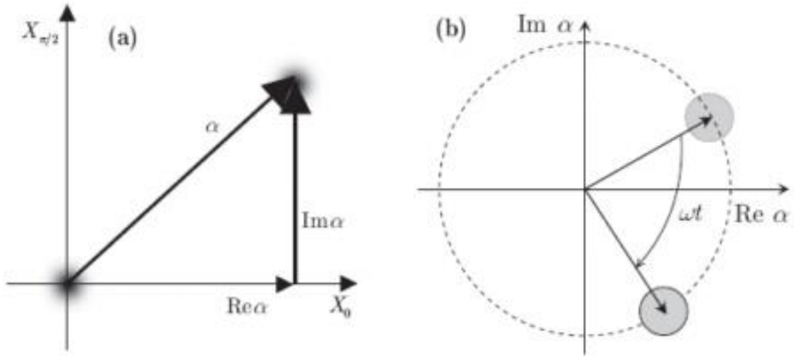
$D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$ unitary displacement

Displacement

For example (in rotating frame of oscillator)

$$H_{signal}(t) = \varepsilon(t)\left(a e^{-i\omega_r t} + a^\dagger e^{i\omega_r t}\right).$$

$$\varepsilon(t) = \Omega_x(t)\cos(\omega_d t) + \Omega_y(t)\sin(\omega_d t)$$



Take $\omega_r \approx \omega_d$ resonance.
And say $\Omega_y(t) = \Omega_y$, $\Omega_x(t) = \Omega_x$

H_{signal} induces a displacement α, with $Re(\alpha) \propto \Omega_y$, $\mathrm{Im}(\alpha) \propto \Omega_x$

Direction of displacement is controlled by $\Omega_x(t)$ and $\Omega_y(t)$.

Some Background

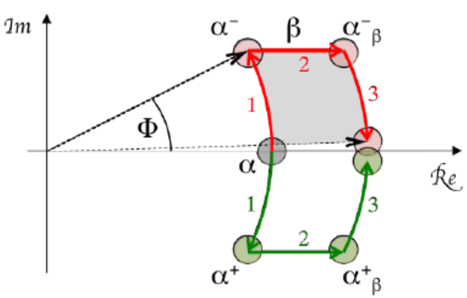
This is not the sensing of a magnetic field or an optical phase.

Penasa *et al. (*2016) (Paris group): *Measurement of the microwave field amplitude beyond the standard quantum limit.*

Using Rydberg atom-cavity mode entangled state.

$$P_g = \frac{1}{2}\left\{1 + \cos(2D\beta)\right\}$$

where $D = 2\alpha \sin\Phi$ is the

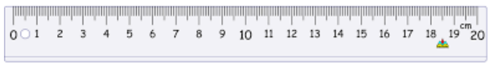


Protocol depends on knowing the direction of the displacement

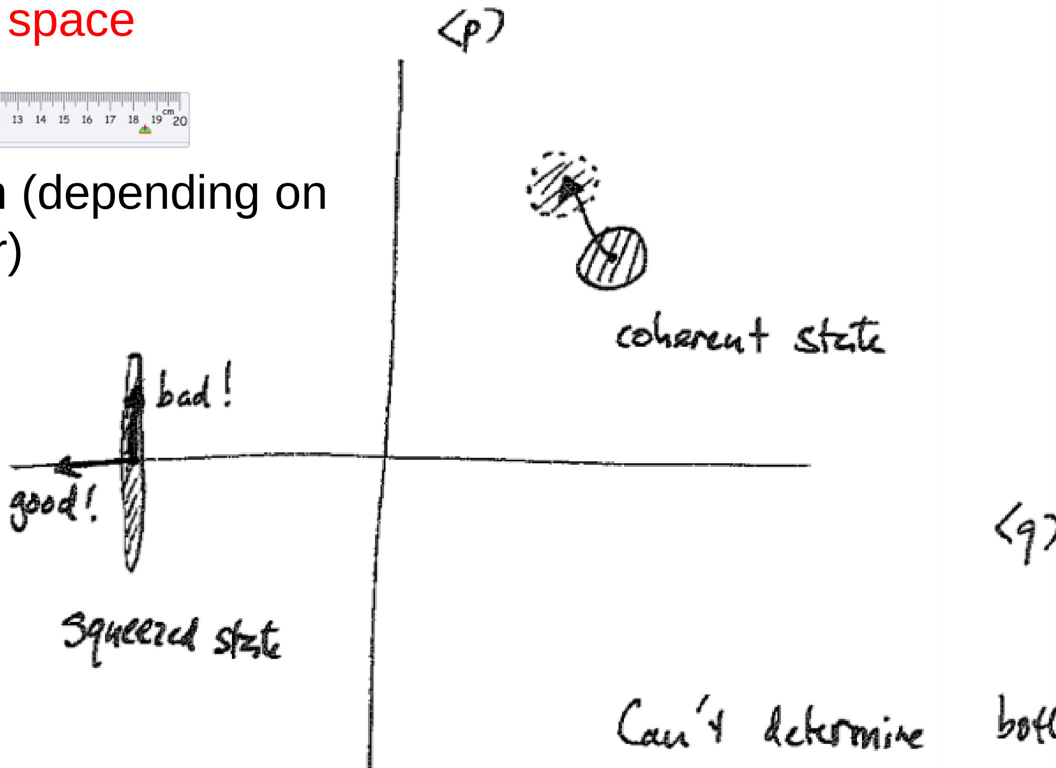
Standard Quantum Limit arguments do not apply since we do not intend to measure a quadrature

