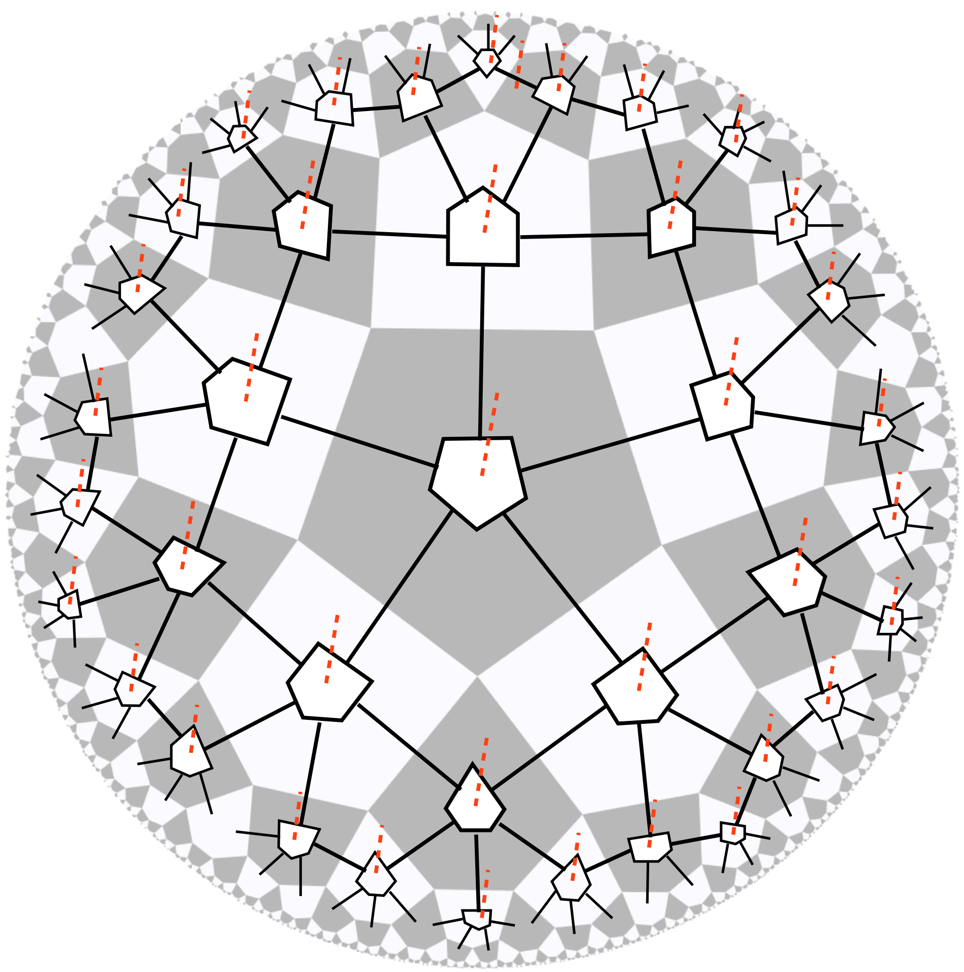
Quantum error-correction in black holes



Beni Yoshida (Perimeter Institute)

@ Taipei (Sep 2016)

[1] Holographic quantum error-correcting codes,

[2] Chaos in quantum channel,

[3] Complexity by design, arXiv:1609:xxxxx

String

Daniel Harlow (Harvard→MIT)

QI

Fernando Pastawski (Caltech→Berlin)

QI

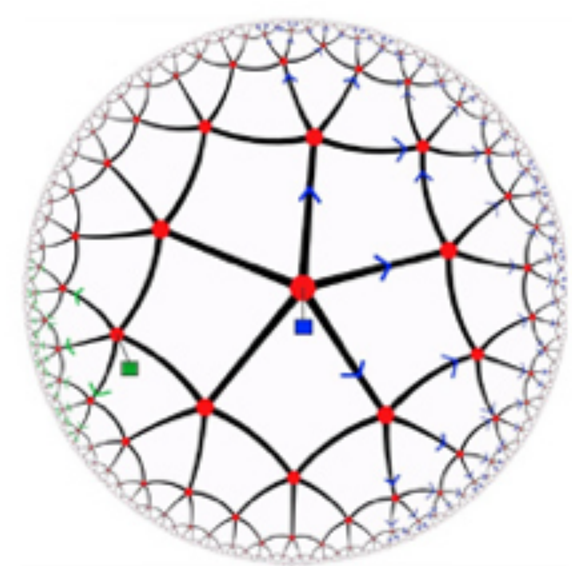
John Preskill (Caltech)

CMT

Xiao-liang Qi (Stanford)

String

Daniel Roberts (MIT→IAS)

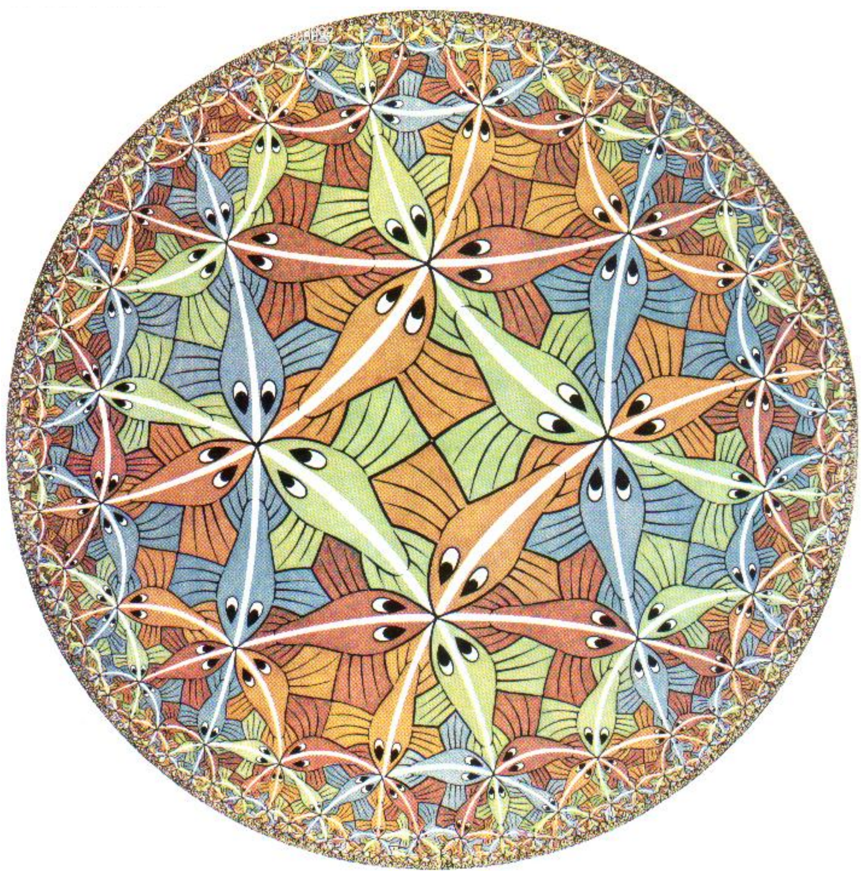


Simons collaborations

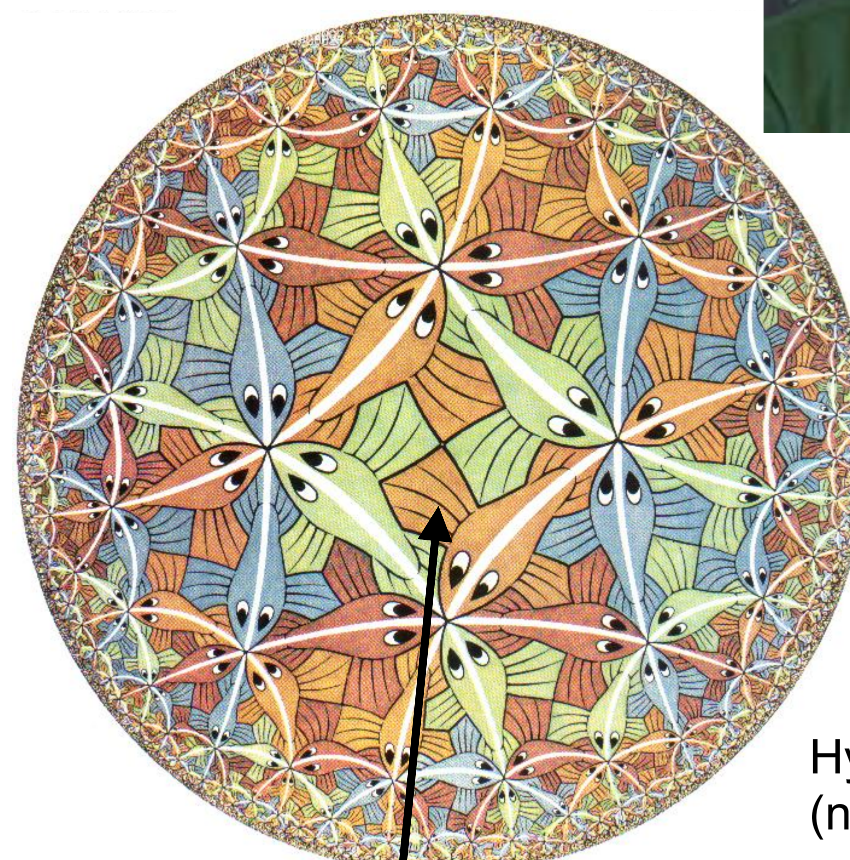
A black hole is a quantum error-correcting code

Anti-de Sitter/Conformal field theory (AdS/CFT) correspondence

- [Conjecture] Equivalence of string (gravity) theory in bulk with CFT on boundary (Maldacena)



Boundary D-dimensional conformal field theory (without gravity)

