

Separation between quantum Lovász number and entanglement-assisted zero-error classical capacity

Xin Wang

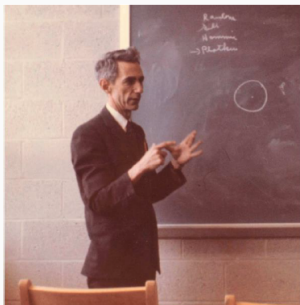
QCIS, University of Technology Sydney (UTS)

Joint work with **Runyao Duan** (UTS), [arXiv: 1608.04508](https://arxiv.org/abs/1608.04508)

AQIS 2016, Taipei

Introduction

Capacity of a communication channel



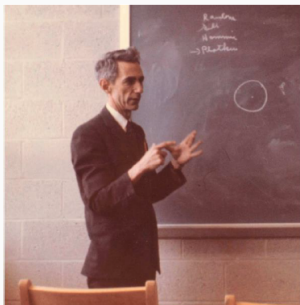
Claude Elwood Shannon

(1916 - 2001)

(photo on 17.04.1961)



Capacity of a communication channel



Claude Elwood Shannon

(1916 - 2001)

(photo on 17.04.1961)



The fundamental problem of communication is that of reproducing at one point, either exactly or approximately, a message selected at another point.

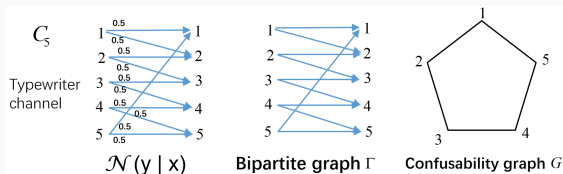
—Claude Elwood Shannon, 1948



- Approximately (small-error) [Shannon-1948]: $\forall m, \hat{m} \approx m$
- Exactly (**zero-error**) [Shannon-1956]: $\forall m, \hat{m} = m$

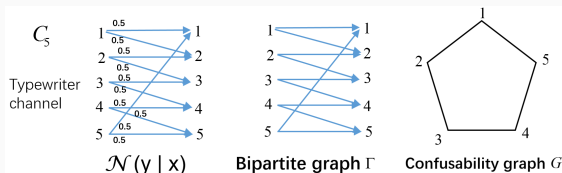
Zero-error capacity (Shannon capacity) of a classical channel

- Classical channel: $\mathcal{N}(y|x)$, e.g.,



Zero-error capacity (Shannon capacity) of a classical channel

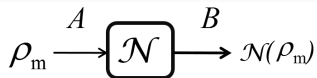
- Classical channel: $\mathcal{N}(y|x)$, e.g.,



- One-shot zero-error capacity $\alpha(\mathcal{N})$: the maximum number of messages can be sent via \mathcal{N} without error, **depends on G** .
- Asymptotic** zero-error capacity: $C_0(\mathcal{N}) = \sup_{n \geq 1} \frac{1}{n} \log \alpha(\mathcal{N}^{\otimes n})$.
- Example:
 $\alpha(\mathcal{C}_5) = 2, \alpha(\mathcal{C}_5^{\otimes 2}) = 5$ [Shannon-1956], $C_0(\mathcal{C}_5) = \log \sqrt{5}$ [Lovász-1979].
- If there is **feedback** assistance, the zero-error capacity depends only on the **bipartite graph** [Shannon-1956].

Zero-error capacity of a quantum channel

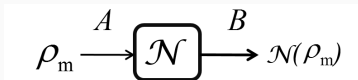
- **One-shot zero-error classical capacity:** The maximum integer n such that $\{\mathcal{N}(\rho_1), \dots, \mathcal{N}(\rho_n)\}$ are orthogonal (perfectly distinguished).