Limitations on sensing

Ruler in phase space



what resolution (depending on photon number)



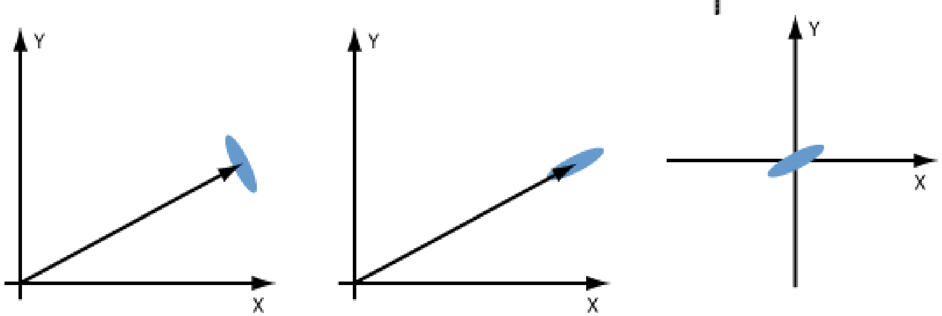
$$[q,p]=i$$

$$q=\frac{1}{\sqrt{2}}(a+a^\dagger)$$

$$p=\frac{i}{\sqrt{2}}(a^\dagger-a)$$

Can't determine both p & q

$$\mathrm{Var}(p)\,\mathrm{Var}(q)\geq\frac{1}{4}$$



amplitude squeezing phase squeezing squeezed vacuum

Displacement notation $D(\beta) = \exp(\beta a^\dagger - \beta^* a)$

Two-mode solutions

- Two modes, one with, say, amplitude-squeezed state and one phase-squeezed state, both undergoing displacement.

- Entanglement: two-mode squeezed state (e.g. Braunstein & Kimble 1999) one of which undergoes a displacement $D(\beta)$

  Trick is to have an eigenstate of both $p_1 + p_2$ and $q_1 - q_2$

Can we do it for a single mode?

Quantum Cramer-Rao Lower Bound

For unbiased estimates $\tilde{u}$ and $\tilde{v}$ (of the parameters u and v in displacement $e^{-i\,u\hat{p}+i\,v\,\hat{q}}$)

$Var(\tilde{u}) + Var(\tilde{v}) \geq 2$ (for coherent/thermal/squeezed states)

In general one has

$$Var(\tilde{u}) + Var(\tilde{v}) \geq \frac{1}{2\bar{n}+1}$$

Derived from $\Sigma \geq F^{-1}$, taking traces, then minimizing r.h.s.

$$F_{ij} \; = \; \frac{1}{2} \langle \psi_{u,v} | \, L_i L_j + L_j L_i \, | \psi_{u,v} \rangle$$

$$\Sigma_{ij} = \sum_x \mathbb{P}(x|\theta_1, \theta_2)(\tilde{\theta}_i(x) - \theta_i)(\tilde{\theta}_j(x) - \theta_j),$$

Is bound achievable/useful?

Genoni *et al*, PRA (2013)

$e^A e^B = e^B e^A e^{[A,B]}, for\ A, B\ lin. comb\ of\ p\ and\ q$

Grid State

Common +1 eigenstate of $S_p = \exp(i\,\hat{p}\,\sqrt{2\pi})$ and $S_q = \exp\left(i\,\hat{q}\,\sqrt{2\pi}\right)$ we call grid state $|\psi_{grid}\rangle$

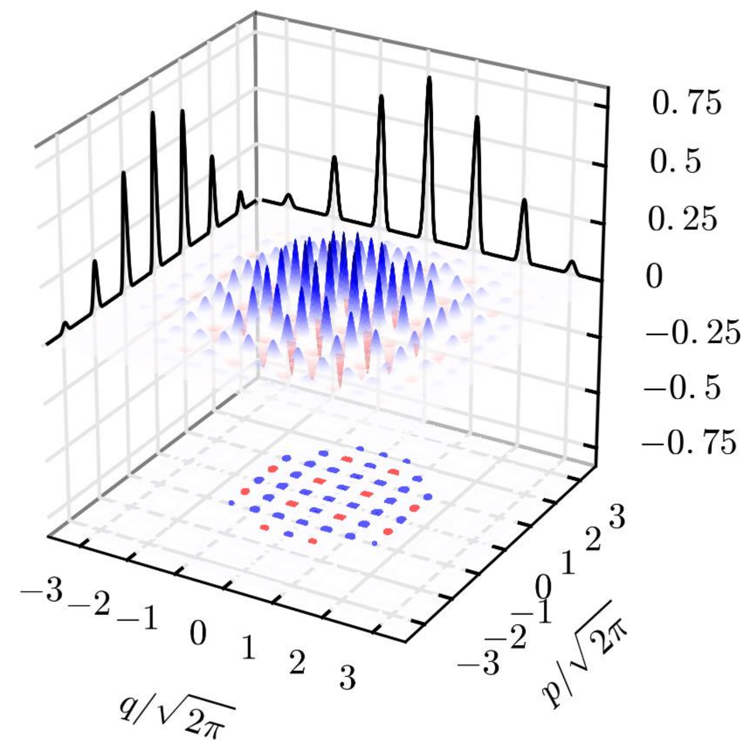
Thus

$$p \approx 0\ mod\ \sqrt{2\pi},\ q \approx 0\ mod\ \sqrt{2\pi}.$$

$$\bar{n} \approx \frac{1}{4\Delta^2}$$

σ of Gaussian envelope $\backsim \frac{1}{\Delta}$
and σ of individual peaks $\sim\Delta$