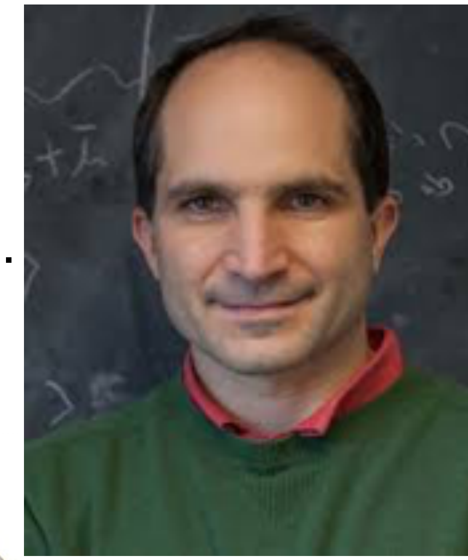


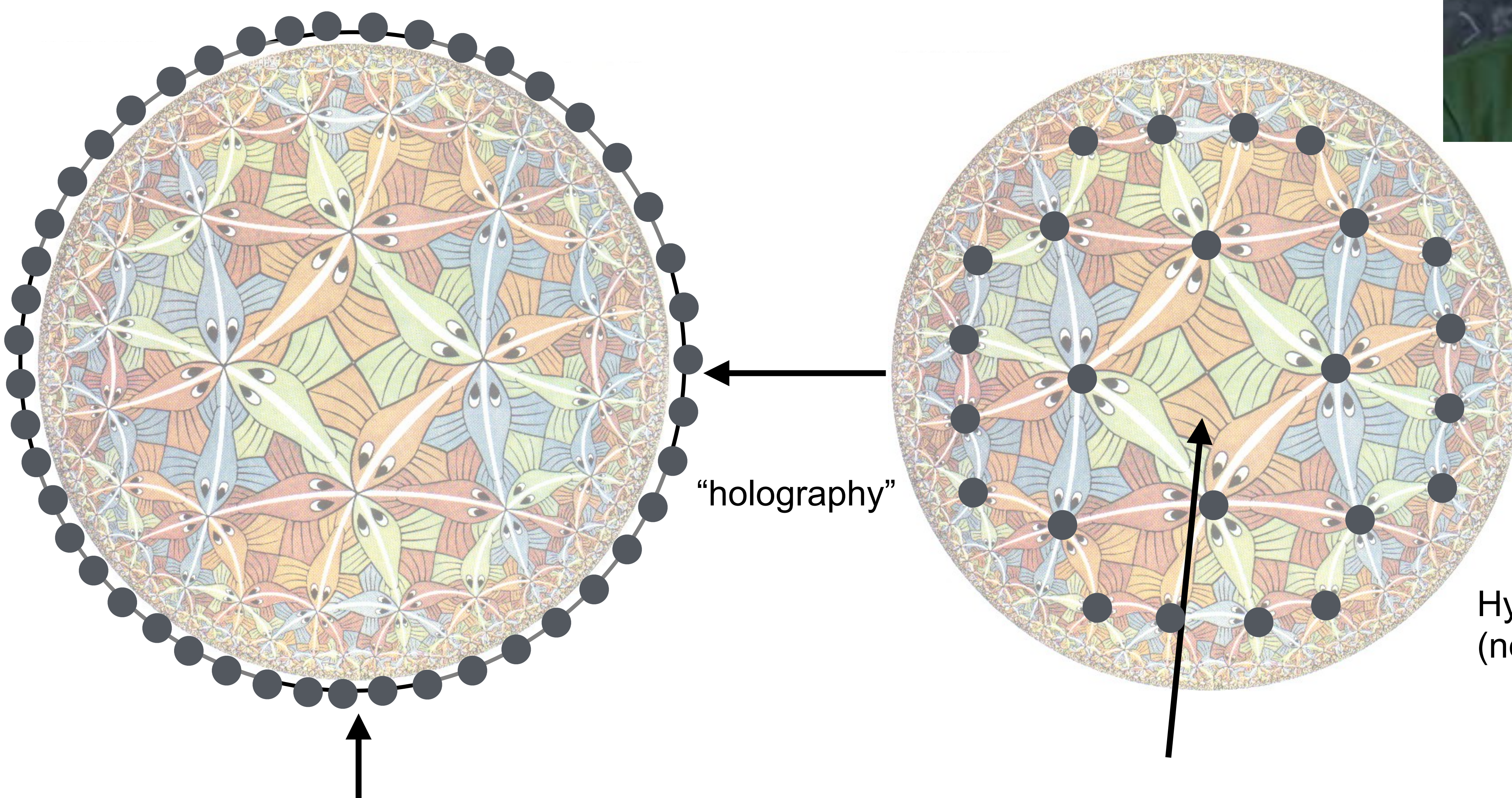
Hyperbolic space (negatively curved)

Bulk (D+1)-dimensional theory with gravity on AdS space

Anti-de Sitter/Conformal field theory (AdS/CFT) correspondence

- [Conjecture] Equivalence of string (gravity) theory in bulk with CFT on boundary (Maldacena)
- [Holography] Bulk degrees of freedom are encoded in boundary, like a hologram.



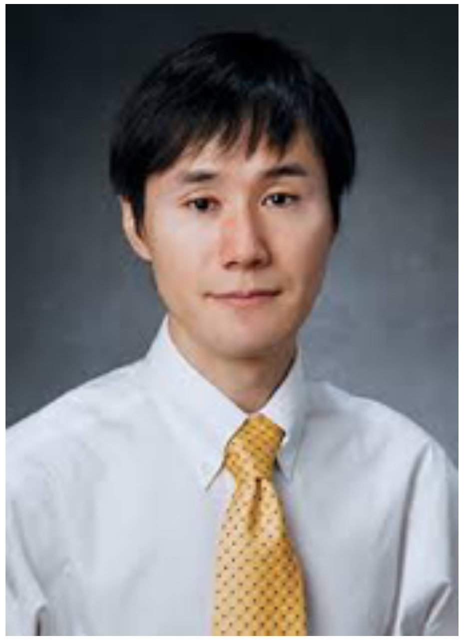


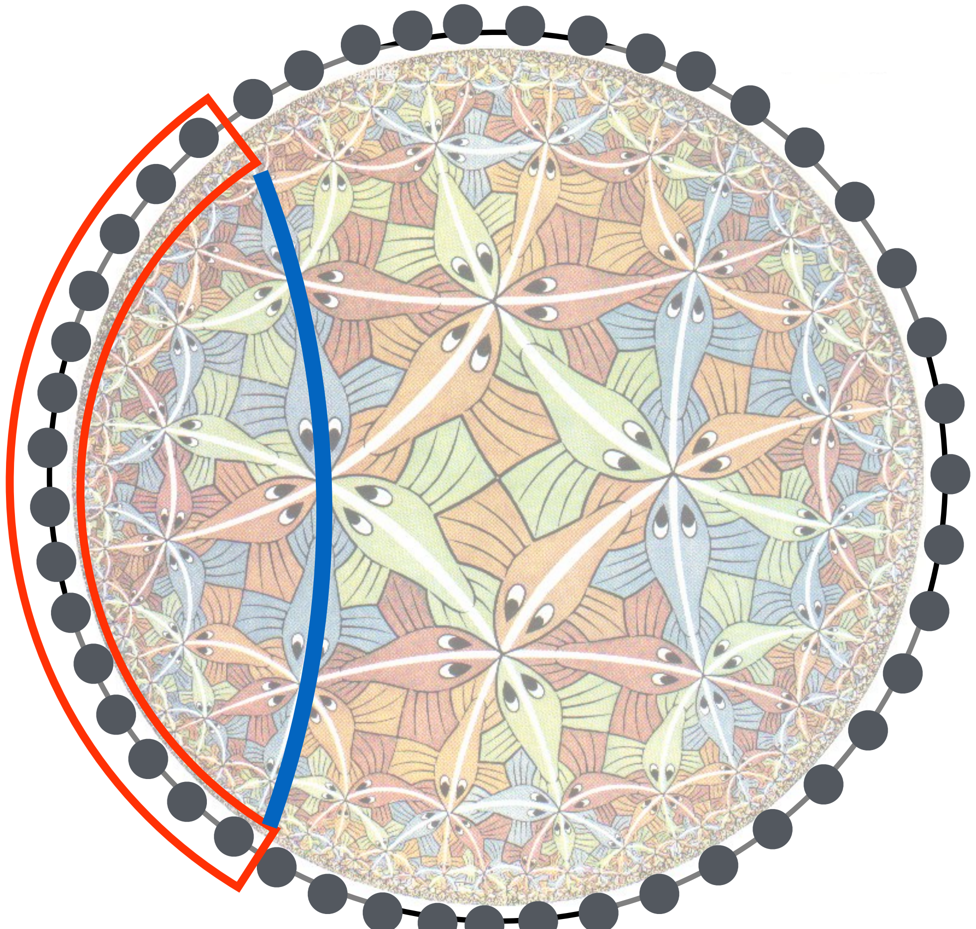
Quantum entanglement in AdS/CFT

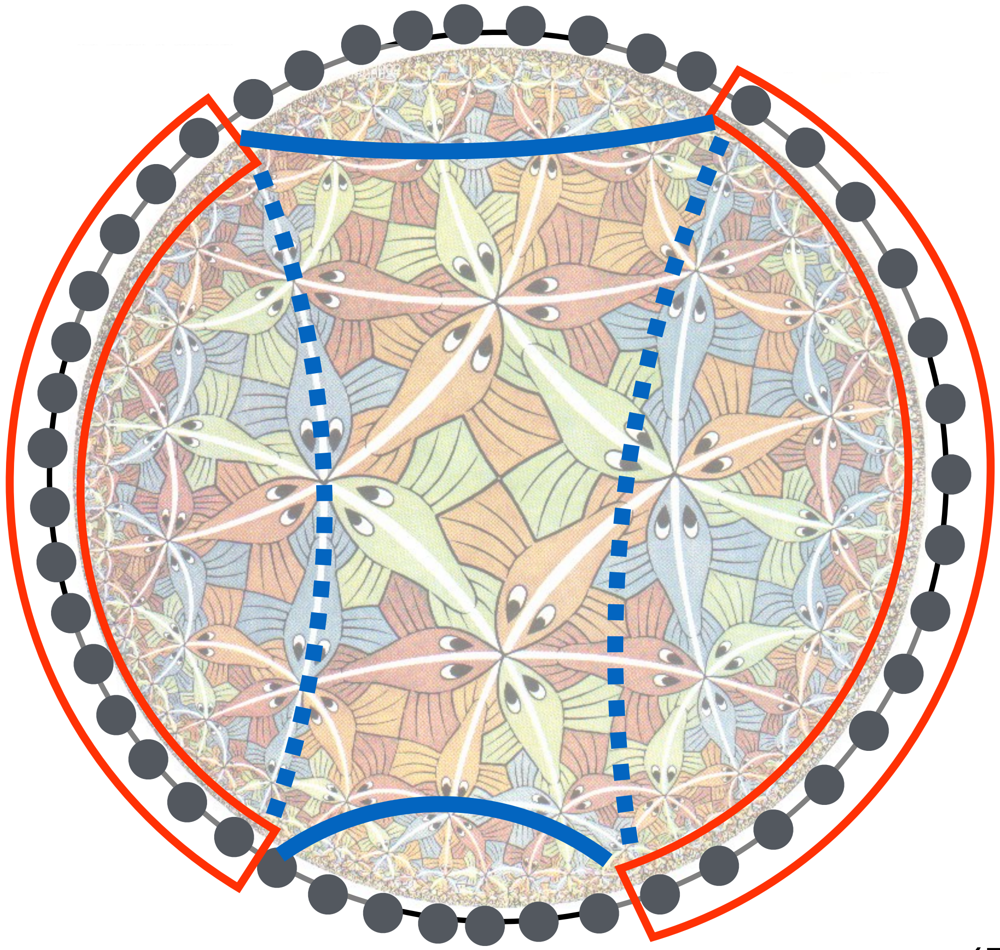
- [Ryu-Takayanagi formula]

QM → $$S(A) = \frac{1}{4G_N} \min_{\gamma_A}(\text{area}(\gamma_A))$$ ← GR

Minimize over spatial bulk
surfaces homologous to A





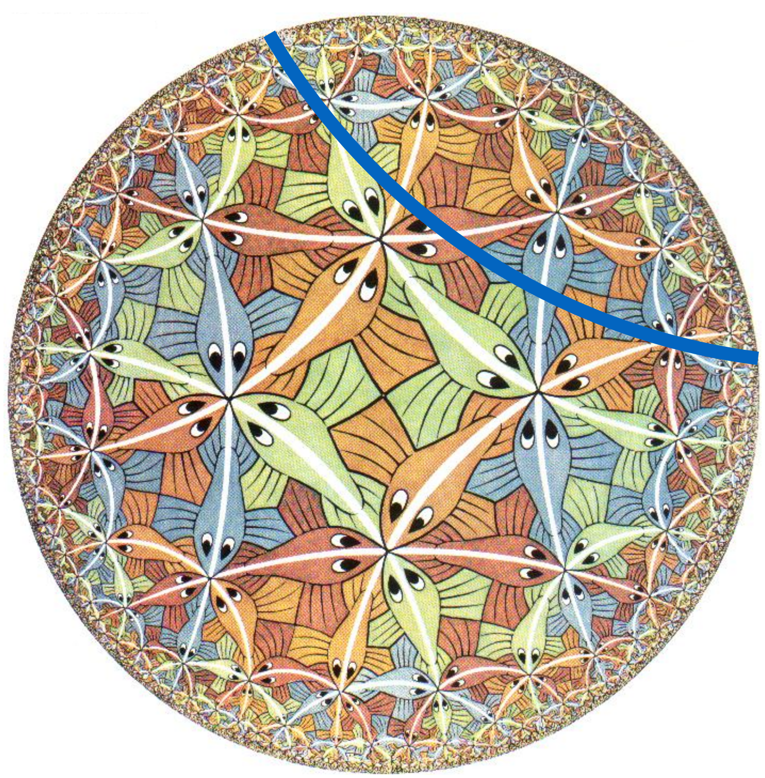


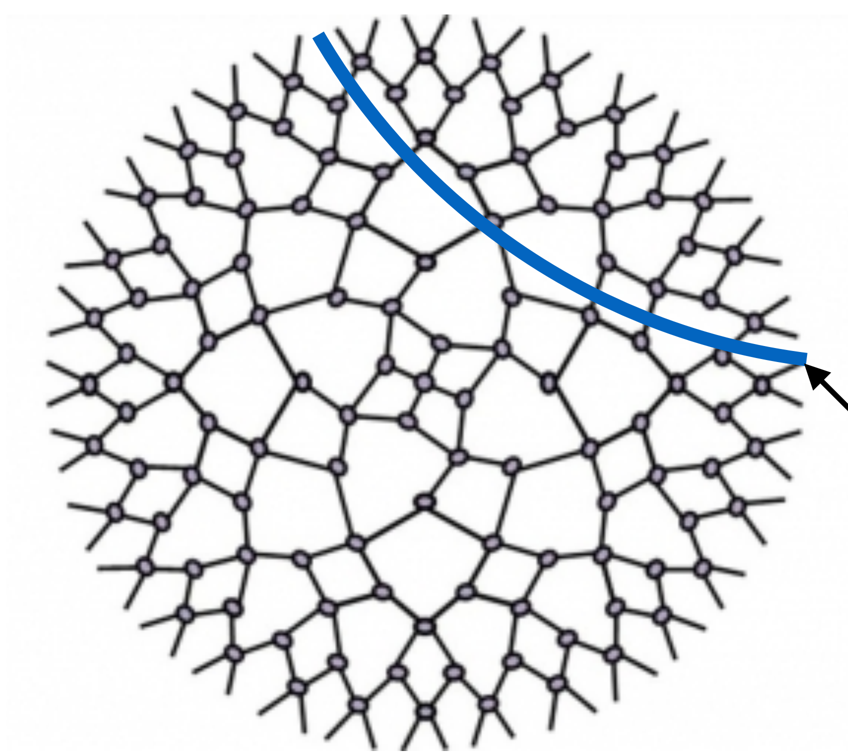
(For "large N")