- **Quantum Channel:** completely positive (CP) and trace-preserving (TP) linear map \mathcal{N} from A to B .
 - Choi-Kraus Representation: $\mathcal{N}(\rho) = \sum_k E_k \rho E_k^\dagger$, $\sum_k E_k^\dagger E_k = \mathbb{1}$
 - Choi-Jamiołkowski matrix:
 $J_{\mathcal{N}} = \sum_{ij} |i\rangle\langle j|_A \otimes \mathcal{N}(|i\rangle\langle j|_{A'}) = (\text{id}_A \otimes \mathcal{N})|\Phi_{AA'}\rangle\langle\Phi_{AA'}|$, with
 $|\Phi_{AA'}\rangle = \sum_k |k_A\rangle|k_{A'}\rangle$

Zero-error capacity of a quantum channel

- **One-shot zero-error classical capacity:** The maximum integer n such that $\{\mathcal{N}(\rho_1), \dots, \mathcal{N}(\rho_n)\}$ are orthogonal (perfectly distinguished).



- **Quantum Channel:** completely positive (CP) and trace-preserving (TP) linear map \mathcal{N} from A to B .

- Choi-Kraus Representation: $\mathcal{N}(\rho) = \sum_k E_k \rho E_k^\dagger$, $\sum_k E_k^\dagger E_k = \mathbb{1}$

- Choi-Jamiołkowski matrix:

$$J_{\mathcal{N}} = \sum_{ij} |i\rangle\langle j|_A \otimes \mathcal{N}(|i\rangle\langle j|_{A'}) = (\text{id}_A \otimes \mathcal{N})|\Phi_{AA'}\rangle\langle\Phi_{AA'}|, \text{ with } |\Phi_{AA'}\rangle = \sum_k |k_A\rangle|k_{A'}\rangle$$

- **Non-commutative (confusability) graph** [Duan-Severini-Winter-13]:

$$S = \text{span}\{E_k^\dagger E_j\} \quad (S = S^\dagger, \mathbb{1} \in S).$$

The zero-error capacity of \mathcal{N} depends only on S .

- **Non-commutative bipartite graph** [Duan-Severini-Winter-16]:

$$K = K(\mathcal{N}) = \text{span}\{E_k\}.$$

If there is **feedback assistance**, the zero-error capacity depends only on K .

Quantum Lovász number

- The zero-error capacity C_0 is **not known** to be **computable**.

Quantum Lovász number

- The zero-error capacity C_0 is **not known** to be **computable**.
- Nevertheless, [Lovász-1979] introduce the **Lovász number** to upper bound the zero-error capacity of a classical channel:

$$2^{C_0(\mathcal{N})} \leq \vartheta(G) = \max\{\|\mathbb{1} + T\| : T_{i,j} = 0 \text{ if } \{i,j\} \in E, T = T^\dagger, \mathbb{1} + T \geq 0\}.$$

Quantum Lovász number

- The zero-error capacity C_0 is **not known** to be **computable**.
- Nevertheless, [Lovász-1979] introduce the **Lovász number** to upper bound the zero-error capacity of a classical channel:

$$2^{C_0(\mathcal{N})} \leq \vartheta(G) = \max\{\|\mathbb{1} + T\| : T_{i,j} = 0 \text{ if } \{i,j\} \in E, T = T^\dagger, \mathbb{1} + T \geq 0\}.$$

- [Duan-Severini-Winter-2013] introduced the **quantum Lovász number** to upper bound the zero-error capacity of a quantum channel:

$$2^{C_0(\mathcal{N})} \leq \tilde{\vartheta}(S) = \max \langle \Phi | (\mathbb{1} \otimes \rho + T) | \Phi \rangle$$

s.t. $T \in S^\perp \otimes \mathcal{L}(A'), \text{Tr } \rho = 1, \mathbb{1} \otimes \rho + T \geq 0, \rho \geq 0.$

Quantum zero-error information theory

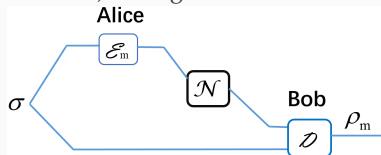
- **Zero-error information theory** has grown into a vast field over the past decades. (See review article [Korner-Orlitsky-1998]).