Maximum strength of displacement
on vacuum input $\bar{n} \leq \pi/2$



$\bar{n} \approx 12$

(Gottesman, Kitaev, Preskill, PRA 2001)

$e^A e^B = e^B e^A e^{[A,B]}, for\ A, B\ lin. comb\ of\ p\ and\ q$

Grid State

Common +1 eigenstate of $S_p = \exp(i\,\hat{p}\,\sqrt{2\pi})$ and $S_q = \exp\left(i\,\hat{q}\,\sqrt{2\pi}\right)$ we call grid state $|\psi_{grid}\rangle$

Thus

$$p \approx 0\ mod\ \sqrt{2\pi},\ q \approx 0\ mod\ \sqrt{2\pi}.$$

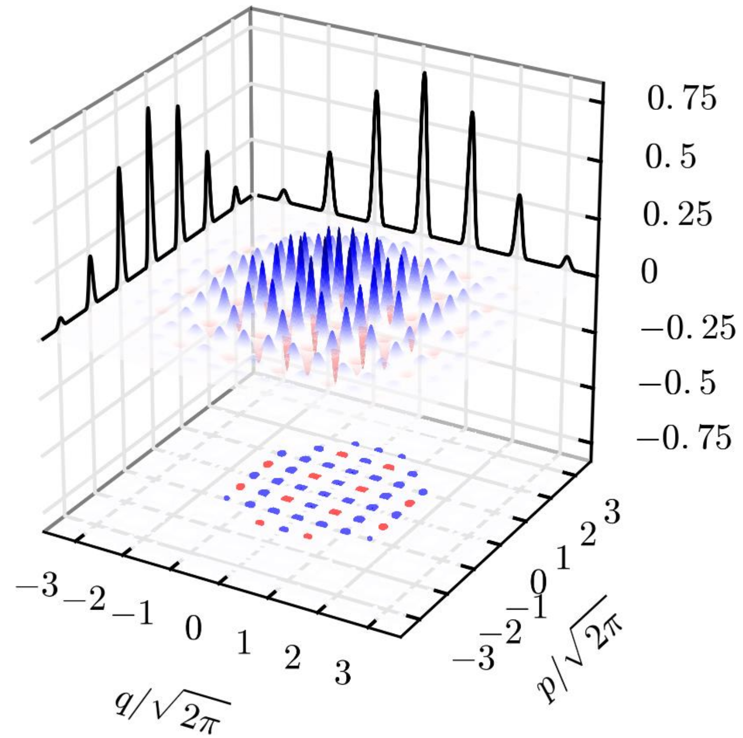
$$|\psi_{grid}\rangle \propto \sum_{t=-\infty}^{\infty} e^{-\pi\Delta^2 t} D(\sqrt{\pi})|sq.vac\rangle$$

$$\bar{n} \approx \frac{1}{4\Delta^2}$$

σ of Gaussian envelope $\backsim \frac{1}{\Delta}$
and σ of individual peaks $\sim\Delta$

Maximum strength of displacement
on vacuum input $\bar{n} \leq \pi/2$

(Gottesman, Kitaev, Preskill, PRA 2001)



$\bar{n} \approx 12$

Note S_p and S_q
slightly different as before
(state versus qubit space)

$e^A e^B = e^B e^A e^{[A,B]}, for\ A, B\ lin. comb\ of\ p\ and\ q$

Qubit into an oscillator

Gottesman, Kitaev, Preskill 2001:

Common +1 eigenstates of $S_p = \exp(-i\,\hat{p}\,2\sqrt{\pi})$ and $S_q = \exp(i\,\hat{q}\,2\sqrt{\pi})$ with $[\hat{q}, \hat{p}] = i$.

Thus

$$p = 0\ mod\ \sqrt{\pi},\ q = 0\ mod\ \sqrt{\pi}.$$

We have $Z = \exp(i\,\hat{q}\,\sqrt{\pi})\,, X = \exp(-i\,\hat{p}\,\sqrt{\pi})$,

$$Z|0\rangle = |0\rangle, Z|1\rangle = -|1\rangle$$

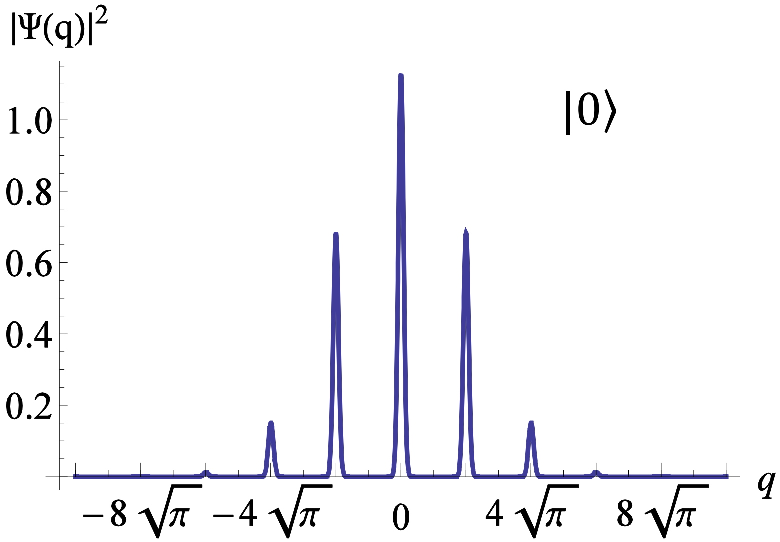
How to prepare a finite-photon number version of these states (using coupling of bosonic mode to a qubit)?