Space-time as a tensor network

- [Swingle's conjecture]

AdS/CFT correspondence can be expressed as a MERA ?





minimal surface

$$S(A) \leq \frac{1}{4G_N} \min_{\gamma_A}(\text{area}(\gamma_A))$$

saturation ?

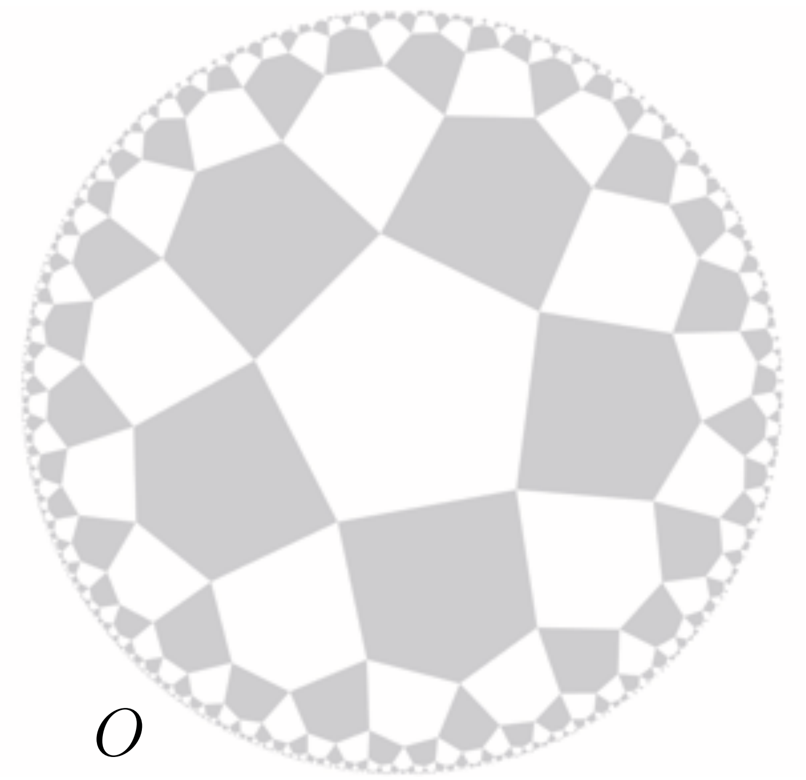
MERA = Multi-scale Entanglement Renormalization Ansatz (Vidal)

Concrete and simple toy tensor network model

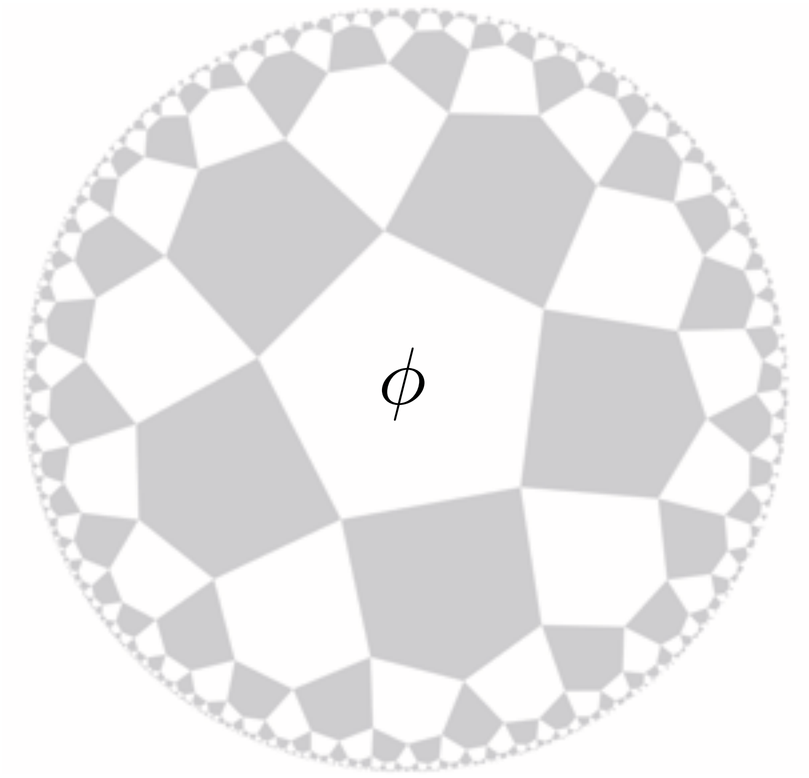
(joint with Harlow, Pastawski and Preskill)

Dictionary : Correspondence of operators

boundary system



bulk system



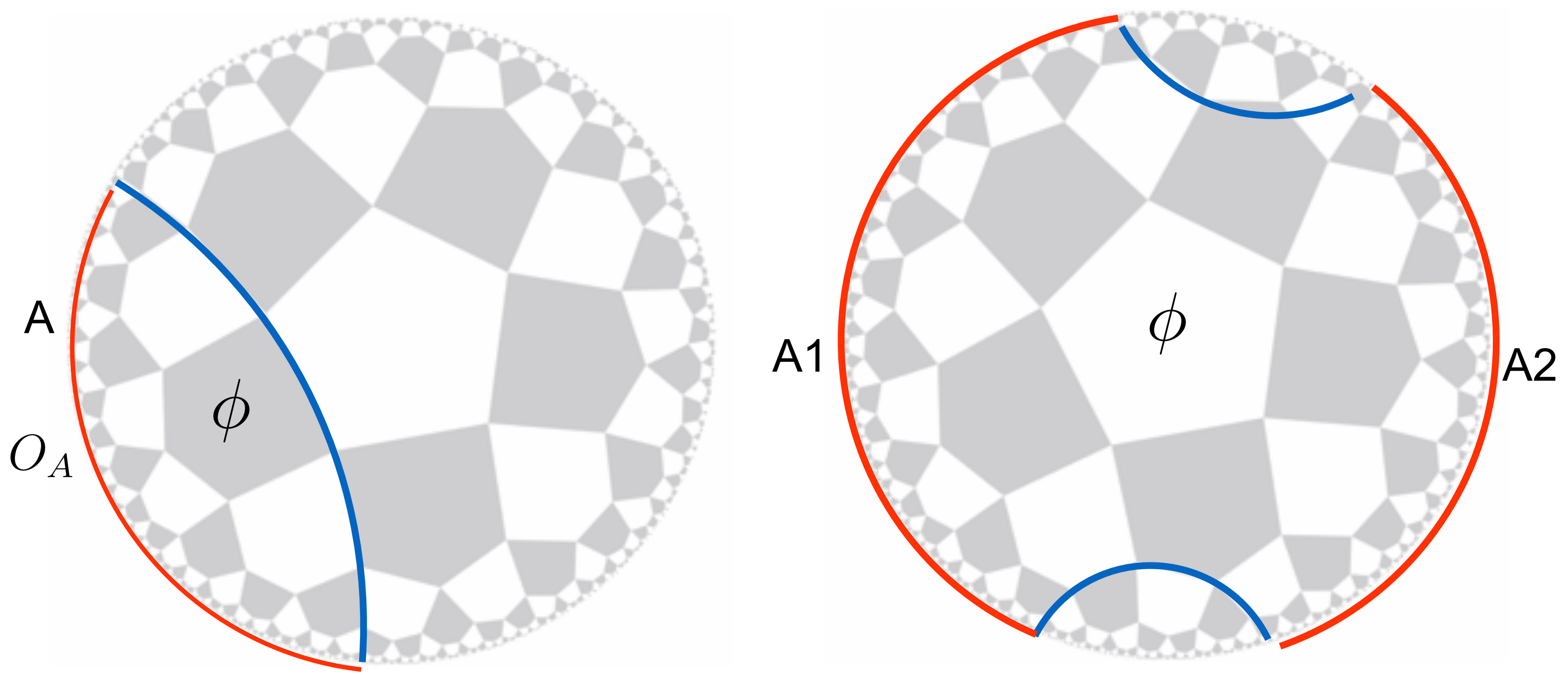
$O_1 \longleftarrow \phi_1$

$O_2 \longleftarrow \phi_2$

Bulk operator vs boundary operator

- [Entanglement wedge reconstruction]

A bulk operator ϕ can be represented by some integral of local boundary operators supported on A if ϕ is contained inside the entanglement wedge of A.



$\phi \sim O_A$ $\qquad$ $\phi \sim O_{A_1 A_2}$

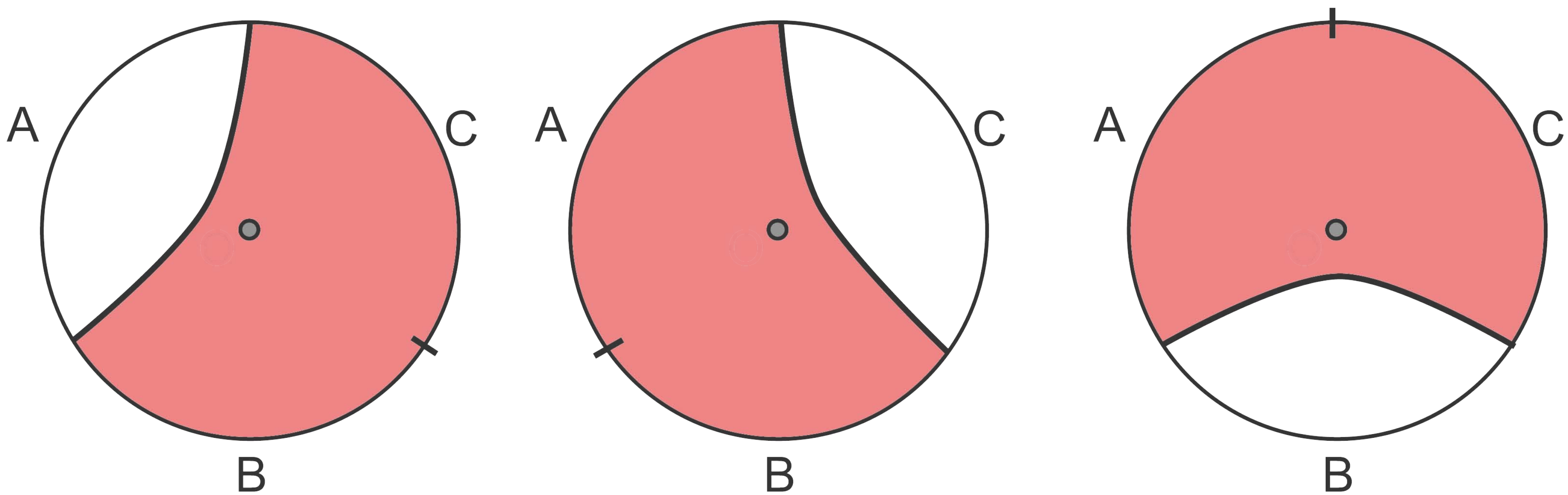
Remarks

- Entanglement wedge may go beyond black hole horizons (i.e. no firewall).
- “Proven” by using a generalized RT formula (Jefferis et al, Dong et al, Bao et al)
- No explicit recipe is known for more than one intervals

Bulk locality puzzle

- The reconstruction recipe leads to a paradox

All the bulk operators must correspond to identity operators on the boundary ?

