Reliable and robust entanglement witness

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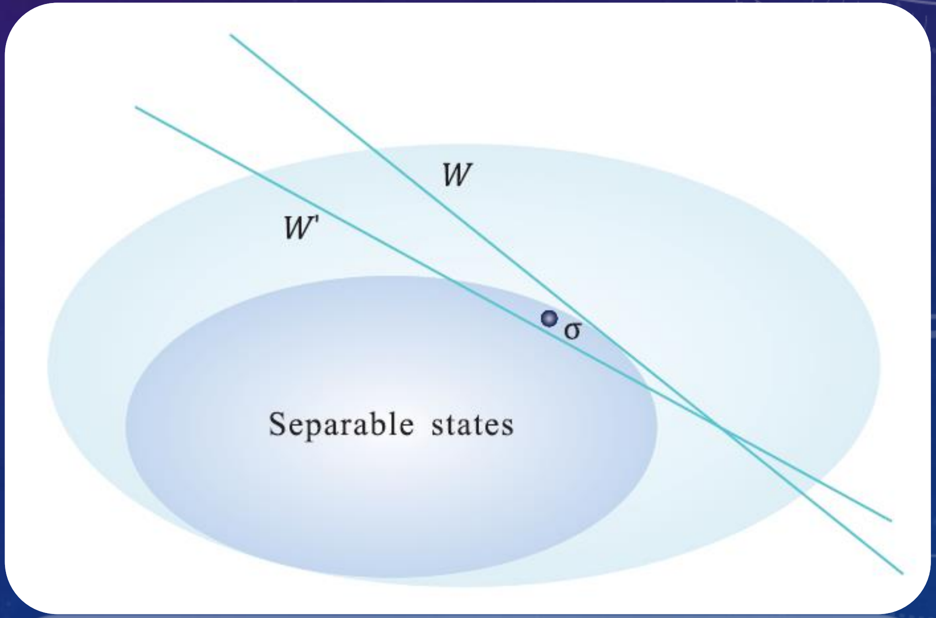
ENTANGLEMENT WITNESS AND THE RELIABILITY PROBLEM

- Separable state

$$\sigma = \sum_i p_i \rho_A^i \otimes \rho_B^i$$

- For any entangled state ρ, there exists a Hermitian operator W such that $\mathrm{tr}[W\rho] < 0$, while $\mathrm{tr}[W\sigma] \geq 0$ for all separable state σ .