Quantum zero-error information theory

- **Zero-error information theory** has grown into a vast field over the past decades. (See review article [Korner-Orlitsky-1998]).
- More capacities in **quantum zero-error information theory**
 - Quantum assistance
 - Shared Entanglement (E)
 - Quantum No-signalling Correlations (QNSC)
 - Feedback (F)
 - Zero-error quantum or private capacity [Leung-Yu-2015, next talk!]

Quantum zero-error information theory

- **Zero-error information theory** has grown into a vast field over the past decades. (See review article [Korner-Orlitsky-1998]).
- More capacities in **quantum zero-error information theory**
 - Quantum assistance
 - Shared Entanglement (E)
 - Quantum No-signalling Correlations (QNSC)
 - Feedback (F)
 - Zero-error quantum or private capacity [Leung-Yu-2015, next talk!]
- **Entanglement-assisted zero-error classical capacity**
 - Make use of (pre-shared) entanglement for communication



- One-shot entanglement-assisted zero-error capacity $\tilde{\alpha}(\mathcal{N})$: the maximum number n such that $\{\rho_1, \dots, \rho_n\}$ are orthogonal.
- Asymptotics: $C_{0E}(\mathcal{N}) = \sup_{n \rightarrow \infty} \frac{1}{n} \log \tilde{\alpha}(\mathcal{N}^{\otimes n})$
- C_{0E} **depends on** the non-commutative graph \mathcal{S} [Duan-Severini-Winter-13].

**Separation between quantum
Lovász number and
entanglement-assisted zero-error
classical capacity**

Problem: $C_{0E}(\mathcal{N}) = \log \tilde{v}(\mathcal{N})$?

Two remarkable phenomena in zero-error information theory

- Entanglement can improve the zero-error capacity
 - Classical channel (no such advantage for the the normal capacity)
 - One-shot: [PRL, Cubitt-Leung-Matthews-Winter-2010]
 - Asymptotic: [CMP, Leung-Mancinska-Matthews-Ozols-Roy-2012]
 - Quantum channel (Superdense Coding)

Problem: $C_{0E}(\mathcal{N}) = \log \tilde{v}(\mathcal{N})$?

Two remarkable phenomena in zero-error information theory

- Entanglement can improve the zero-error capacity
 - Classical channel (no such advantage for the the normal capacity)
 - One-shot: [PRL, Cubitt-Leung-Matthews-Winter-2010]
 - Asymptotic: [CMP, Leung-Mancinska-Matthews-Ozols-Roy-2012]
 - Quantum channel (Superdense Coding)
- C_{0E} is upper bounded by the quantum Lovász number

$$C_{0E}(\mathcal{N}) \leq \log \tilde{v}(\mathcal{N}).$$

- Classical channel: [PRA, Beigi-2010]
- Quantum channel: [IT, Duan-Severini-Winter-2013, arXiv-2010]

Problem: $C_{0E}(\mathcal{N}) = \log \tilde{\vartheta}(\mathcal{N})$?

Two remarkable phenomena in zero-error information theory

- Entanglement can improve the zero-error capacity
 - Classical channel (no such advantage for the the normal capacity)
 - One-shot: [PRL, Cubitt-Leung-Matthews-Winter-2010]
 - Asymptotic: [CMP, Leung-Mancinska-Matthews-Ozols-Roy-2012]
 - Quantum channel (Superdense Coding)
- C_{0E} is upper bounded by the quantum Lovász number

$$C_{0E}(\mathcal{N}) \leq \log \tilde{\vartheta}(\mathcal{N}).$$

- Classical channel: [PRA, Beigi-2010]
- Quantum channel: [IT, Duan-Severini-Winter-2013, arXiv-2010]

A pressing problem in zero-error information theory

- $C_{0E}(\mathcal{N}) = \log \tilde{\vartheta}(\mathcal{N})$? (mentioned in the above articles)
- Open since 2010.
- If this is true, then C_{0E} is additive.