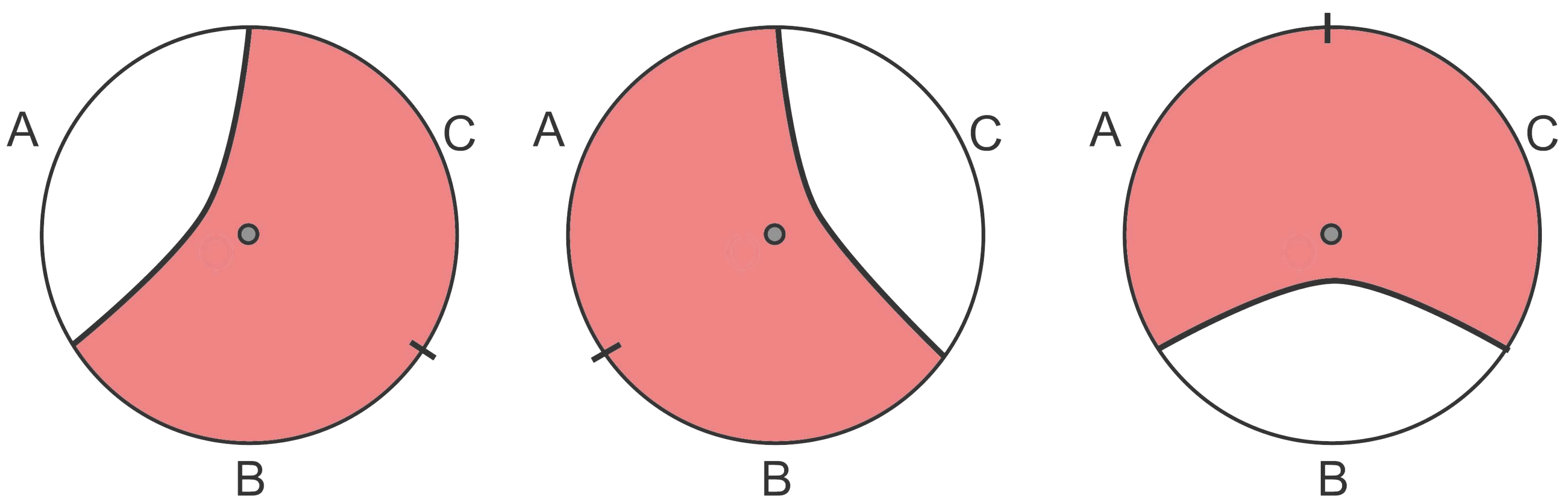


If so, the AdS/CFT seems very boring …

Quantum error-correction in AdS/CFT ?

- The AdS/CFT correspondence can be viewed as a quantum error-correcting code.

[Almheiri-Dong-Harlow].



O_{BC} O_{AB} O_{CA}

They are different operators, but act in the same manner in a low energy subspace.

cf. Quantum secret-sharing code

Let’s construct a toy model

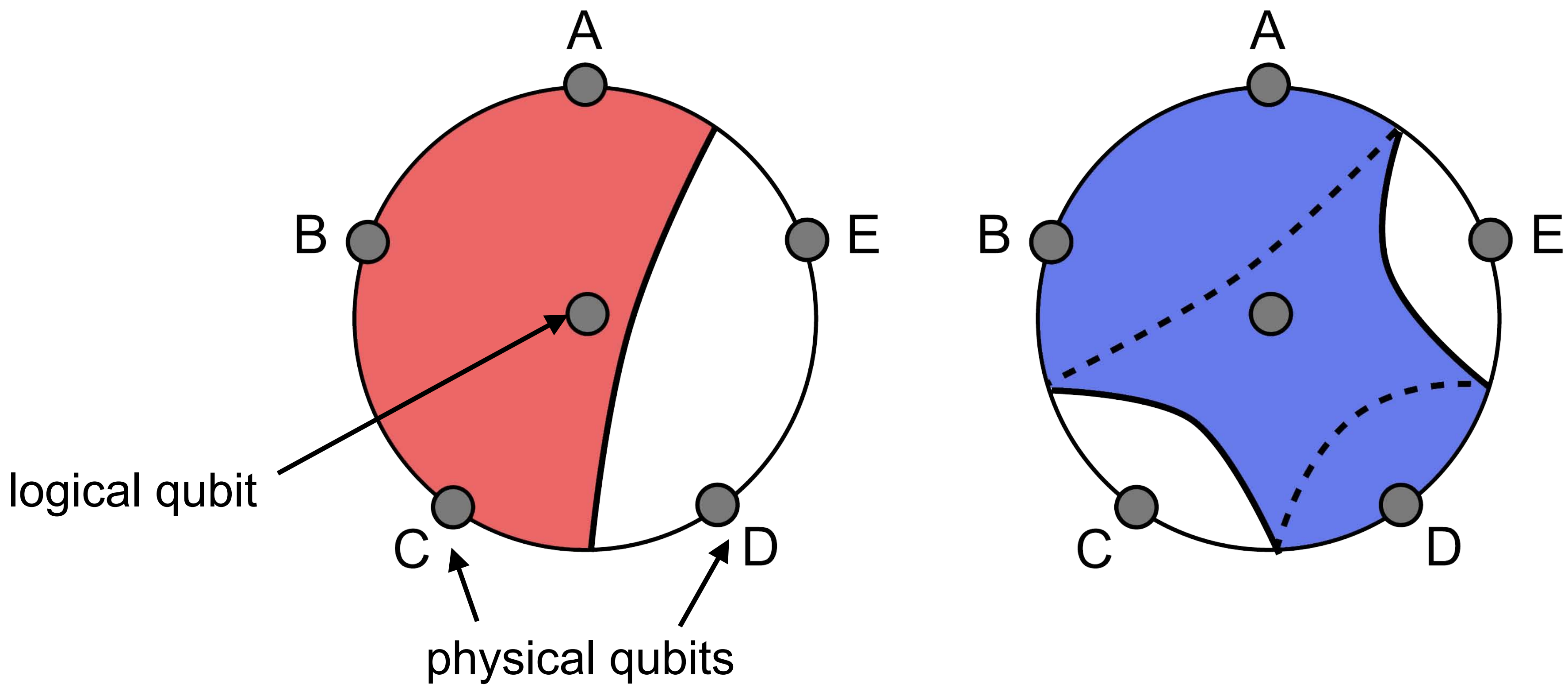
A simple toy model

1 bulk qubit

5 boundary qubits

in total, just 6 qubits

A bulk operator must have representations on any region with three qubits.



Entanglement wedge reconstruction !

Five-qubit code

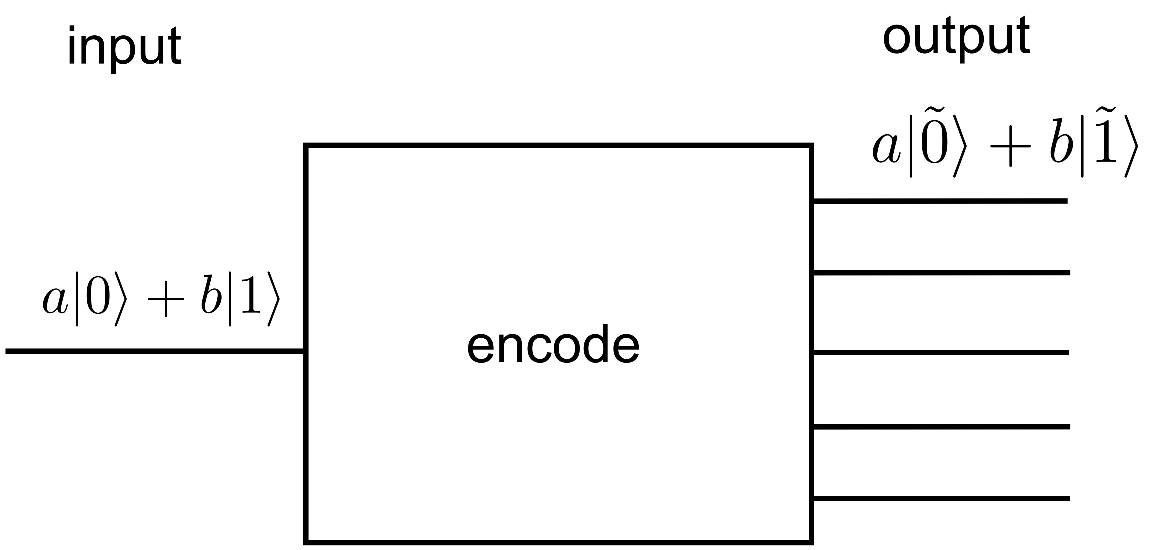
- Encode a single logical qubit into a system of five qubits.

$$S_1 = X \otimes Z \otimes Z \otimes X \otimes I$$

$$S_2 = I \otimes X \otimes Z \otimes Z \otimes X$$

$$S_3 = X \otimes I \otimes X \otimes Z \otimes Z$$

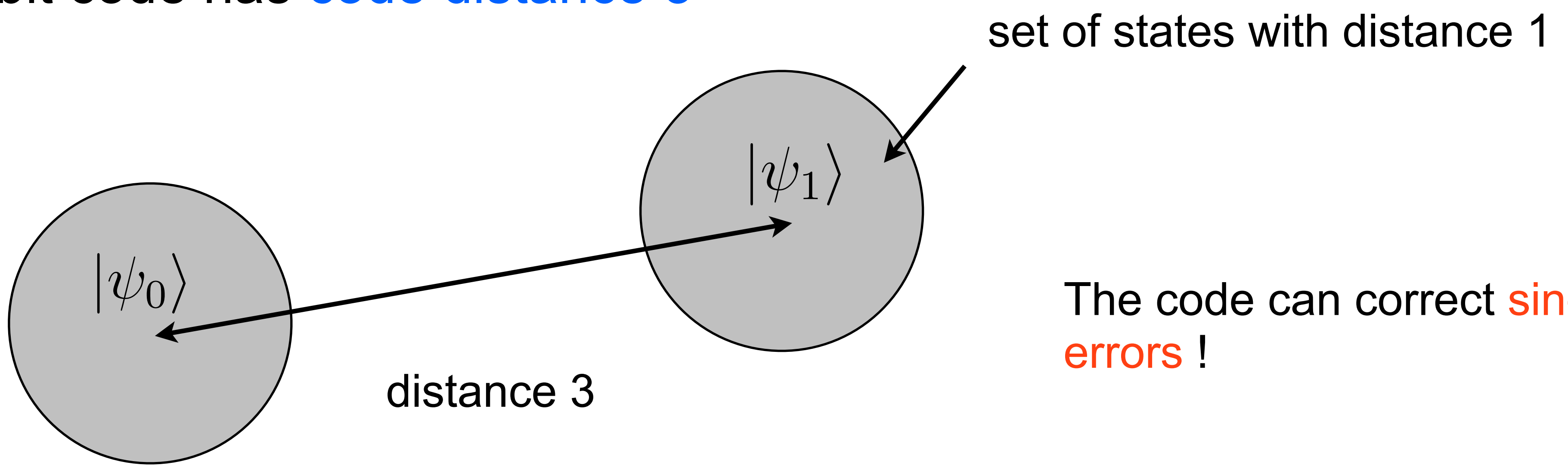
$$S_4 = Z \otimes X \otimes I \otimes X \otimes Z$$