TIME-SHIFT ATTACK

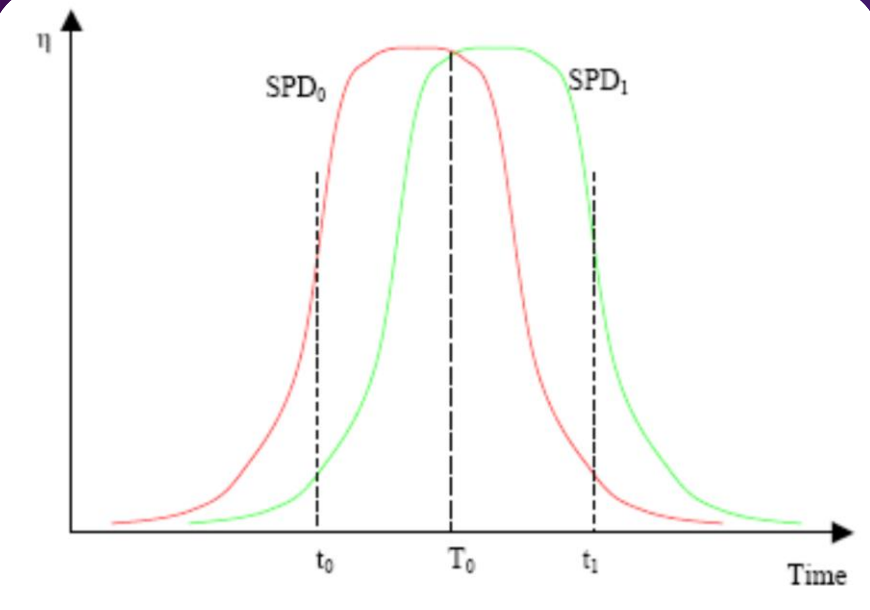
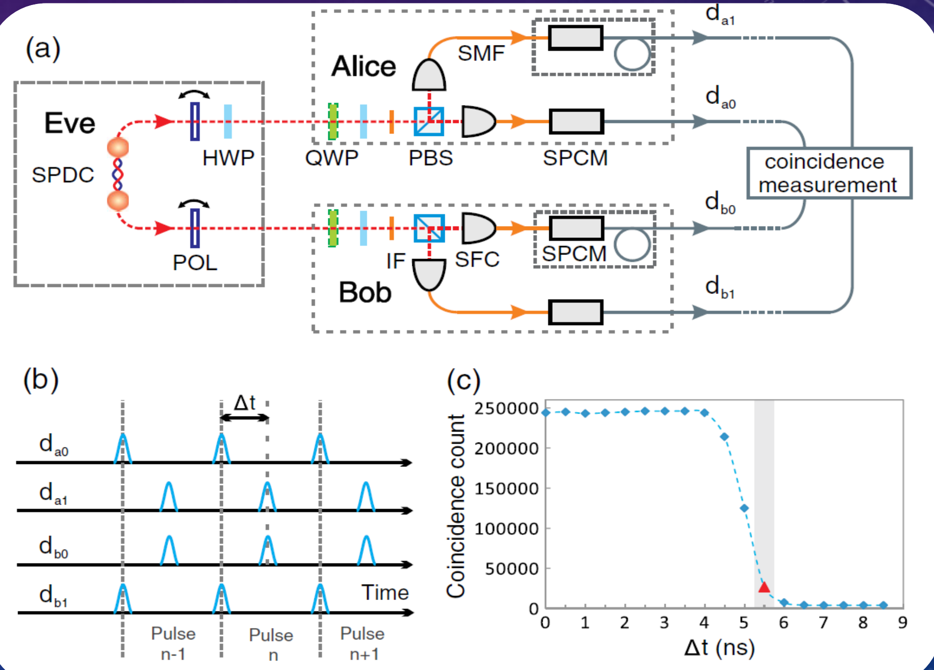


Fig. 1. The time-dependence efficiencies of SPDs.



B. Qi, et al., Quant. Info. Comp. 7, 73 (2007).

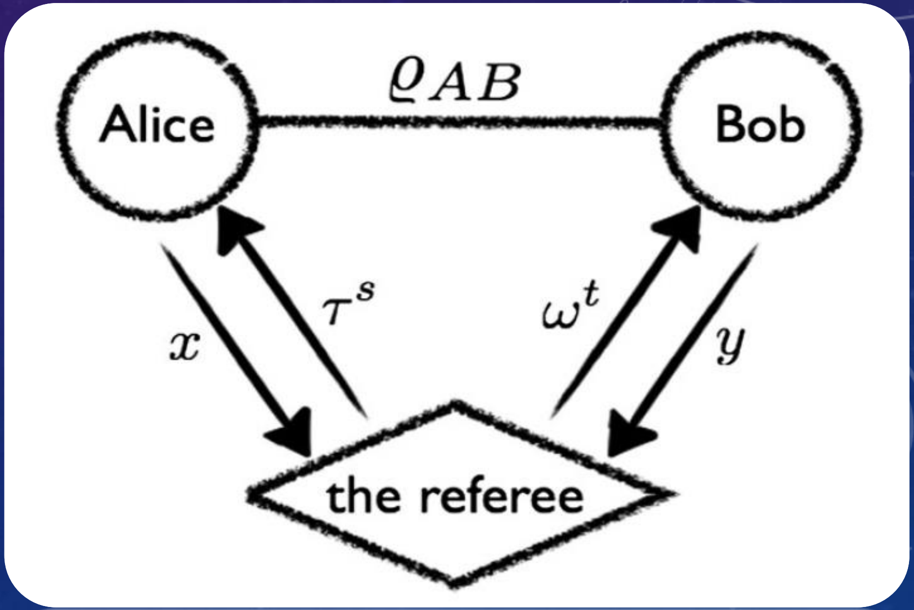
P. Xu, X. Yuan, et al., PRL 112, 140506 (2014)

NONLOCAL GAMES

- Quantum inputs and classical outputs with probability $P(a,b|\tau_s,\omega_t)$
- There must exist a linear combination of probabilities

$$I(p) = \sum \beta_{s,t,a,b}\, P(a,b|\tau_s,\omega_t)$$

such that $I(P_\rho) < 0$ and $I(P_\rho) \geq 0$ for all separable states.