Approximate States

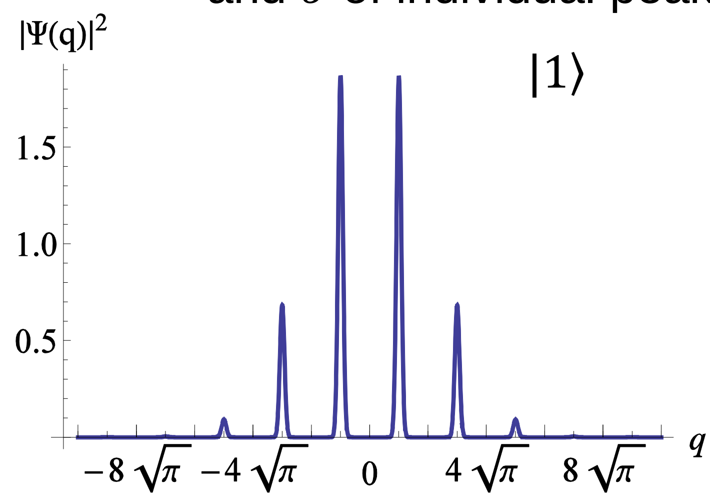
Common +1 eigenstates of $S_p = \exp(-i\,\hat{p}\,2\sqrt{\pi})$ and $S_q = \exp(i\,\hat{q}\,2\sqrt{\pi})$ are $|0\rangle$ and $|1\rangle$ so that $\mathrm{Z} = \exp(i\,\hat{q}\,\sqrt{\pi})$ with $Z|0\rangle = |0\rangle, Z|1\rangle = -|1\rangle$.

$\bar{n} \approx \frac{1}{2\Delta^2}$

σ of Gaussian envelope $\backsim \frac{1}{\Delta}$ and σ of individual peaks $\sim\Delta$



Squeezed peaks at even multiples of $\sqrt{\pi}$

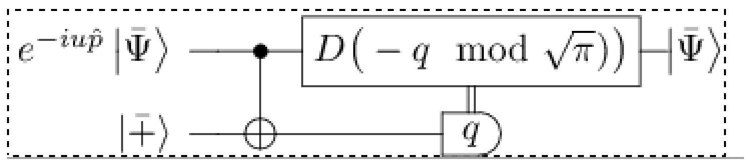


Squeezed peaks at odd multiples of $\sqrt{\pi}$

Error correction means detecting small displacements (measuring p and q mod $\sqrt{\pi}$) and reversing this displacement: this works for with $|u|, |v| \leq \sqrt{\pi}/2$.

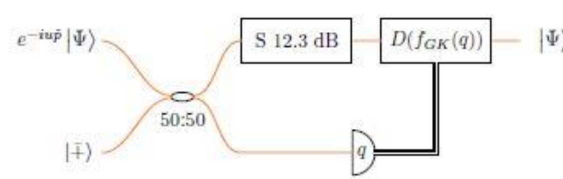
Preservation or Error Correction

- All errors (photon loss etc.) can be expanded in terms of linear combinations of displacements, $e^{i\,u\,\hat{p}}\;e^{i\,v\,\hat{q}}$ for real *u,v*.
- Assume n_{max} photons in oscillator. Expand $(\kappa\;ta)^p$ in terms of small displacements when $\kappa t\sqrt{n_{max}} \ll 1$.

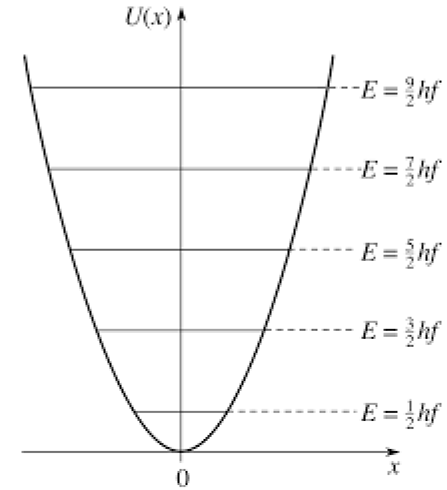


Error correction can be done with an ancilla oscillator state and linear optics (e.g. Glancy, Knill 2006) and can be repeated indefinitely as long as the approximate states can be viewed as a perfect state with only 'small' displacement errors $e^{i\,u\,\hat{p}}\;e^{i\,v\,\hat{q}}$ with $|u|, |v| \le \sqrt{\pi}/6$

'$\frac{\sqrt{\pi}}{6}$ Threshold'



Qubit into an oscillator



What states offer 'protection', form a code?