- codeword space

$$\mathcal{C} = \{|\psi\rangle : S_j|\psi\rangle = |\psi\rangle \ \forall j\}$$

- Five-qubit code has code distance 3

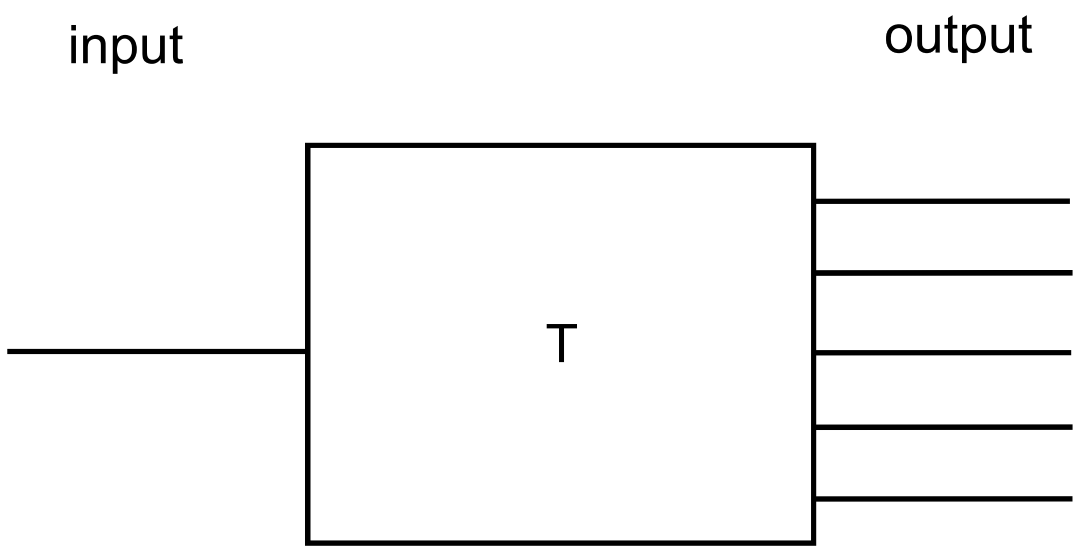


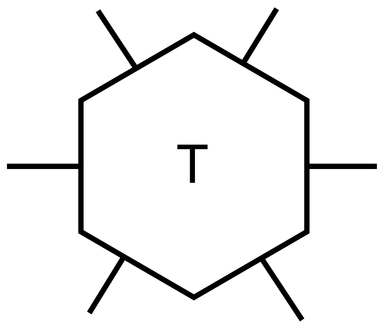
The code can correct single-qubit errors !

Five-qubit code is a quantum gravity

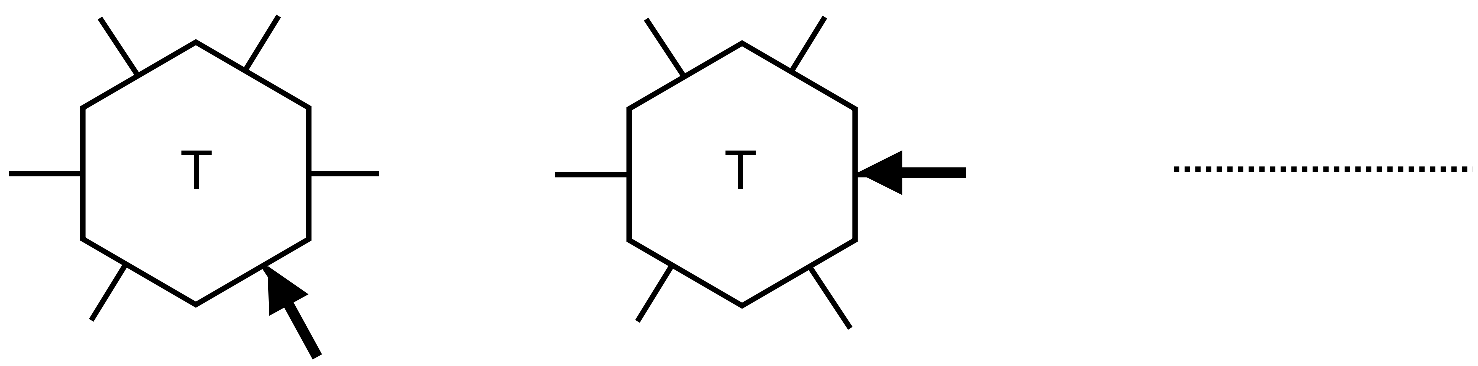
Perfectness of five qubit code

- Let's view the five-qubit code as a six-leg tensor.



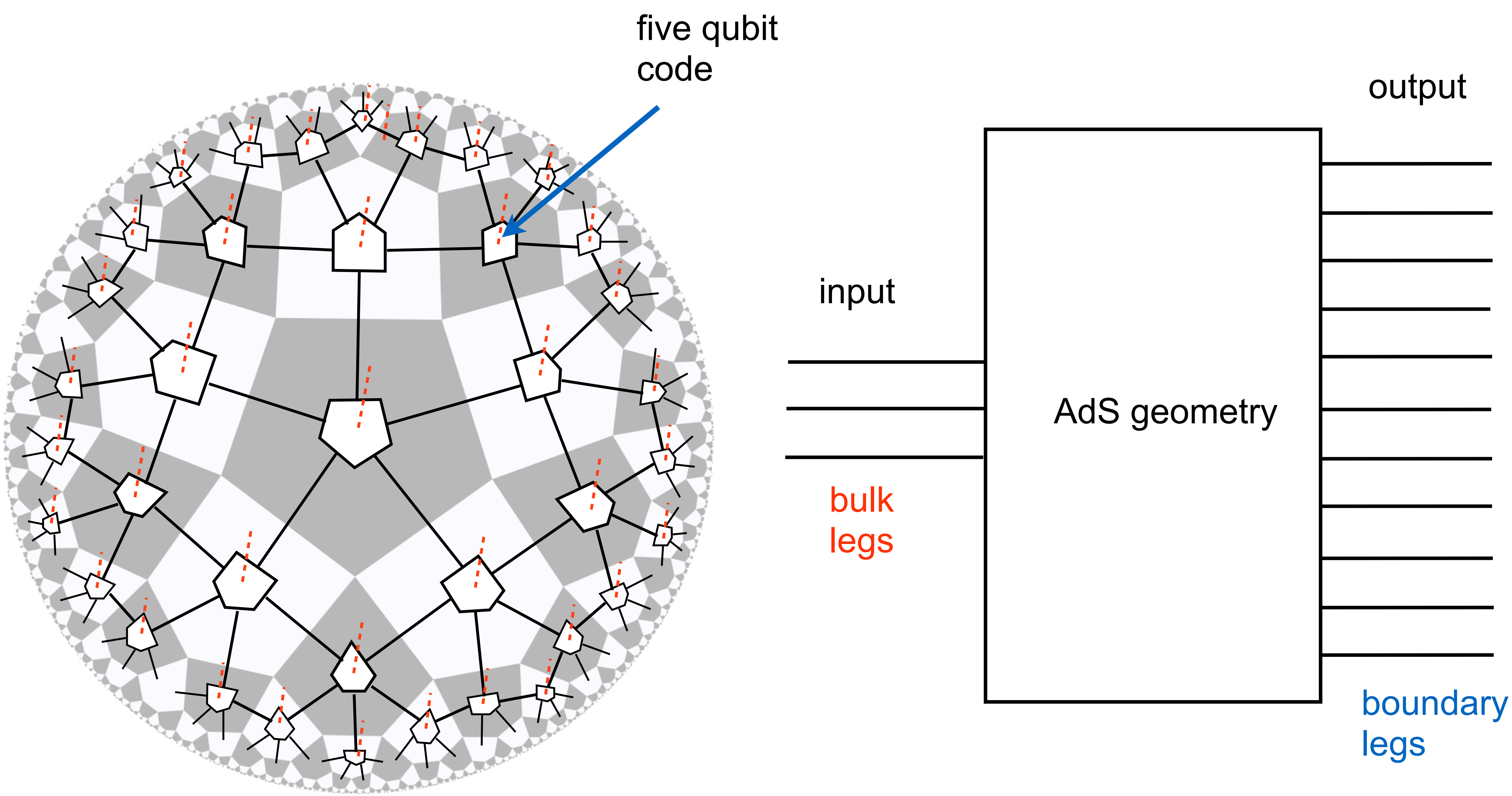


- Any leg can be used as an input of quantum codes.



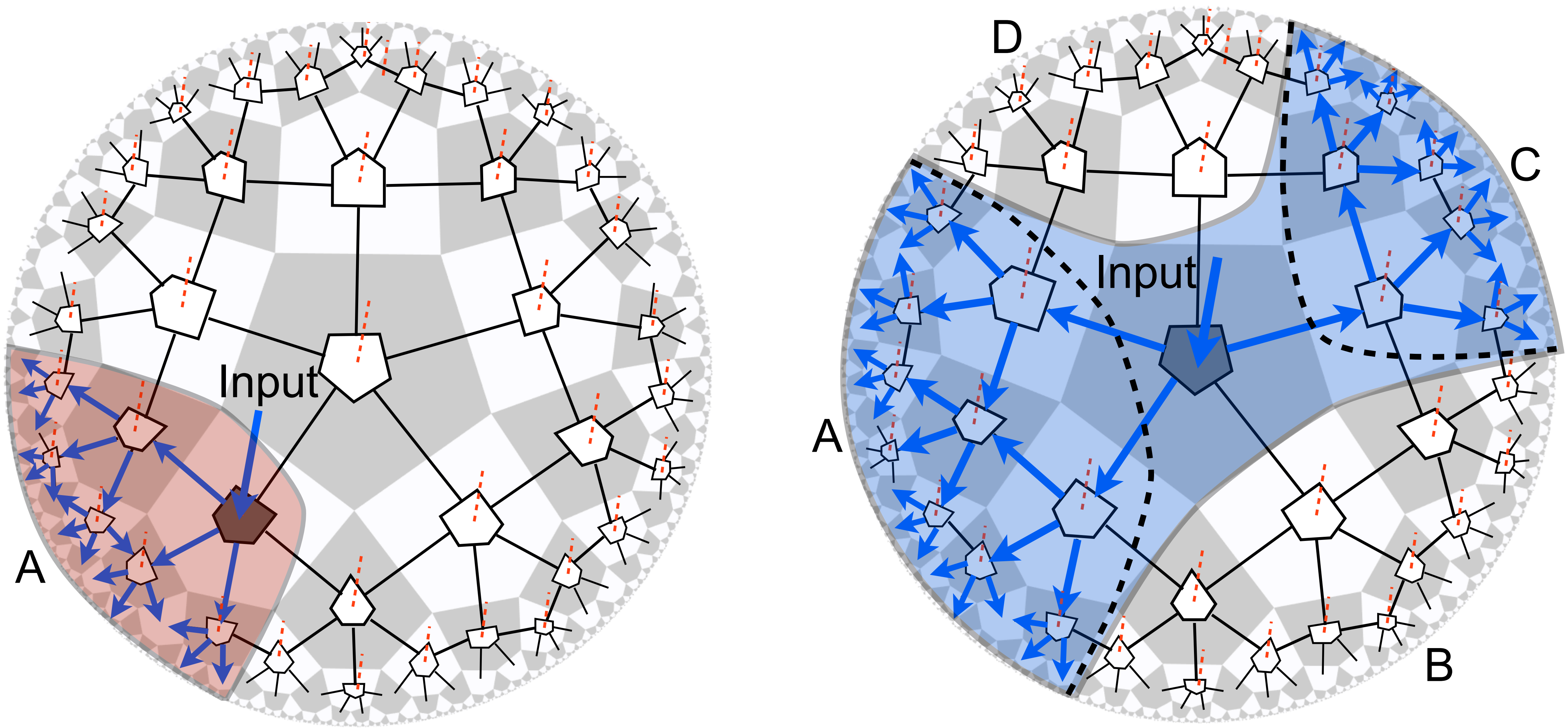
A holographic quantum error-correcting code