F. Buscemi, PRL 108, 200401 (2012)

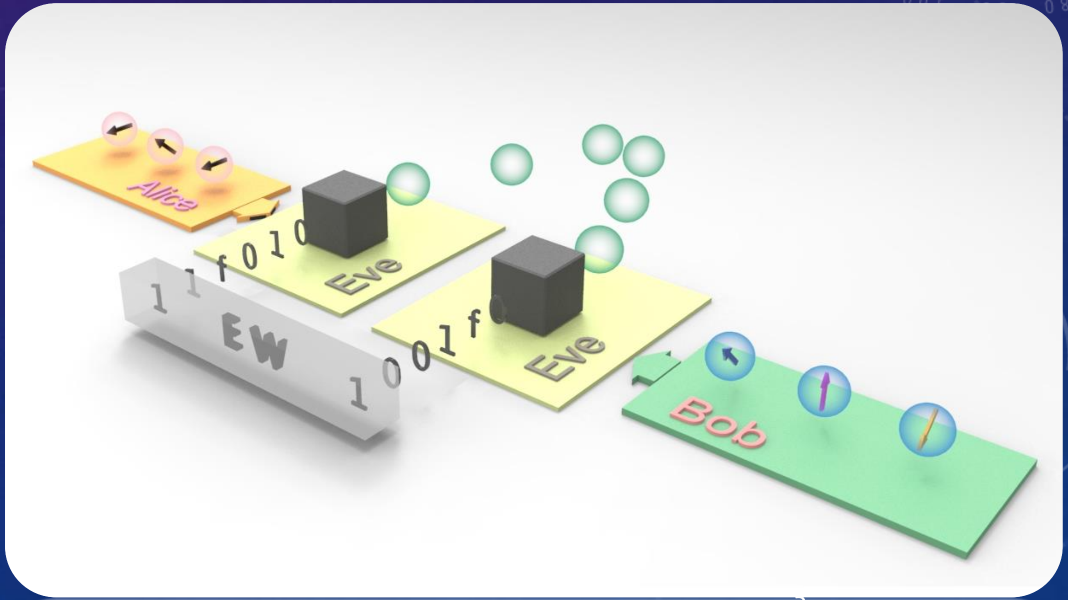
RELIABLE EW: MDIEW

- Suppose W to be an EW for ρ_{AB}. Given two measurement bases $\{\tau_s^T \in H_A\}$ and $\{\omega_t^T \in H_B\}$, W can be decomposed as