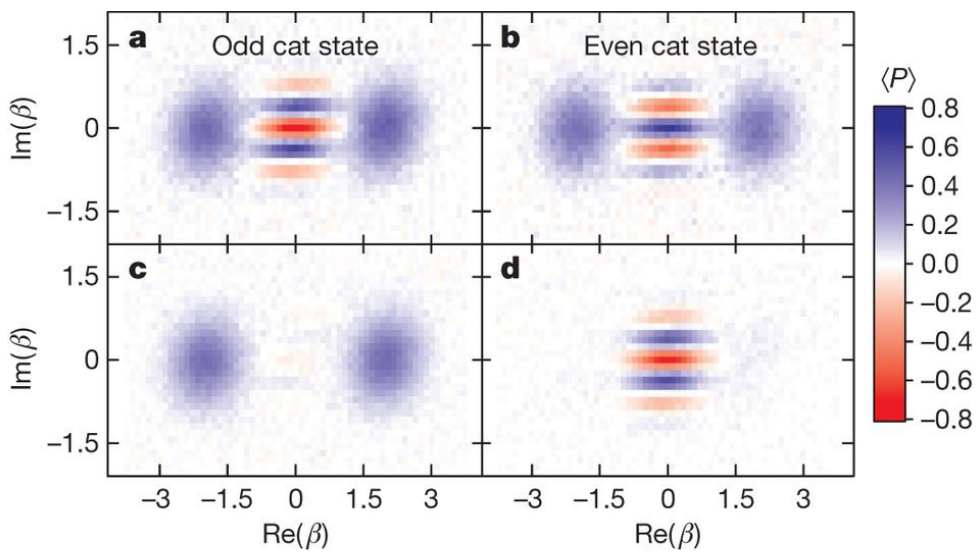
Small displacement errors

Cat code (Yale group)

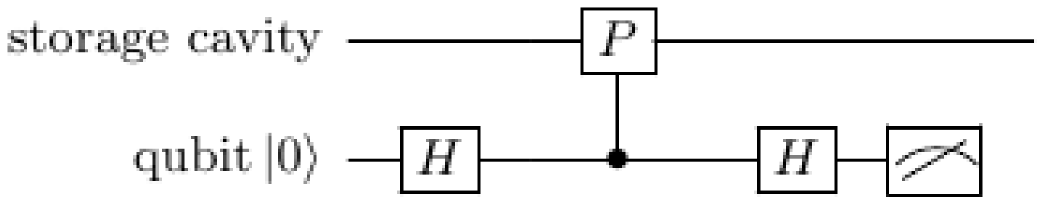
$|0\rangle \propto |\alpha\rangle + |-\alpha\rangle, |1\rangle \propto |i\alpha\rangle + |-i\alpha\rangle$.

Single photon loss (action of annihilation operator a) can be detected by photon parity measurement ($P = e^{i\pi a^\dagger a}$).

Single photon loss $a|0\rangle = |\alpha\rangle - |-\alpha\rangle$ gives odd parity cat state.



Qubit into an oscillator



Measuring the parity operator P using an ancilla qubit.

Controlled-P uses dispersive qubit-cavity coupling $\chi Z a^\dagger a$ for time t with $\chi t = \pi/2$.

Grid states can also be prepared by repeatedly performing ‘Ramsey Phase Estimation’ Experiments.

Displacement notation $D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$

Preparation of |0>

Approximate +1 eigenstate of $S_q = \exp(i\,\hat{q}\,2\sqrt{\pi})$
(and $\mathrm{Z} = \exp(i\,\hat{q}\,\sqrt{\pi})$) is squeezed vacuum $q \approx 0$.
How to make this into an +1 eigenstate of $S_p = \exp(-i\,\hat{p}\,2\sqrt{\pi})$?

Measure the eigenvalue $e^{i\theta}$ of $S_p = D(\sqrt{2\pi})$!
Phase Estimation

Phase Estimation:

Protocol which take eigenstate $|\psi_\theta\rangle$ of unitary U with
$U|\psi_\theta\rangle = e^{i\theta}|\psi_\theta\rangle$ and estimates θ as some $\tilde{\theta}$.

Only finite precision/approximate projection onto eigenstate.

No post-selection…

Phase Estimation (partial list)

Textbook PE: $l = 2^k, k = M-1, \ldots 0$ and use the circuit with adaptive phases φ (semi-classical implementation of Fourier Transform, 1999). One bit of phase per round.

Kitaev PE: $l = 2^k, k = M-1, \ldots 0$ and use the circuit with $\varphi = 0$ and $\varphi = \pi/2$

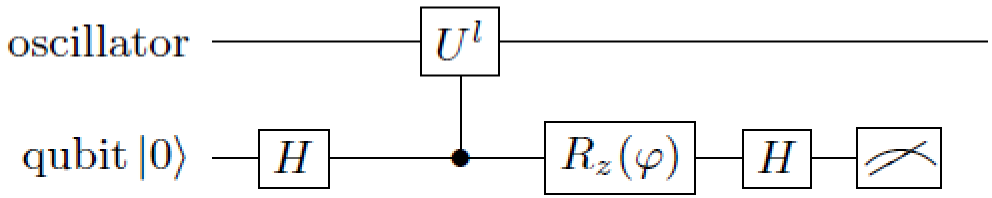
Heisenberg-limited PE without adaptive phases (Higgins et al. NJP 2009): Kitaev PE with round repetition depending on k. Only useful if resources (#photons or time) for doing U^l scale with l (no true here! it scales with l^2)

'Homer-Simpson' Non-adaptive PE: l=1

M/2 times with phase $\varphi = 0$, M/2 times with phase $\varphi = \pi/2$.

Adaptive protocol with feedback (Berry, Wiseman, Breslin, PRA 2001): l=1 phase adaptively φ in each round.