- A tiling of the five qubit code



Entanglement wedge reconstruction

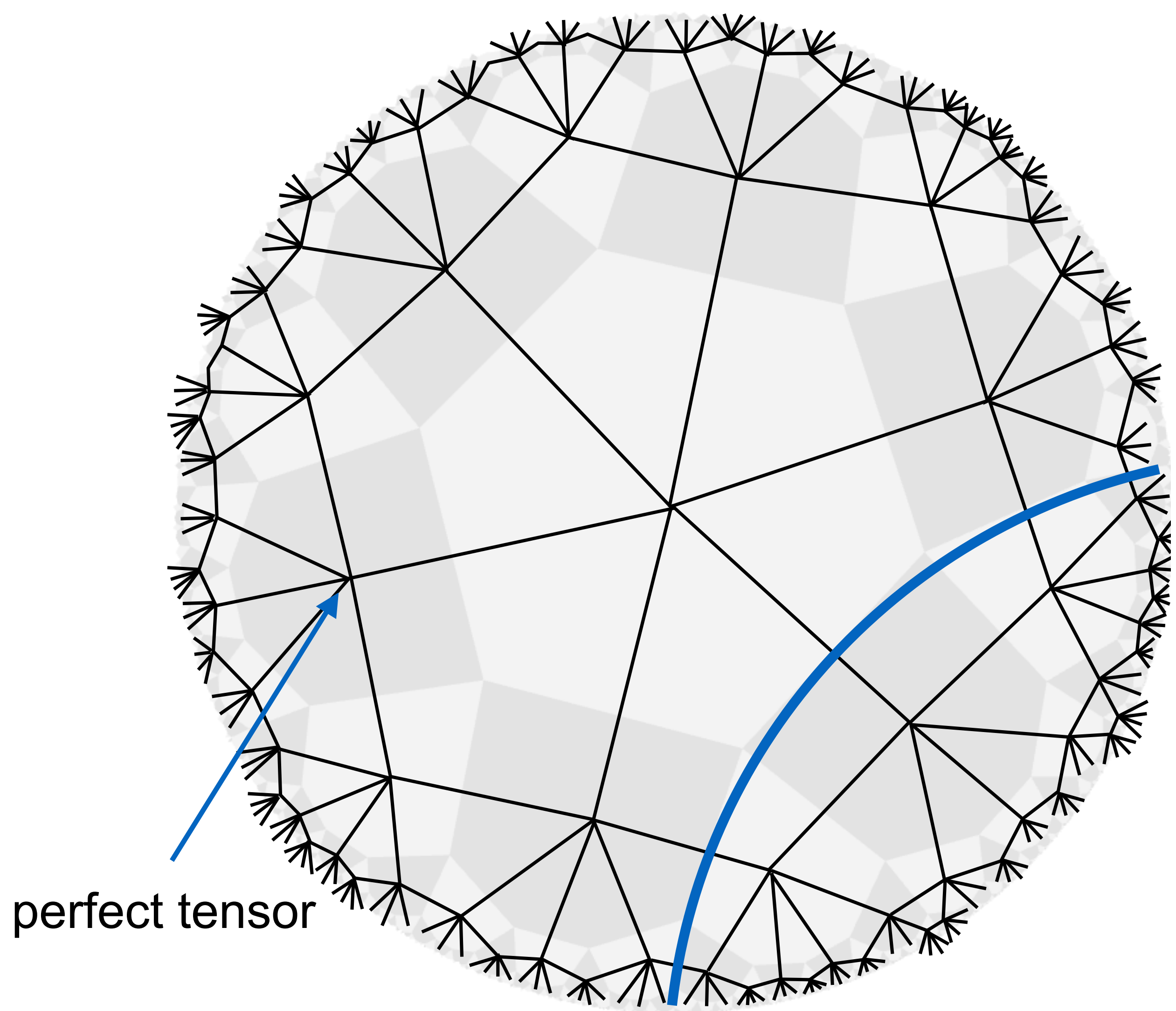
- 1 in & 3 out (operator pushing)

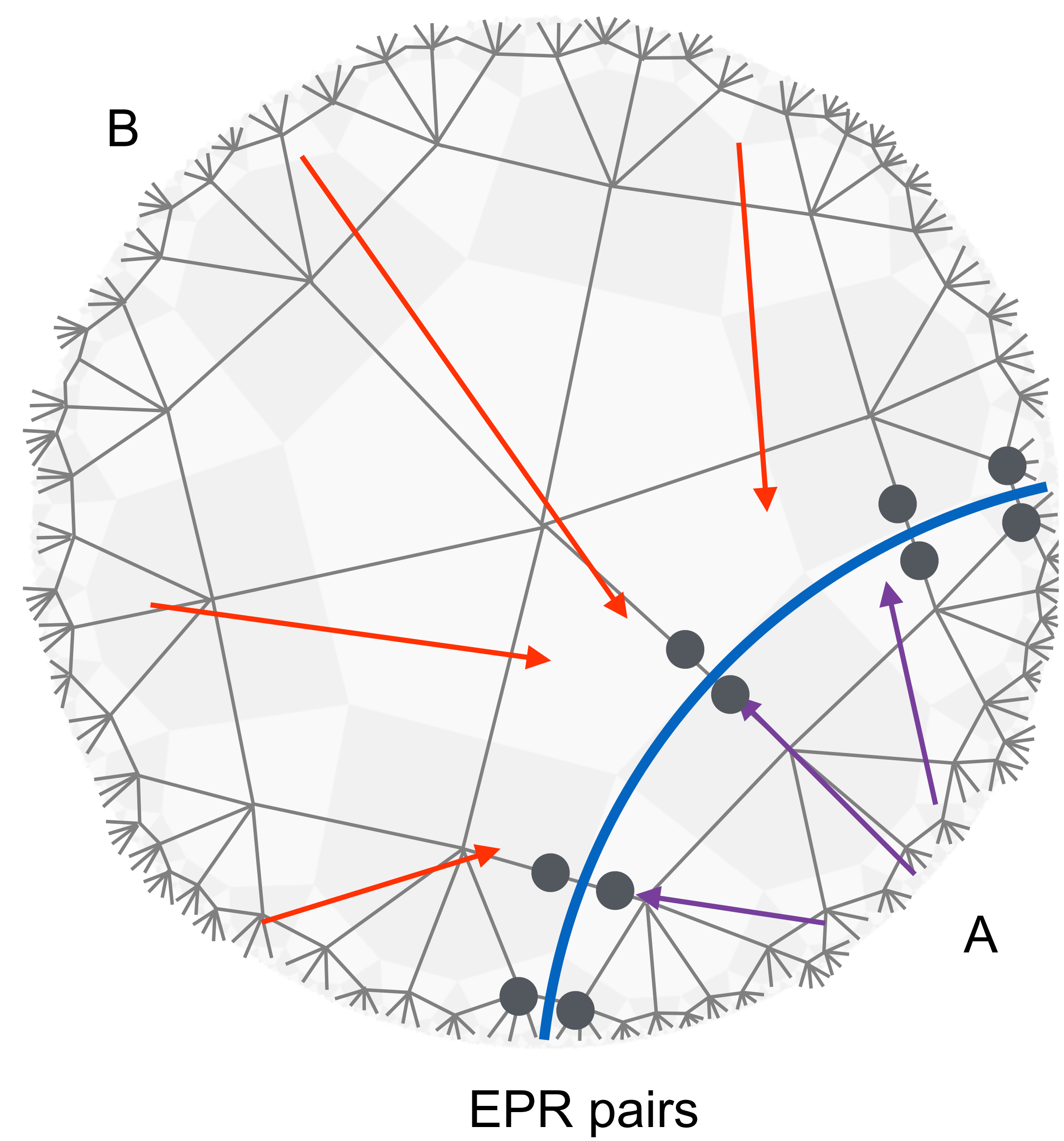


Holographic state

- The Ryu-Takayanagi formula holds exactly (tiling of perfect tensors)

Coarse-graining (RG transformation) = Distillation of EPR pairs along the geodesic

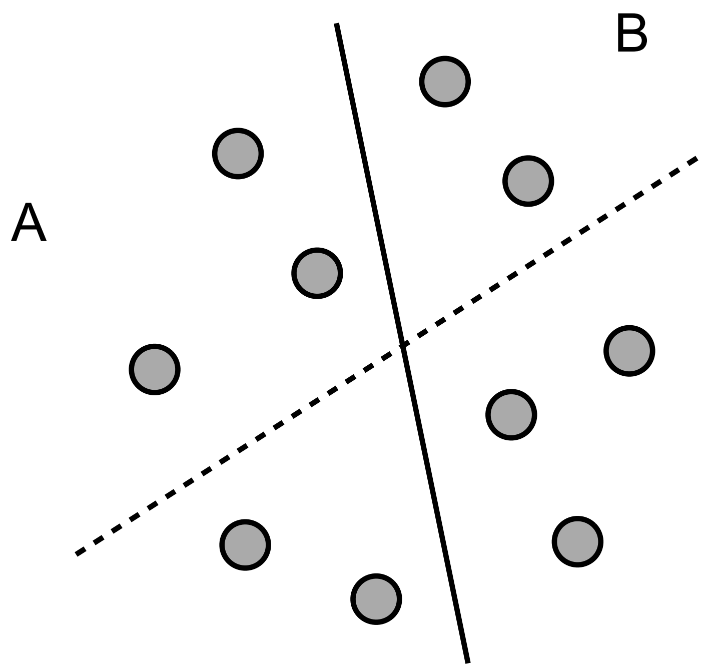




Perfect tensors

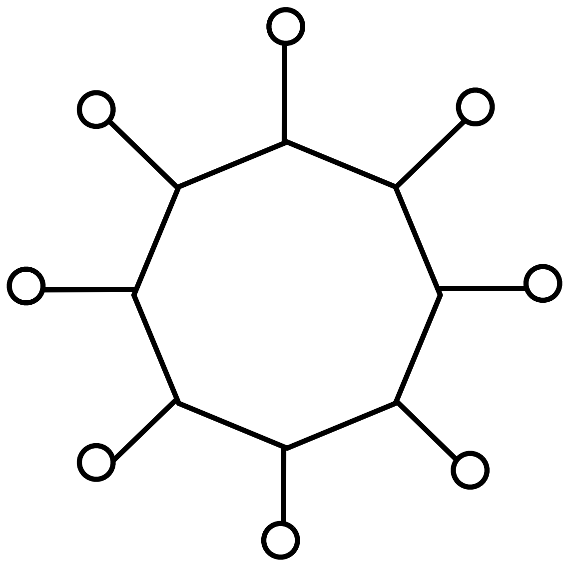
- A pure state with maximal entanglement in any bipartition

Perfect state (2n spins)



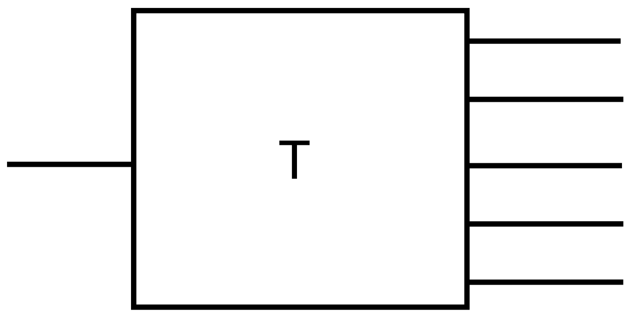
$$\rho_A \propto I_A \qquad \text{for all} \quad |A| \leq n$$

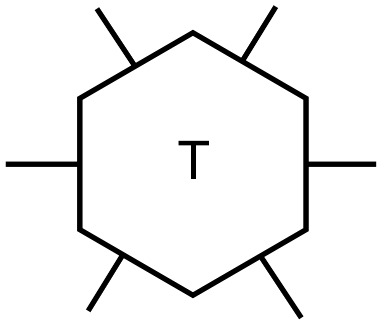
Perfect tensor (2n legs)



$$|\psi\rangle = \sum_{i_1=0}^{v-1} \sum_{i_2=0}^{v-1} \cdots \sum_{i_n=0}^{v-1} T_{i_1 i_2 \dots i_n} |i_1 i_2 \dots i_n\rangle$$

- Five qubit code





6 leg perfect tensor

Random tensors

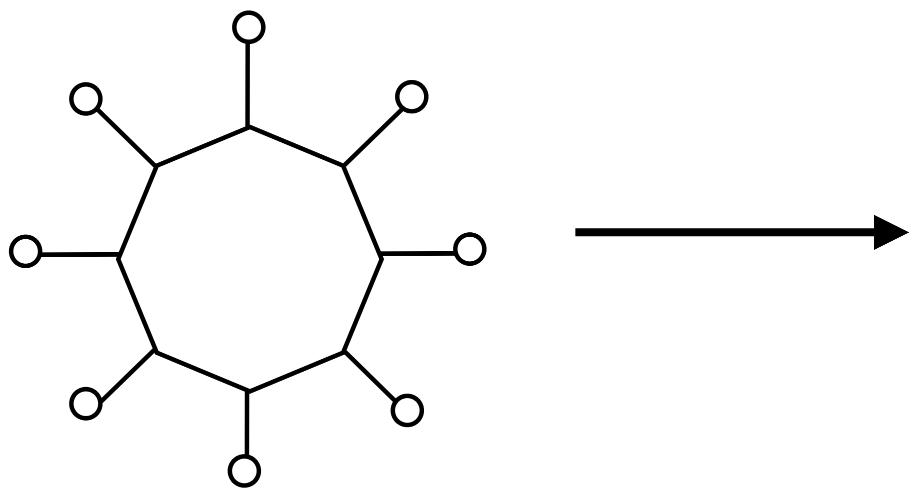
- Perfect tensors are very rare…

- But almost perfect tensors are pretty common !