$$W = \sum \beta_{s,t} \tau_s^T \otimes \omega_t^T$$

- Then the MDIEW can be given by

$$I(p) = \sum \beta_{s,t} P(1,1|\tau_s, \omega_t)$$



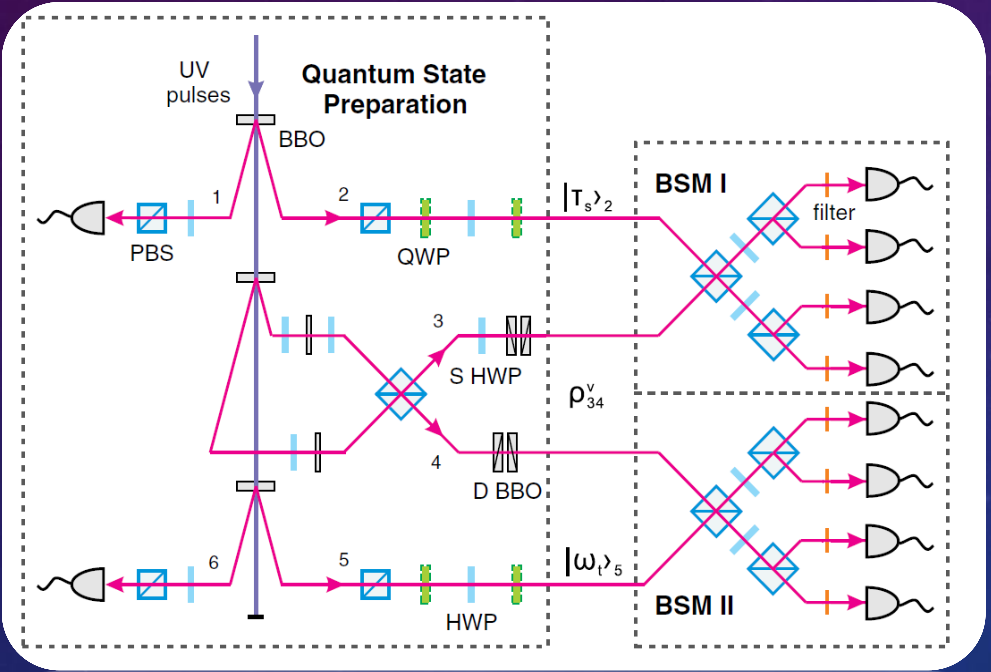
C. Branciard et al., PRL 110, 060405 (2013)

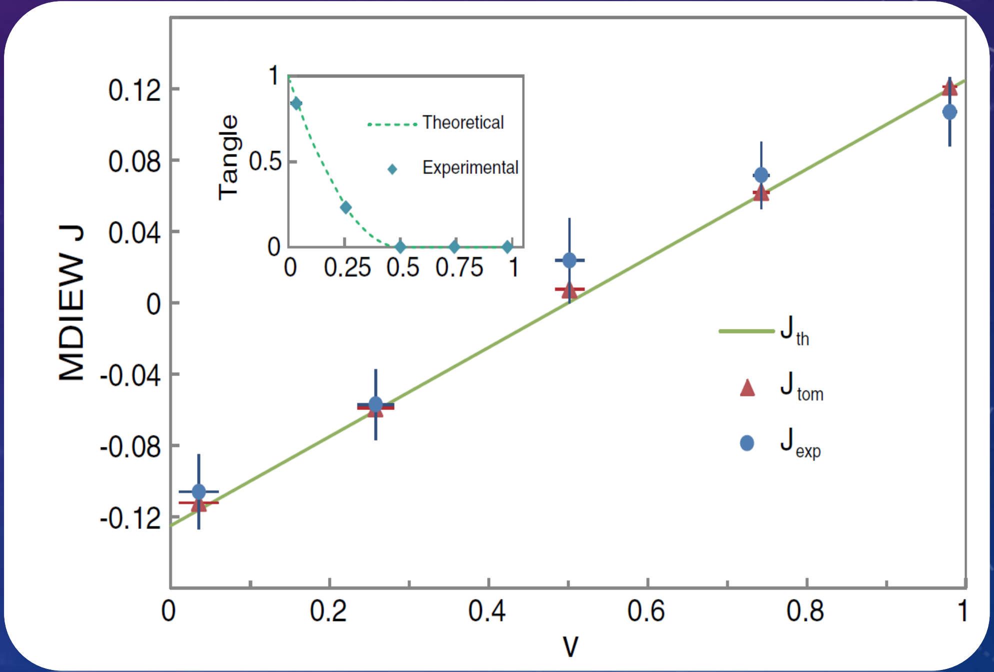
MDIEW EXAMPLE

- Consider a 2-qubit Werner state $\rho_{AB}^{v} = v|\Psi^-\rangle\langle\Psi^-| + (1-v)I/4$ with $v \in [0,1]$ and $|\Psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$
- ρ_{AB} is entangled if and only if $v > 1/3$, it can be detected by $W = \frac{1}{2}I - |\Psi^-\rangle\langle\Psi^-|$, where $tr[W\rho] = \frac{1-3v}{4} < 0$ for $v > 1/3$
- Define $\sigma_0 = I$ and $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ as the Pauli matrix. For $s, t = 0, \dots, 3$, $\tau_s = \sigma_s \frac{I + \vec{n}\cdot\vec{\sigma}}{2} \sigma_s$, $\omega_t = \sigma_t \frac{I + \vec{n}\cdot\vec{\sigma}}{2} \sigma_t$
- Then $\beta_{s,t} = \begin{cases} \frac{5}{8} & s = t \\ -\frac{1}{8} & s \neq t \end{cases}$, and the MDI-EW is