Difficulties of the problem and our approach

Difficulties

- Entanglement assistance is difficult to characterize
- Understanding about C_{0E} is limited

Difficulties of the problem and our approach

Difficulties

- Entanglement assistance is difficult to characterize
- Understanding about C_{0E} is limited

Our approach

- Construct a class of qutrit-to-qutrit channel \mathcal{N}_α
- Consider the zero-error communication properties of \mathcal{N}_α with the assistance of quantum no-signalling correlations (QNSC), which is a broader class of resources than entanglement.
- Compare $C_{0,NS}$ to quantum Lovász number

Construction of a class of qutrit-to-qutrit channel \mathcal{N}_α

- The class of channels we use is

$$\mathcal{N}_\alpha(\rho) = E_\alpha \rho E_\alpha^\dagger + D_\alpha \rho D_\alpha^\dagger \quad (0 < \alpha \leq \pi/4)$$

with

$$E_\alpha = \sin \alpha |0\rangle\langle 1| + |1\rangle\langle 2|,$$

$$D_\alpha = \cos \alpha |2\rangle\langle 1| + |1\rangle\langle 0|.$$

Construction of a class of qutrit-to-qutrit channel \mathcal{N}_α

- The class of channels we use is

$$\mathcal{N}_\alpha(\rho) = E_\alpha \rho E_\alpha^\dagger + D_\alpha \rho D_\alpha^\dagger \quad (0 < \alpha \leq \pi/4)$$

with

$$E_\alpha = \sin \alpha |0\rangle\langle 1| + |1\rangle\langle 2|,$$

$$D_\alpha = \cos \alpha |2\rangle\langle 1| + |1\rangle\langle 0|.$$

- The **non-commutative graph** of \mathcal{N}_α is $S = \text{span}\{F_1, F_2, F_3, F_4\}$ with

$$F_1 = |0\rangle\langle 0| + \cos^2 \alpha |1\rangle\langle 1|,$$

$$F_2 = \sin^2 \alpha |1\rangle\langle 1| + |2\rangle\langle 2|,$$

$$F_3 = |0\rangle\langle 2| \text{ and } F_4 = |2\rangle\langle 0|.$$

- The Choi-Jamiołkowski matrix of \mathcal{N}_α is given by

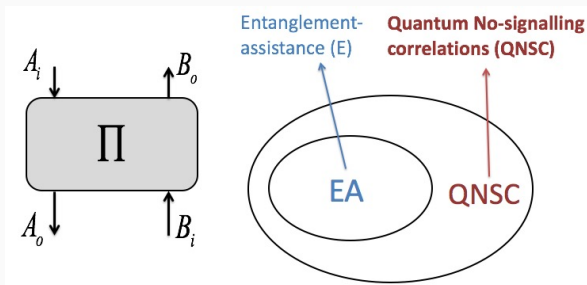
$$J_\alpha = (1 + \sin^2 \alpha) |u\rangle\langle u| + (1 + \cos^2 \alpha) |v\rangle\langle v|,$$

where

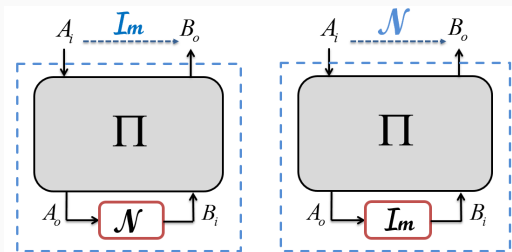
$$|u\rangle = \frac{\sin \alpha}{\sqrt{1 + \sin^2 \alpha}} |10\rangle + \frac{1}{\sqrt{1 + \sin^2 \alpha}} |21\rangle, |v\rangle = \frac{\cos \alpha}{\sqrt{1 + \cos^2 \alpha}} |12\rangle + \frac{1}{\sqrt{1 + \cos^2 \alpha}} |01\rangle.$$

Quantum no-signalling correlations (QNSC)

- Two-input and two-output quantum channels (bipartite CPTP linear map) $\Pi : \mathcal{L}(\mathcal{A}_i) \otimes \mathcal{L}(\mathcal{B}_i) \rightarrow \mathcal{L}(\mathcal{A}_o) \otimes \mathcal{L}(\mathcal{B}_o)$
- No-signalling correlations [Beckman-Gottesman-Nielsen-Preskill-01, Eggeling-Schlingemann-Werner-02, Piani-Horodecki et al.-06], **A and B cannot use the channel to communicate classical information.**
- The structure of QNSC is more **mathematically tractable** than the entanglement assistance.