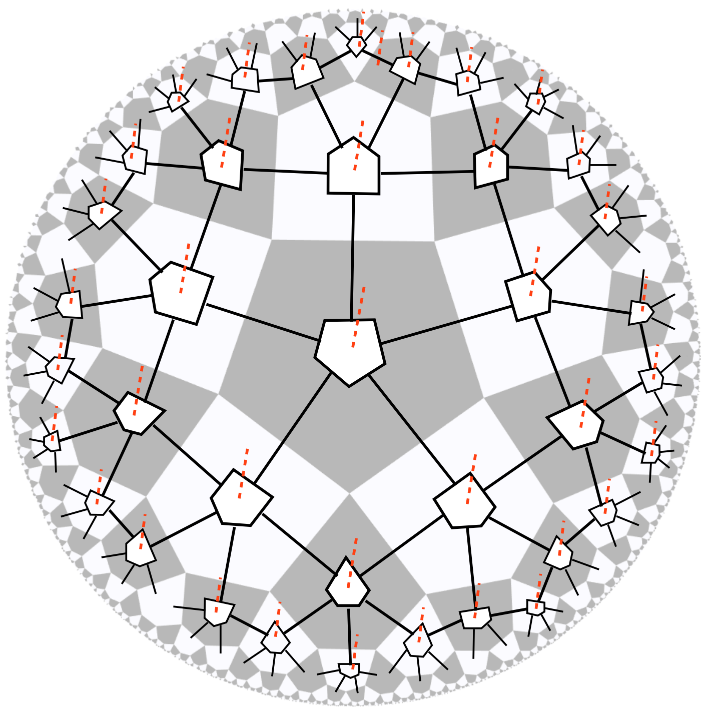
  Pick a Haar random state.

  Due to the Page's theorem, the state is almost maximally entangled along any cut.

- To construct a holographic code/state, just pick tensors randomly.



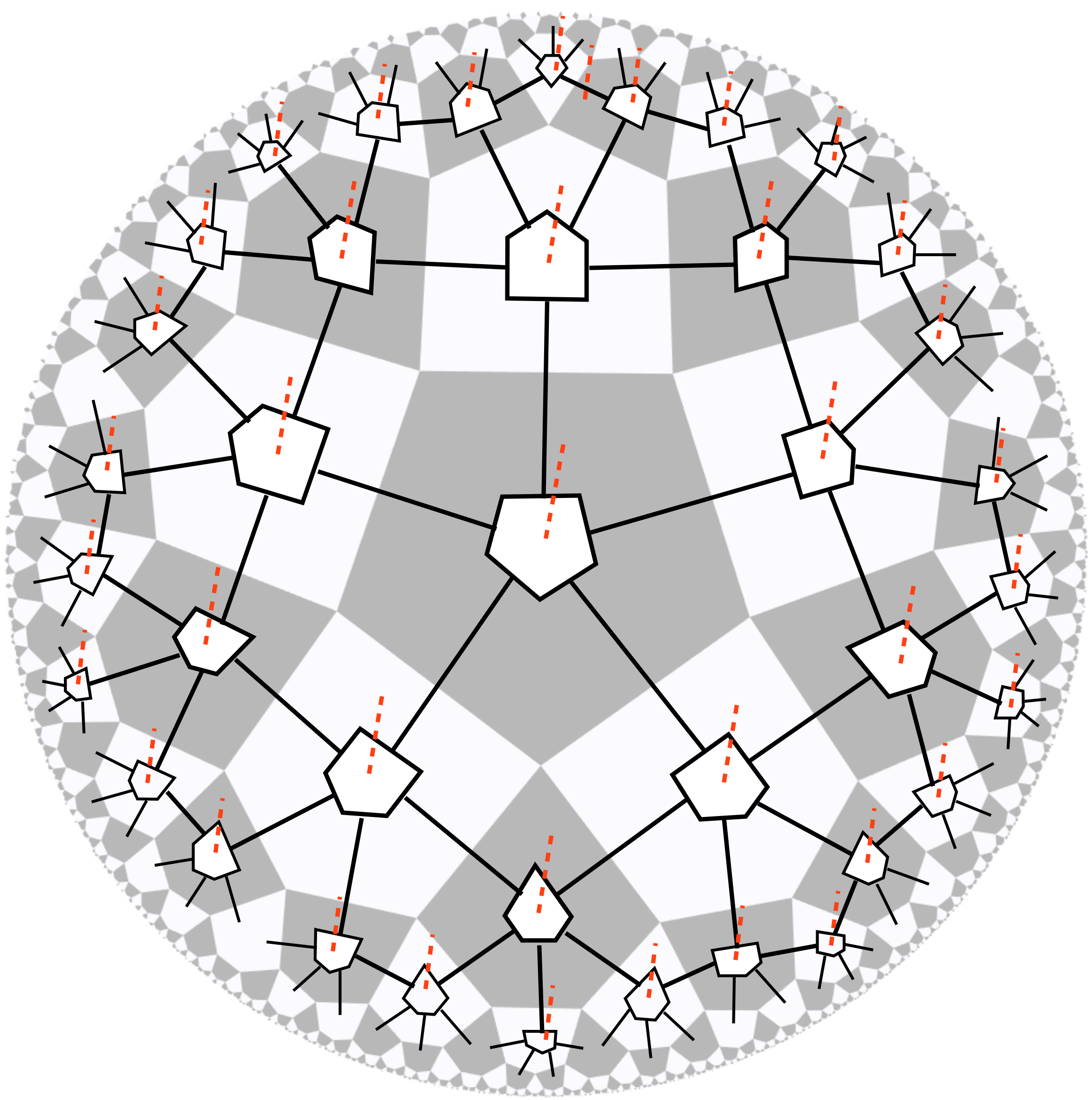
Random tensor



Holographic code

Coding properties: erasure threshold

- Remove qubits with probability $p = \frac{1}{2} - \epsilon$



A central bulk leg is contained in the entanglement wedge of A (if |A|>|B|)

Erasure threshold = 1/2

New quantum codes from quantum gravity ?

So far, no black holes…

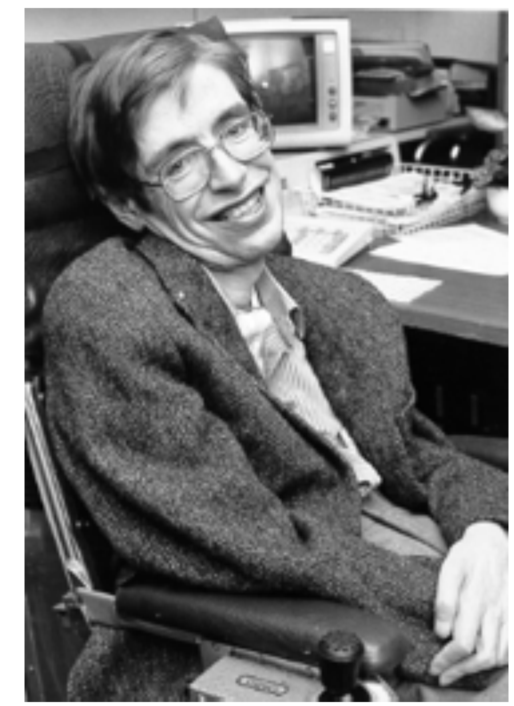
Information loss puzzle

- Is quantum information lost ?

$|\psi\rangle$

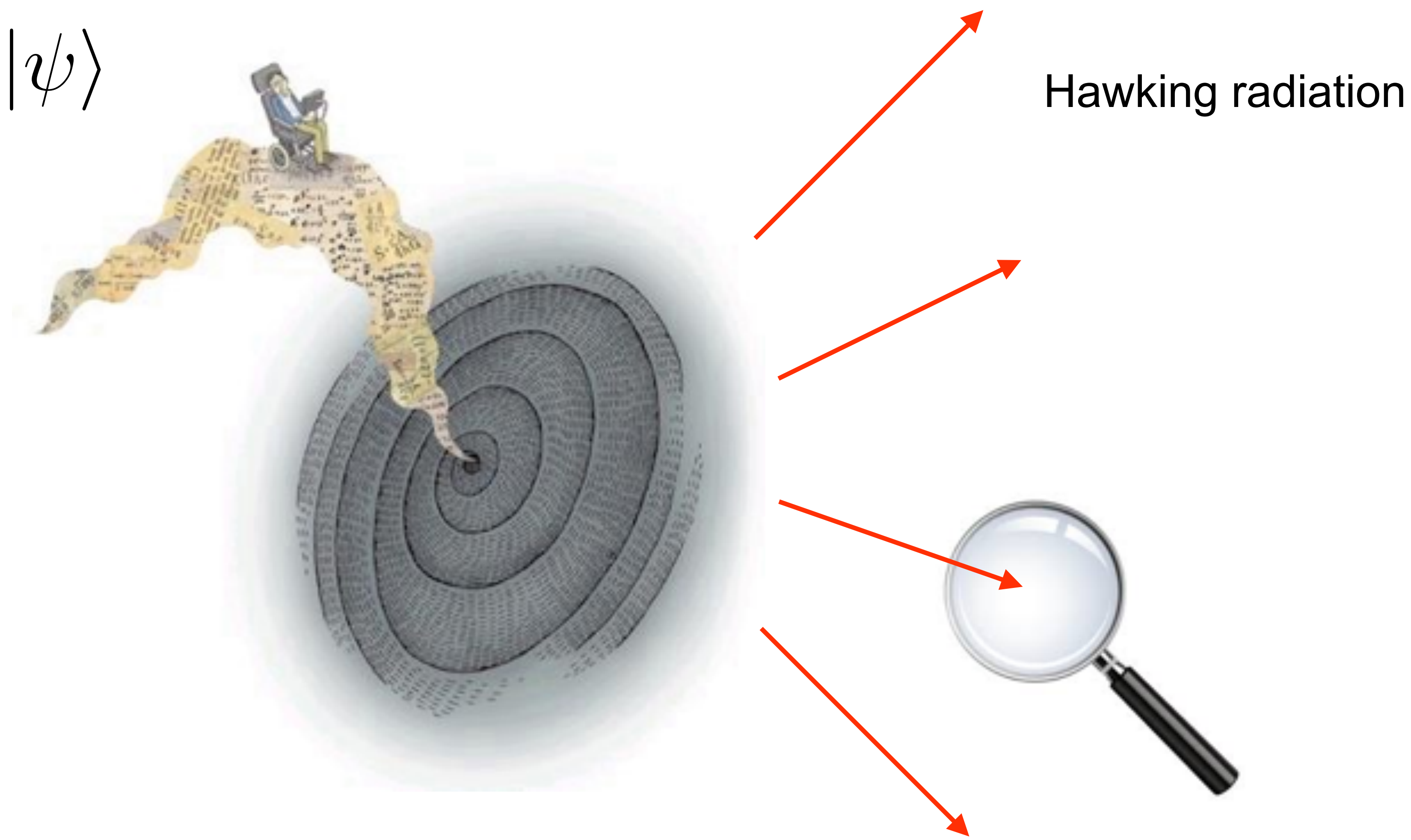
Hawking radiation

“$e^{-\beta H}$”



Information loss puzzle

- Or hidden into some non-local degrees of freedom ?