U is displacement, need controlled-displacement



$$U|\psi_\theta\rangle = e^{i\theta}|\psi_\theta\rangle$$

$$P(0) = \frac{1}{2}(1 + \cos(\theta l + \varphi))$$

Phase Estimation (partial list)

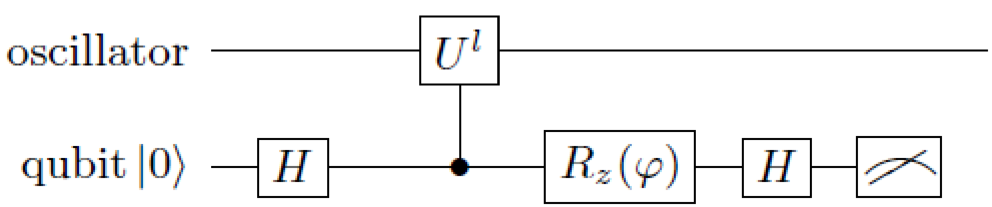
Too many photons, not the right Gaussian envelope

Kitaev PE ... nd use the circuit with
$\varphi = 0$ and ...

Heisenb ... al. NJP 2009):
Kitaev PE w ... l if resources (#photons or time) for ...

Homer-Simpson Non-adaptive PE: l=1
M/2 times with phase $\varphi = 0$, M/2 times with phase $\varphi = \pi/2$.

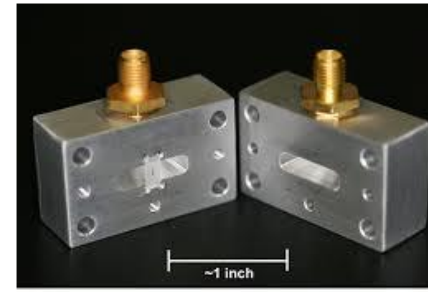
Adaptive protocol with feedback (Berry, Wiseman, Breslin, PRA 2001):
l=1 phase adaptively φ in each round.



$$U|\psi_\theta\rangle = e^{i\theta}|\psi_\theta\rangle$$
$$P(0) = \frac{1}{2}(1 + \cos(\theta l + \varphi))$$

Cats in cavities, e.g. Vlastakis *et al.,* Science 2013, Ofek *et al.:* arXiv.org:1602.04768

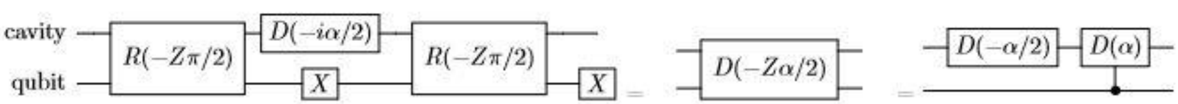
Circuit-QED paradigm



- High-Q micro-cavity, say,1 $msec$ or more.
- High quality qubit, say, $T_1, T_2 \approx O(10)\ \mu sec$
- Strong dispersive qubit-cavity coupling $\chi Z a^\dagger a$

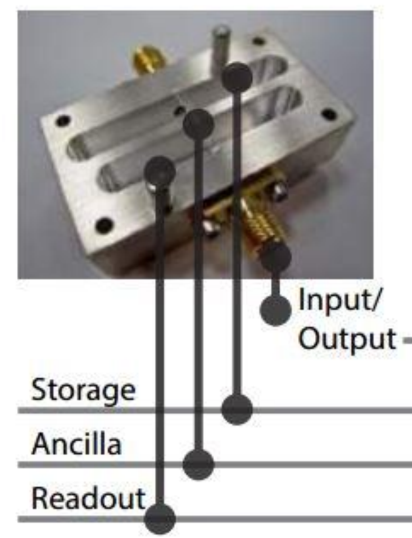
(e.g. $\frac{\chi}{2\pi} = 2.5MHz$, cavity/qubit detuning *1 GHz*, nonlinearities *O(1) kHz*)

- Dispersive coupling allows for qubit-controlled cavity rotation $(R(\theta Z) = \exp(-i\theta\ a^\dagger a\ Z))$ which can be directly used for qubit-controlled displacement.



- Controlled-rotations take $T = {}^{\pi}/_{\chi} = 200\ nanosec$.

- Use no more than 50 photons



Numerical Simulation Results

- Start with squeezed vacuum with 8.3 dB of squeezing.

- M=8 protocol is executed in $4\ \mu sec$.

(number of photons in state $\bar{n} \approx 25 \pm 25$)