$$I(p) = \frac{5}{8} \sum_{s=t} P(1,1|\tau_s, \omega_t) - \frac{1}{8} \sum_{s \neq t} P(1,1|\tau_s, \omega_t)$$

MDIEW EXPERIMENT





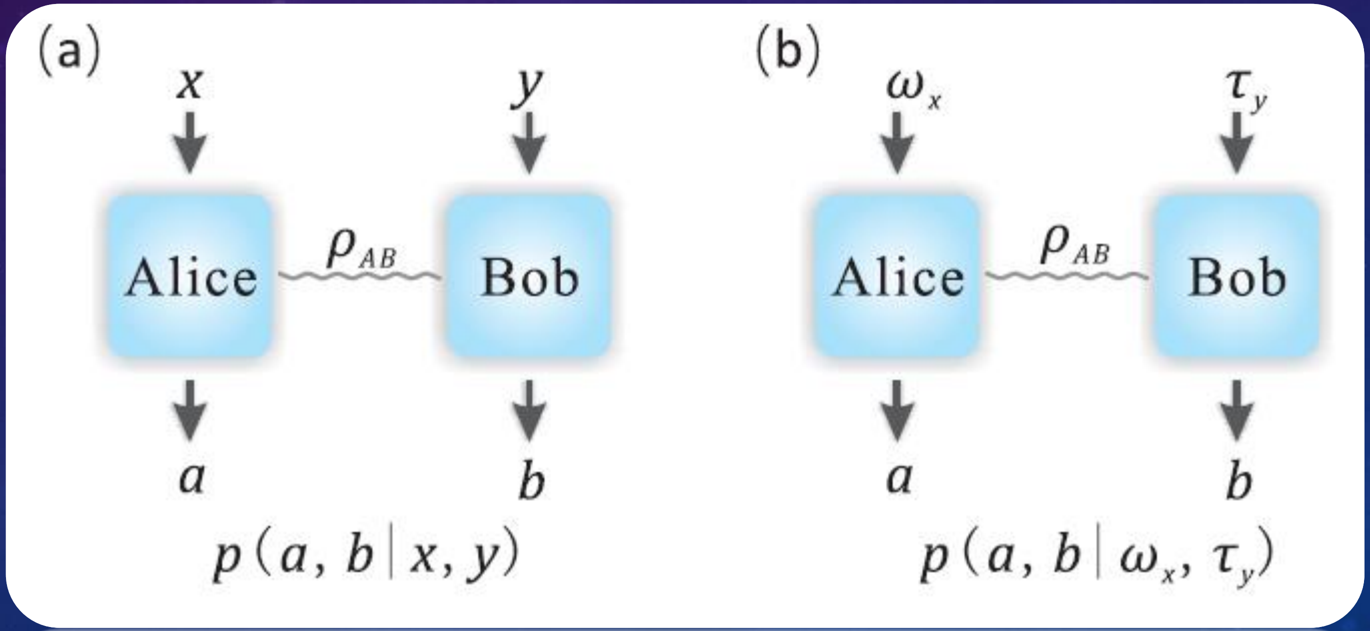
P. Xu, X. Yuan, et al., PRL 112, 140506 (2014)

THE ROBUSTNESS PROBLEM

- In MDIEW, we have $I(p) = \mathrm{tr}[W\rho]/d_A d_B$ with perfect measurement.
- The implemented witness, which may although be designed optimally in the first place, can become a bad one with imperfect measurement, which merely detects no entanglement.
- The observed experimental data may still have enough information for detecting entanglement.
- The key problem is to find the best estimation of entanglement given the observed experimental data.