QNSC-assisted zero-error capacity and simulation cost



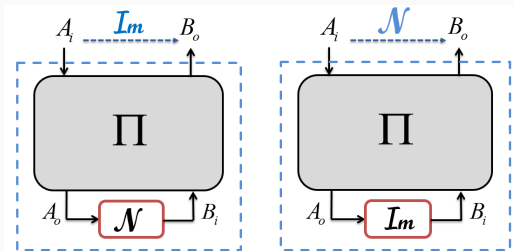
- One-shot QNSC-assisted zero-error capacity is given by [Duan-Winter-2016]:

$$\Upsilon(\mathcal{N}) = \max \text{Tr } S_A \text{ s.t. } 0 \leq U_{AB} \leq S_A \otimes \mathbb{1}_B, \quad (1)$$

$$\text{Tr}_A U_{AB} = \mathbb{1}_B, \text{Tr } J_{AB}(S_A \otimes \mathbb{1}_B - U_{AB}) = 0.$$

- Asymptotic: $C_{0,\text{NS}}(\mathcal{N}) = \sup_{n \geq 1} \frac{1}{n} \log \Upsilon(\mathcal{N}^{\otimes n})$

QNSC-assisted zero-error capacity and simulation cost



- **One-shot QNSC-assisted zero-error capacity** is given by [Duan-Winter-2016]:

$$\Upsilon(\mathcal{N}) = \max \text{Tr } S_A \text{ s.t. } 0 \leq U_{AB} \leq S_A \otimes \mathbb{1}_B, \quad (1)$$

$$\text{Tr}_A U_{AB} = \mathbb{1}_B, \text{Tr } J_{AB}(S_A \otimes \mathbb{1}_B - U_{AB}) = 0.$$

- Asymptotic: $C_{0,NS}(\mathcal{N}) = \sup_{n \geq 1} \frac{1}{n} \log \Upsilon(\mathcal{N}^{\otimes n})$
- **One-shot QNSC-assisted zero-error simulation cost** [Duan-Winter-2016]:

$$\Sigma(\mathcal{N}) = 2^{-H_{\min}(A|B)_J} = \min \text{Tr } T_B, \text{ s.t. } J_{AB} \leq \mathbb{1}_A \otimes T_B,$$

where $H_{\min}(A|B)_J$ is the **conditional min-entropy** [Koenig-Renner-Schaffner-09].

- $\log \Upsilon(\mathcal{N}) \leq C_{0,NS}(\mathcal{N}) \leq C_E(\mathcal{N}) \leq S_{0,NS}(\mathcal{N}) = \log \Sigma(\mathcal{N})$

QNSC-assisted zero-error capacity of \mathcal{N}_α

For the channel \mathcal{N}_α ($0 < \alpha \leq \pi/4$),

- $C_{0,NS}(\mathcal{N}_\alpha) \geq 2$
 - Construct a feasible solution to the prime SDP of $\Upsilon(\mathcal{N}_\alpha)$:

$$R_A = 2(\cos^2 \alpha |0\rangle\langle 0| + |1\rangle\langle 1| + \sin^2 \alpha |2\rangle\langle 2|)$$

$$U_{AB} = \cos^2 \alpha |01\rangle\langle 01| + \sin^2 \alpha |21\rangle\langle 21| + |10\rangle\langle 10| + |12\rangle\langle 12| \\ + \sin \alpha (|10\rangle\langle 21| + |21\rangle\langle 10|) + \cos \alpha (|01\rangle\langle 12| + |12\rangle\langle 01|).$$