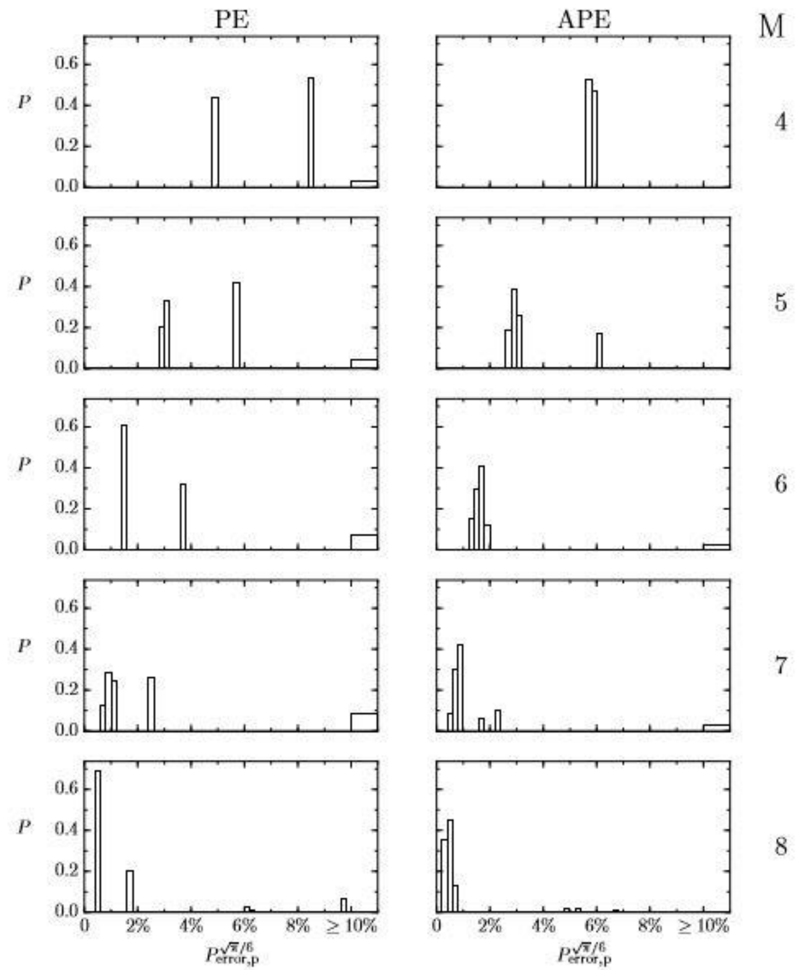
Adaptive protocol with M=8 is best

Gives a 94% (heralded) chance of preparing a state for which ‘probability for p-shift errors beyond $\sqrt{\pi}/6$’ on state is less than 1%.

Biggest source of concern are nonlinearities

$$K\,(a^\dagger a)^2, \chi' Z (a^\dagger a)^2.$$

and bad qubits. But Yale group can use numerical techniques to prepare cavity states such as GKP states.



Displacement Sensor Theory Analysis

Preparation of grid sensor state using textbook phase estimation

Displacement comes by…

Measurement of S_p and S_q using phase estimation.

Result: $\mathrm{MSD}(\tilde{u}) + \mathrm{MSD}(\tilde{v}) = O({}^{1}/_{\sqrt{\bar{n}}})$

Mean-square-deviation defined as $\mathrm{MSD}(\tilde{u}) = \sum_{\tilde{u}} P(\tilde{u}|u)(\tilde{u} - u)^2$

Puzzle: $\sqrt{n}$ versus n? Intuitively, grid state with Δ parameter is squeezed so that $Var(p) \sim \Delta^2$ and $\bar{n} \sim 1/\Delta^2$, so $1/\bar{n}$ scaling expected….
Remember bound $Var(\tilde{u}) + Var(\tilde{v}) \geq \frac{1}{2\bar{n}+1}$

Other sensor: quantum compass state

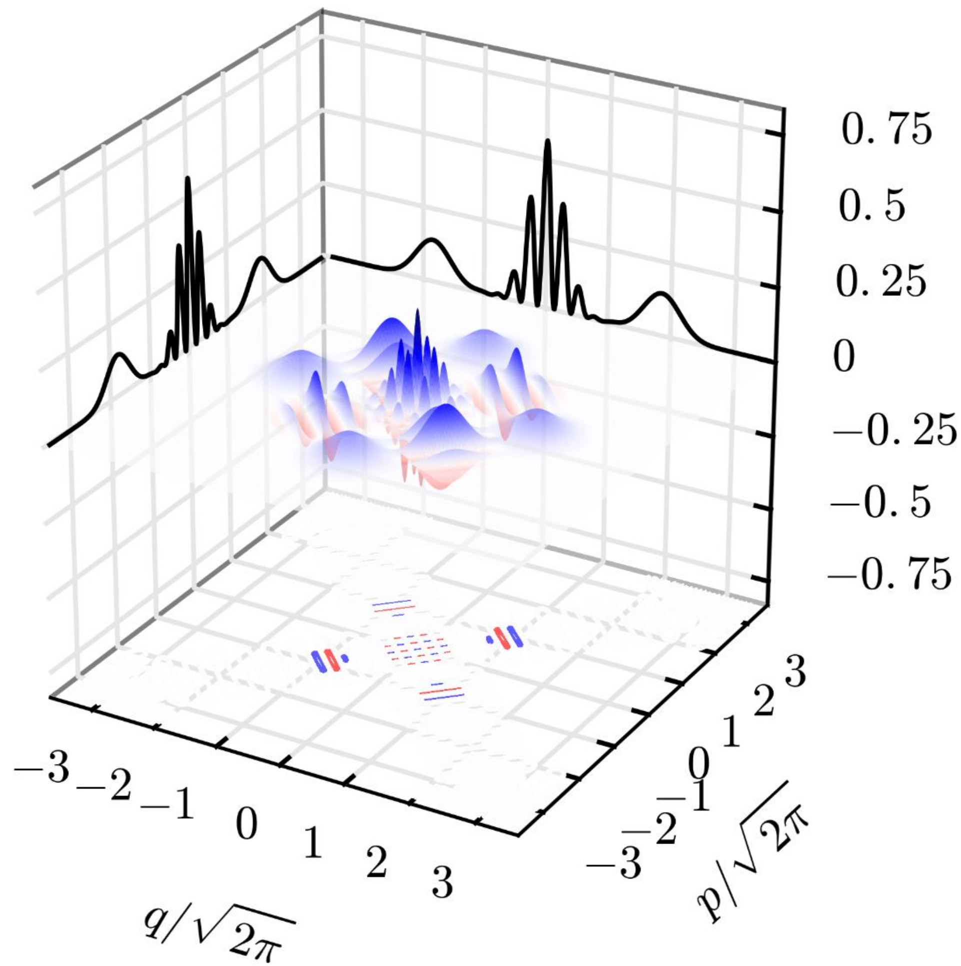
Zurek, Nature 2001: state is $\propto |\alpha\rangle + |-\alpha\rangle + |i\alpha\rangle + |-i\alpha\rangle$