COMPARISON WITH BELL INEQUALITY

- Bell inequality: $I(p) = \sum \beta_{s,t,a,b} \, P(a, b|x, y)$, linear programming.
- MDIEW: $I(p) = \sum \beta_{s,t,a,b} \, P(a, b|\tau_s, \omega_t)$, NP-hard.

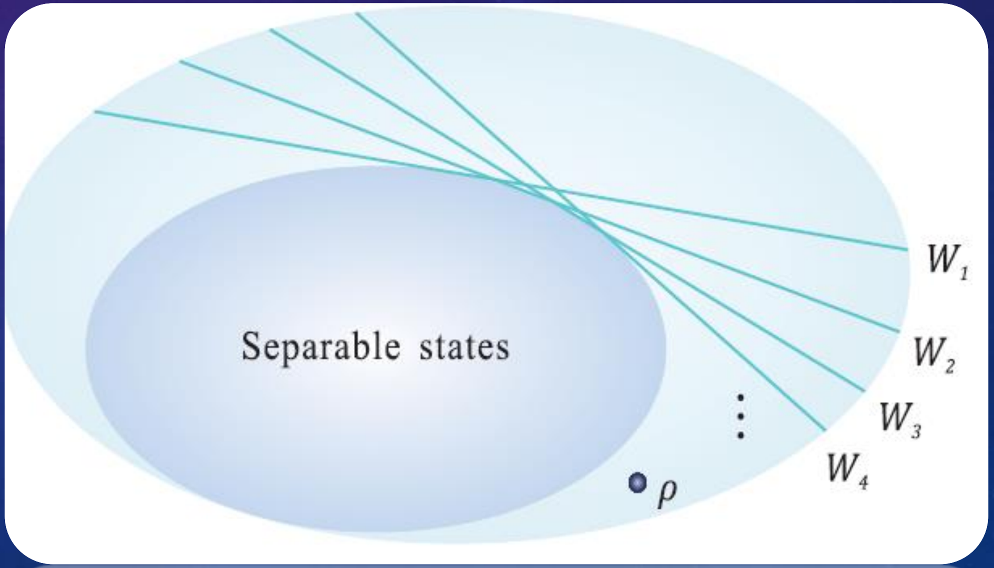


ROBUST MDIEW

- Problem to solve: find the optimal coefficients $\beta_{s,t}$ for the observed probability distribution $P(1,1|\tau_s, \omega_t)$.
- Minimize : $I(p) = \sum \beta_{s,t}\, P(1,1|\tau_s, \omega_t)$
- Constraints:
    - Tr[W]=1,
    - $W = \sum \beta_{s,t} \tau_s^T \otimes \omega_t^T$ is an EW, that is,

  $\langle\psi|_A \langle\phi|_B W |\psi\rangle_A |\phi\rangle_B \geq 0,$

  for all pure states $|\psi\rangle_A |\phi\rangle_B$.



ϵ-LEVEL MDIEW

- A Hermitian operator W_ϵ is defined as an ϵ–level entanglement witness, when

$$\mathrm{Prob}\{\mathrm{Tr}[\sigma W_\epsilon] < 0 | \sigma \in S\} \leq \epsilon,$$

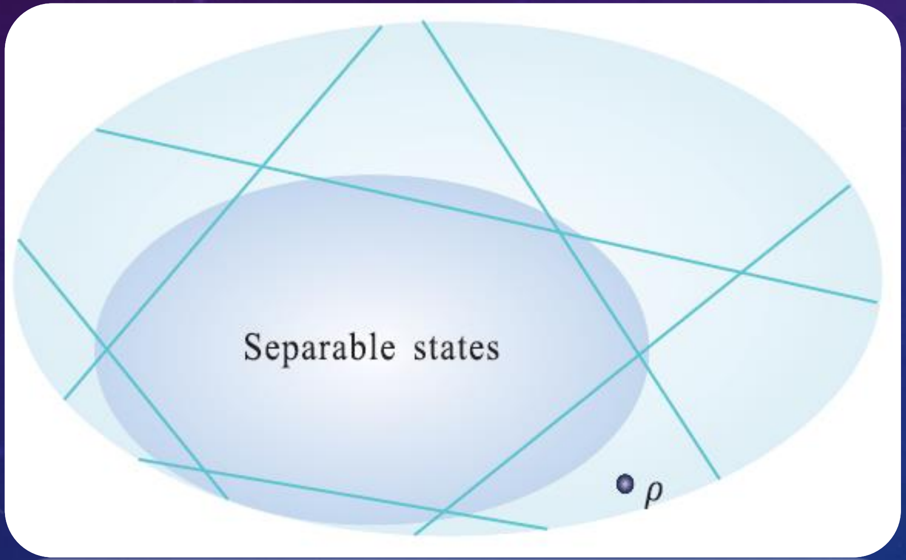
where S is the set of separable states.