- $C_{0,NS}(\mathcal{N}_\alpha) \geq \log \Upsilon(\mathcal{N}_\alpha) \geq \log \text{Tr } R_A = 2$

QNSC-assisted zero-error capacity of \mathcal{N}_α

For the channel \mathcal{N}_α ($0 < \alpha \leq \pi/4$),

- $C_{0,NS}(\mathcal{N}_\alpha) \geq 2$
 - Construct a feasible solution to the prime SDP of $\Upsilon(\mathcal{N}_\alpha)$:

$$R_A = 2(\cos^2 \alpha |0\rangle\langle 0| + |1\rangle\langle 1| + \sin^2 \alpha |2\rangle\langle 2|)$$

$$U_{AB} = \cos^2 \alpha |01\rangle\langle 01| + \sin^2 \alpha |21\rangle\langle 21| + |10\rangle\langle 10| + |12\rangle\langle 12| \\ + \sin \alpha (|10\rangle\langle 21| + |21\rangle\langle 10|) + \cos \alpha (|01\rangle\langle 12| + |12\rangle\langle 01|).$$

- $C_{0,NS}(\mathcal{N}_\alpha) \geq \log \Upsilon(\mathcal{N}_\alpha) \geq \log \text{Tr } R_A = 2$
- Similarly, we can prove that

$$S_{0,NS}(\mathcal{N}_\alpha) = -H_{\min}(A|B)_{J_{AB}} = \log \Sigma(\mathcal{N}) = 2.$$

- Recall $C_{0,NS}(\mathcal{N}) \leq C_E(\mathcal{N}) \leq S_{0,NS}(\mathcal{N})$

Proposition:

$$C_{0,NS}(\mathcal{N}_\alpha) = C_E(\mathcal{N}_\alpha) = S_{0,NS}(\mathcal{N}_\alpha) = 2.$$

Quantum Lovász number of \mathcal{N}_α

Proposition: For the channel \mathcal{N}_α ($0 < \alpha \leq \pi/4$),

$$\tilde{\vartheta}(\mathcal{N}_\alpha) = 2 + \cos^2 \alpha + \cos^{-2} \alpha > 4.$$

Quantum Lovász number of \mathcal{N}_α

Proposition: For the channel \mathcal{N}_α ($0 < \alpha \leq \pi/4$),

$$\tilde{\vartheta}(\mathcal{N}_\alpha) = 2 + \cos^2 \alpha + \cos^{-2} \alpha > 4.$$

Outline of proof:

- Recall the **primal SDP** of $\tilde{\vartheta}(\mathcal{N}_\alpha)$:

$$\tilde{\vartheta}(S) = \max \langle \Phi | (\mathbb{1} \otimes \rho + T) | \Phi \rangle$$

$$\text{s.t. } T \in S^\perp \otimes \mathcal{L}(A'), \mathbb{1} \otimes \rho + T \geq 0, \text{Tr } \rho = 1, \rho \geq 0.$$

- Suppose that $\rho = \frac{\cos^2 \alpha}{1 + \cos^2 \alpha} |0\rangle\langle 0| + \frac{1}{1 + \cos^2 \alpha} |1\rangle\langle 1|$ and $T = T_1 \otimes T_2 + R$ with

$$T_1 = \frac{1}{1 + \cos^2 \alpha} (|0\rangle\langle 0| - \frac{1}{\cos^2 \alpha} |1\rangle\langle 1| + \frac{\sin^2 \alpha}{\cos^2 \alpha} |2\rangle\langle 2|),$$

$$T_2 = \cos^4 \alpha |0\rangle\langle 0| - |1\rangle\langle 1|, R = |00\rangle\langle 11| + |11\rangle\langle 00|.$$