Interference tiles in the middle have an area scaling as $\frac{1}{\bar{n}}$

Displacement $D(\beta)$ of strength

$$|\beta| \sim \frac{1}{\sqrt{\bar{n}}}$$

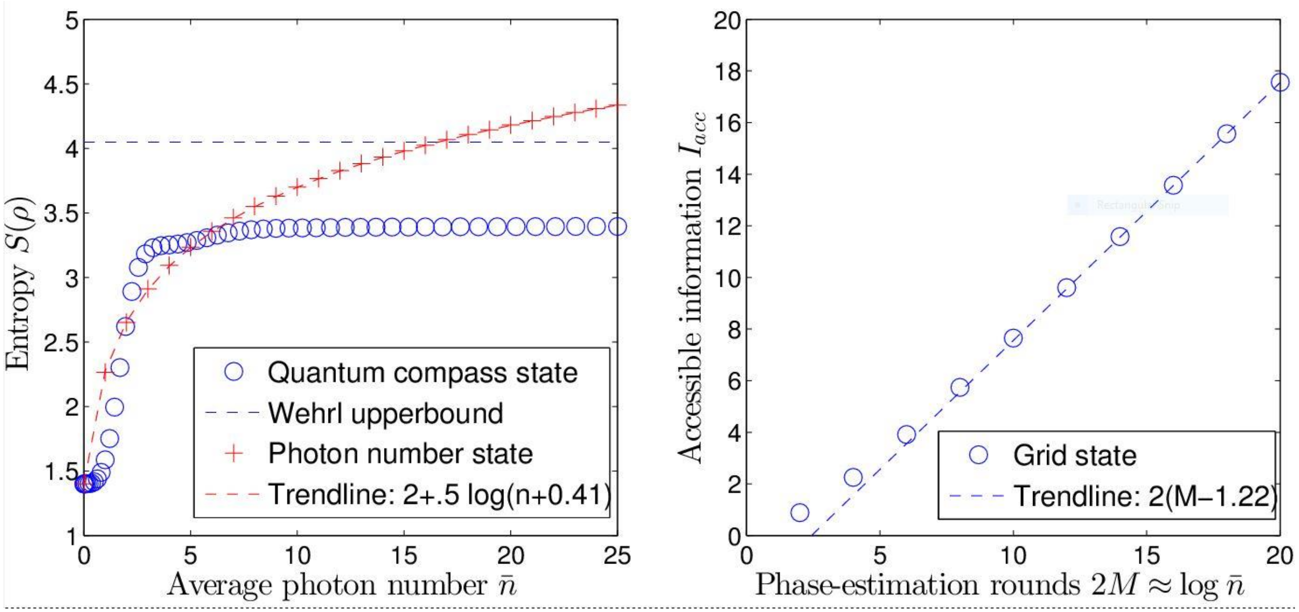
can map it onto orthogonal state.



Information

Assume u and v are uniformly distributed in interval $[-\sqrt{\frac{\pi}{2}}, \sqrt{\frac{\pi}{2}})$. What information about u and v can be obtained by measurement?

Our finding: Quantum compass state has O(1) upper bound on information while information in grid state scales as $\Theta(\log \bar{n})$.



Conclusion

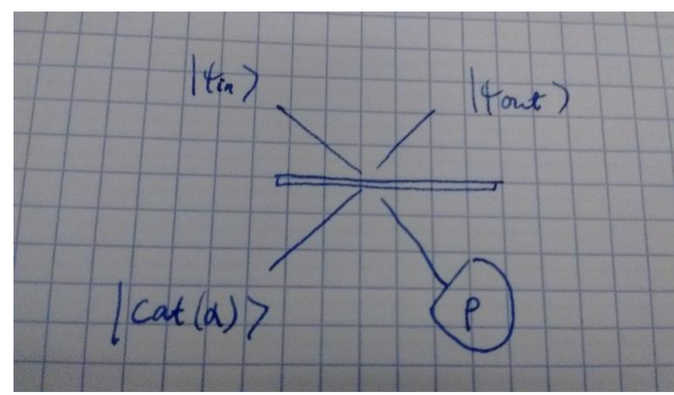
Creation of Grid or GKP code states may be experimentally feasible (using some heralding/post-selection).

They could be useful for encoding a qubit into an oscillator as well as for displacement sensing. Phase estimation can be used for non-fault tolerant error correction.

Current work on protocol for breeding grid states from cat states via beamsplitters and homodyne detection. M rounds is equivalent to M rounds of ‘Ramsey phase-estimation’.

Bad: Large cats needed! (but fix..)

Good: grid states on the fly, no nonlinearities.



(improvement on scheme by Vasconcelos, Sanz, Glancy in 2010)