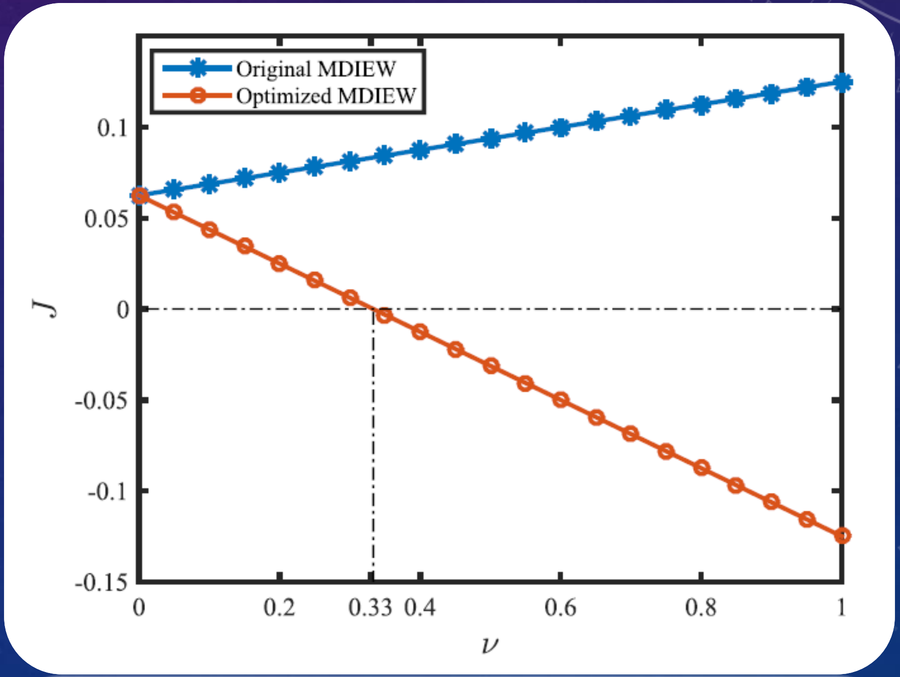
- Constraints: randomly generate N separable states $|\psi\rangle_A^i |\phi\rangle_B^i$ and require the average $\langle W_\epsilon \rangle \geq 0$ only for these states.
- To do so, we need to set $N \geq \frac{r}{\epsilon\beta} - 1$, where r is the number of optimization variables, β is the success probability.

F. G. S. L. Brandao and R. O. Vianna, PRL 93, 220503 (2004).

INTUITION AND EXAMPLE

- Two-qubit Werner state $\rho_{AB}^{v} = v|\Psi^-\rangle\langle\Psi^-| + (1-v)I/4$ with $v \in [0,1]$ and $|\Psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$
- Entanglement witness $W = \frac{1}{2}I - |\Psi^-\rangle\langle\Psi^-|$





X. Yuan et al. PRA 93, 042317 (2016)

SUMMARY

- The optimization is only a post-processing of experiment data, thus can be easily applied to existing experiment.
- The optimization program finds the ϵ-level optimal EW W_ϵ, which as its name indicates, has a probability less than or equal to ϵ to detect a separable state to be entangled. To decrease ϵ, one can increase N or calculate α such that $\mathrm{W} = W_\epsilon + \alpha I$ is an EW.
- A different approach to the robustness problem is given in [E. Verbaniset al., Phys. Rev. Lett. 116, 190501, 2016].

THANK YOU!

- Xiao Yuan, 2016