- $\{\rho, T\}$ is a feasible solution to primal SDP of $\tilde{\vartheta}(\mathcal{N}_\alpha)$. Hence,

$$\tilde{\vartheta}(\mathcal{N}_\alpha) \geq \text{Tr}[\Phi \langle \Phi | (\mathbb{1} \otimes \rho + T)] = 2 + \cos^2 \alpha + 1/\cos^2 \alpha.$$

- Similarly, we can use the **dual SDP** to show

$$\tilde{\vartheta}(\mathcal{N}_\alpha) \leq 2 + \cos^2 \alpha + 1/\cos^2 \alpha.$$

$$C_{0E}(\mathcal{N}) \neq \log \tilde{\vartheta}(\mathcal{N})!!!$$

Theorem: For the channel \mathcal{N}_α ($0 < \alpha \leq \pi/4$), the quantum Lovász number is **strictly larger** than the entanglement-assisted zero-error capacity (or even with quantum no-signalling assistance), i.e.,

$$\log \tilde{\vartheta}(\mathcal{N}_\alpha) > C_{0,NS}(\mathcal{N}_\alpha) \geq C_{0E}(\mathcal{N}_\alpha).$$

- $C_{0E}(\mathcal{N}_\alpha) \leq C_{0,NS}(\mathcal{N}_\alpha) = C_E(\mathcal{N}_\alpha) = S_{0,NS}(\mathcal{N}_\alpha) = 2$
- $\log \tilde{\vartheta}(\mathcal{N}_\alpha) = \log(2 + \cos^2 \alpha + \cos^{-2} \alpha) > 2$

Quantum fractional packing number and feedback-assisted or QNSC-assisted zero-error capacity

- For **classical channel** \mathcal{N} [Shannon-1956, Cubitt-Leung-Matthews-Winter-2011],

$$C_{0F}(\mathcal{N}) = C_{0,NS}(\mathcal{N}) = \log \alpha^*(\mathcal{N}),$$

where $\alpha^*(\mathcal{N})$ is the fractional packing number:

$$\alpha^*(\mathcal{N}) = \max \sum_x v_x \text{ s.t. } \sum_x v_x [\mathcal{N}(y|x)] \leq 1 \forall y, 0 \leq v_x \leq 1 \forall x.$$

Quantum fractional packing number and feedback-assisted or QNSC-assisted zero-error capacity

- For **classical channel** \mathcal{N} [Shannon-1956, Cubitt-Leung-Matthews-Winter-2011],

$$C_{0F}(\mathcal{N}) = C_{0,NS}(\mathcal{N}) = \log \alpha^*(\mathcal{N}),$$

where $\alpha^*(\mathcal{N})$ is the fractional packing number:

$$\alpha^*(\mathcal{N}) = \max \sum_x v_x \text{ s.t. } \sum_x v_x [\mathcal{N}(y|x)] \leq 1 \forall y, 0 \leq v_x \leq 1 \forall x.$$

- For **classical-quantum channel** \mathcal{N} [Duan-Winter-2016],

$$C_{0,NS}(\mathcal{N}) = \log A(\mathcal{N}),$$

where $A(\mathcal{N})$ is the **quantum fractional packing number**:

$$A(\mathcal{N}) = \max \text{Tr } R_A \text{ s.t. } 0 \leq R_A, \text{Tr}_A P_{AB}(R_A \otimes \mathbb{1}_B) \leq \mathbb{1}_B,$$

Quantum fractional packing number and feedback-assisted or QNSC-assisted zero-error capacity

- For **classical channel** \mathcal{N} [Shannon-1956, Cubitt-Leung-Matthews-Winter-2011],

$$C_{0F}(\mathcal{N}) = C_{0,NS}(\mathcal{N}) = \log \alpha^*(\mathcal{N}),$$

where $\alpha^*(\mathcal{N})$ is the fractional packing number:

$$\alpha^*(\mathcal{N}) = \max \sum_x v_x \text{ s.t. } \sum_x v_x [\mathcal{N}(y|x)] \leq 1 \forall y, 0 \leq v_x \leq 1 \forall x.$$

- For **classical-quantum channel** \mathcal{N} [Duan-Winter-2016],

$$C_{0,NS}(\mathcal{N}) = \log A(\mathcal{N}),$$

where $A(\mathcal{N})$ is the **quantum fractional packing number**:

$$A(\mathcal{N}) = \max \text{Tr } R_A \text{ s.t. } 0 \leq R_A, \text{Tr}_A P_{AB}(R_A \otimes \mathbb{1}_B) \leq \mathbb{1}_B,$$

- **However**, for **quantum channel** \mathcal{N}_α ($0 < \alpha \leq \pi/4$), we have that $A(\mathcal{N}_\alpha) = 2 + \cos^2 \alpha + \cos^{-2} \alpha$ and

$$C_{0F}(\mathcal{N}_\alpha) < \log A(\mathcal{N}_\alpha), C_{0,NS}(\mathcal{N}_\alpha) < \log A(\mathcal{N}_\alpha).$$

Conclusion

Summary

- For the channel $\mathcal{N}_\alpha = E_\alpha \rho E_\alpha^\dagger + D_\alpha \rho D_\alpha^\dagger$ ($0 < \alpha \leq \pi/4$) with

$$E_\alpha = \sin \alpha |0\rangle\langle 1| + |1\rangle\langle 2|, D_\alpha = \cos \alpha |2\rangle\langle 1| + |1\rangle\langle 0|,$$

we have that

- $C_{0,NS}(\mathcal{N}_\alpha) = C_E(\mathcal{N}_\alpha) = S_{0,NS}(\mathcal{N}_\alpha) = 2$ (A kind of reversibility)
- $\tilde{\vartheta}(\mathcal{N}_\alpha) = 2 + \cos^2 \alpha + \cos^{-2} \alpha > 4$
- $A(K_\alpha) = 2 + \cos^2 \alpha + \cos^{-2} \alpha > 4$

Summary

- For the channel $\mathcal{N}_\alpha = E_\alpha \rho E_\alpha^\dagger + D_\alpha \rho D_\alpha^\dagger$ ($0 < \alpha \leq \pi/4$) with

$$E_\alpha = \sin \alpha |0\rangle\langle 1| + |1\rangle\langle 2|, D_\alpha = \cos \alpha |2\rangle\langle 1| + |1\rangle\langle 0|,$$

we have that

- $C_{0,NS}(\mathcal{N}_\alpha) = C_E(\mathcal{N}_\alpha) = S_{0,NS}(\mathcal{N}_\alpha) = 2$ (A kind of reversibility)
- $\tilde{\vartheta}(\mathcal{N}_\alpha) = 2 + \cos^2 \alpha + \cos^{-2} \alpha > 4$
- $A(K_\alpha) = 2 + \cos^2 \alpha + \cos^{-2} \alpha > 4$
- A **separation** between quantum Lovász number and entanglement-assisted zero-error classical capacity:

$$\log \tilde{\vartheta}(\mathcal{N}_\alpha) > C_{0,NS}(\mathcal{N}_\alpha) \geq C_{0E}(\mathcal{N}_\alpha).$$

- A **gap** between quantum fractional packing number and feedback-assisted or QNSC-assisted zero-error capacity:

$$C_{0F}(\mathcal{N}_\alpha) < \log A(\mathcal{N}_\alpha), C_{0,NS}(\mathcal{N}_\alpha) < \log A(\mathcal{N}_\alpha).$$

Open Questions:

- It remains **unknown** whether Lovász number coincides with C_{0E} for every **classical channel**, i.e.,

$$C_{0E}(G) = \log \vartheta(G)?$$

- Other communication properties of \mathcal{N}_α ?
- Characterize the channels which have **reversibility** in the sense that

$$C_{0,NS}(\mathcal{N}) = S_{0,NS}(\mathcal{N}).$$

- **Additivity** for $C_{0E}(\mathcal{N})$ or $C_{0,NS}(\mathcal{N})$?

Thank you!
Questions?