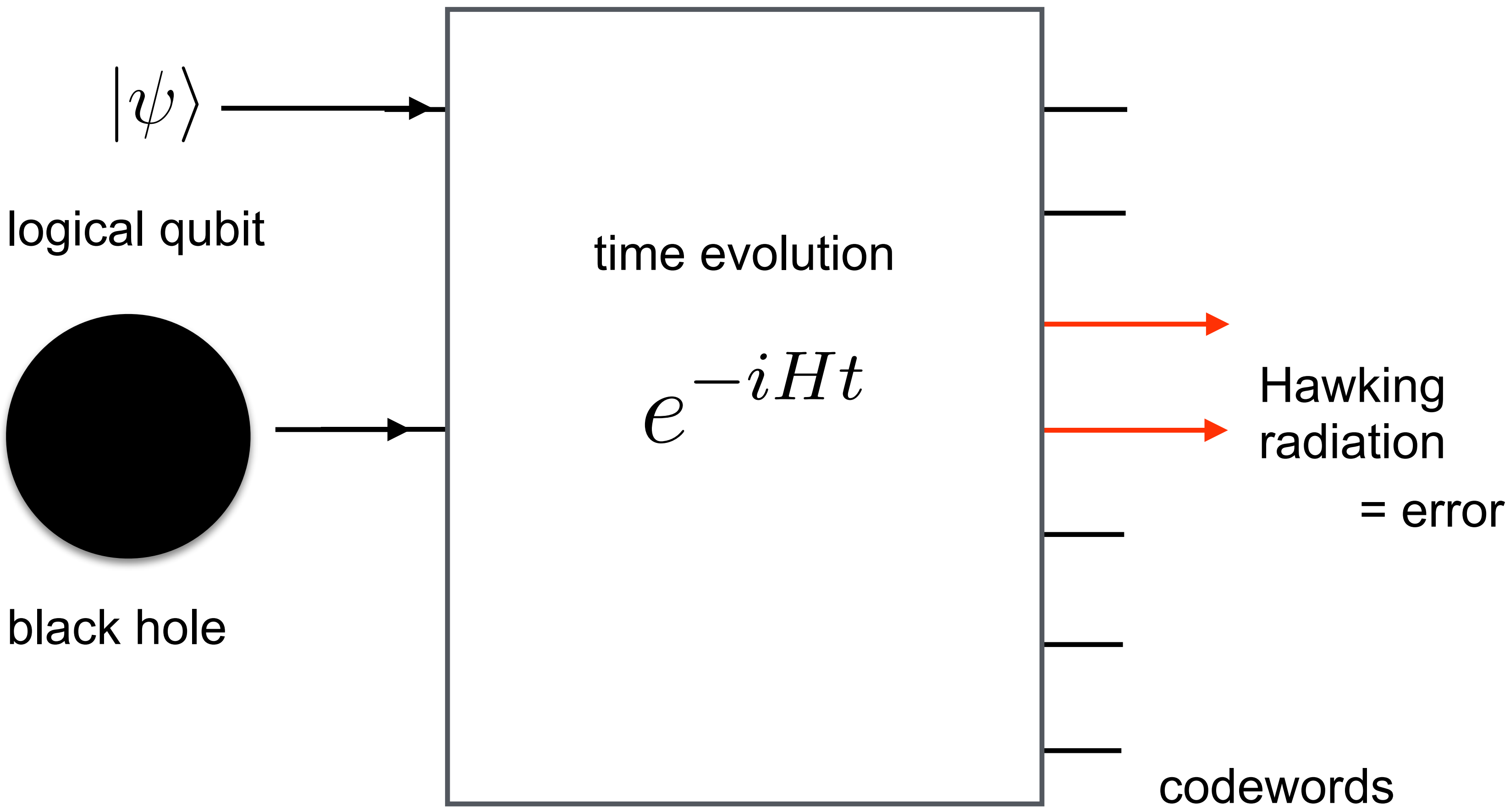
Locally it looks like "$e^{-\beta H}$", but globally it is not .

Quantum error-correction

- Scrambling is very similar to how quantum error-correcting codes work.

- Local indistinguishability.

$$\rho_R \simeq \rho'_R$$

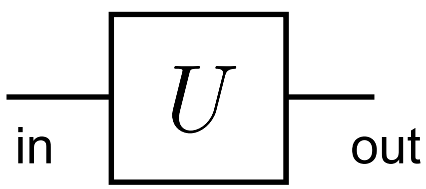


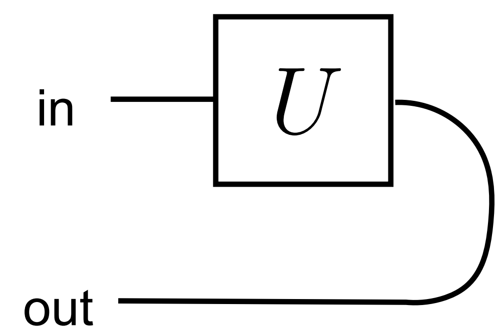
Choi-Jamilkowski isomorphism

- Quantum channel on n qubits can be viewed as a state on 2n qubits.

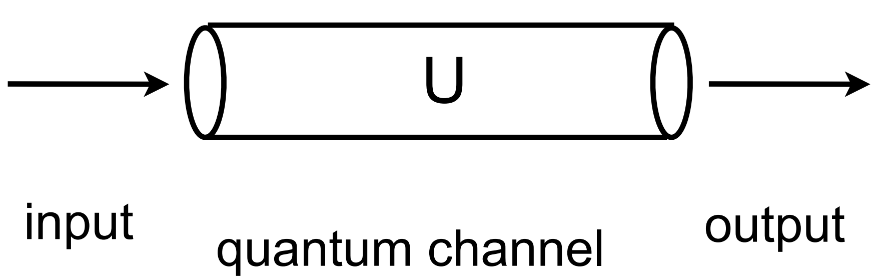
unitary operator as a state

$$U = \sum_{i,j} U_{i,j} |i\rangle\langle j| \qquad |U\rangle = \sum_{i,j} U_{i,j} |i\rangle \otimes |j\rangle$$





$$\rho_{in} = \{p_j, |\psi_j\rangle\} \qquad \rho_{out} = \{p_j, |\phi_j\rangle\}$$



$$|\Psi\rangle = \sum_j \sqrt{p_j} |\psi_j\rangle \otimes |\phi_j\rangle$$

[Choi-Jamilkowski, Hayden-Preskill,Hartman-Maldacena]

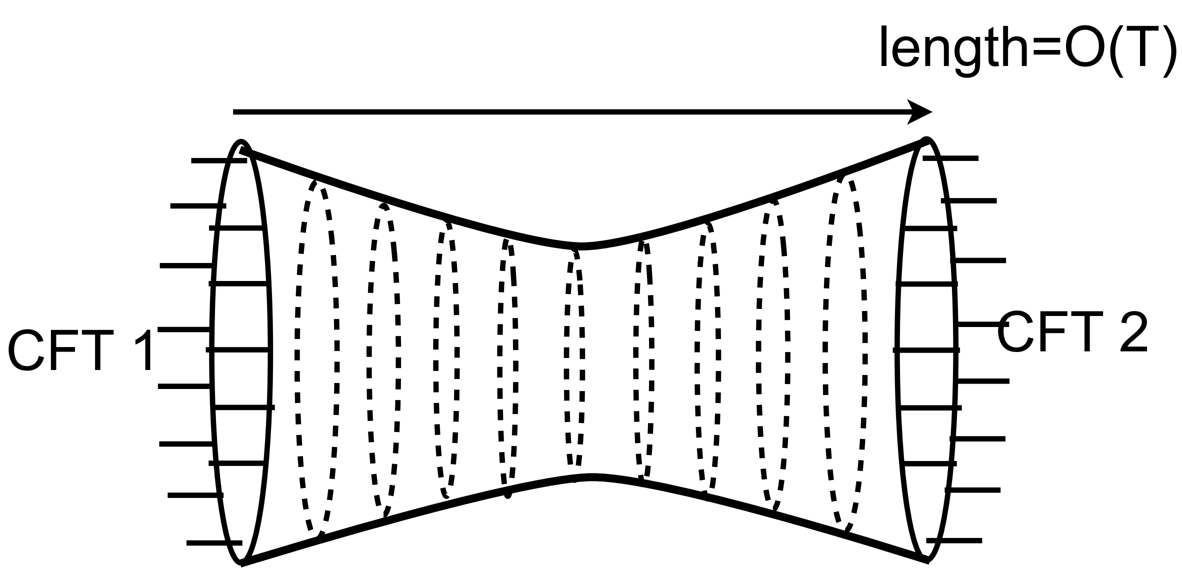
Scrambling in a black hole

- Thermofield double state (finite T)

$$\rho_{in} = \frac{e^{-\beta H}}{\mathrm{Tr}\ e^{-\beta H}} \qquad \rho_{out} = \frac{e^{-\beta H}}{\mathrm{Tr}\ e^{-\beta H}}$$

$$|\Psi\rangle = \sum_j e^{-\beta E_j/2} \boxed{e^{-iE_j t}} |\psi_j\rangle \otimes |\psi_j\rangle$$

- A black hole geometry for the TFD state is the two-sided hole (AdS/CFT prediction)

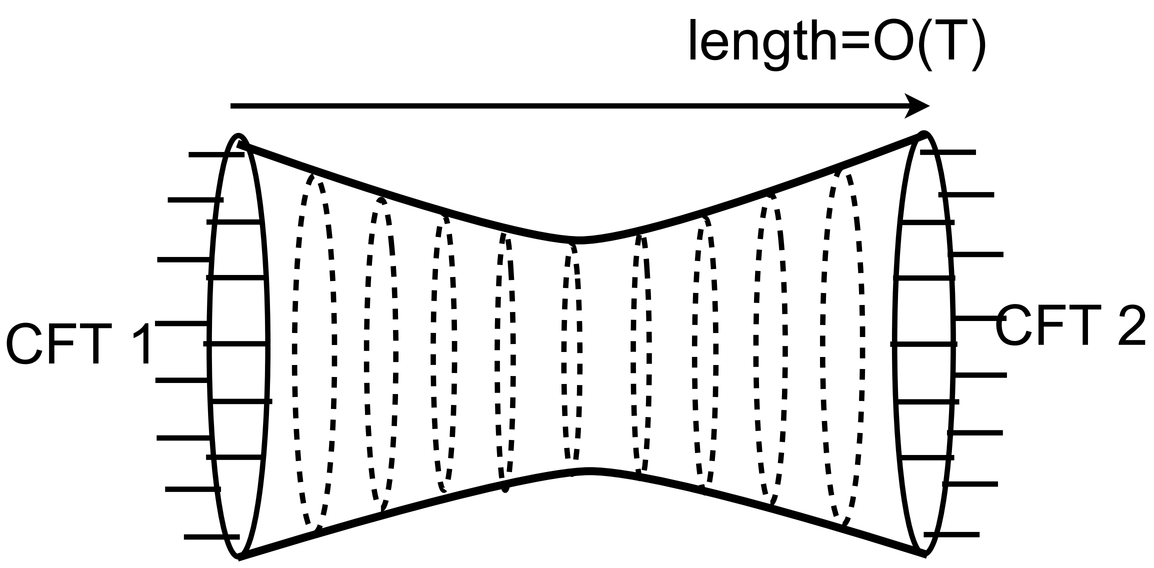


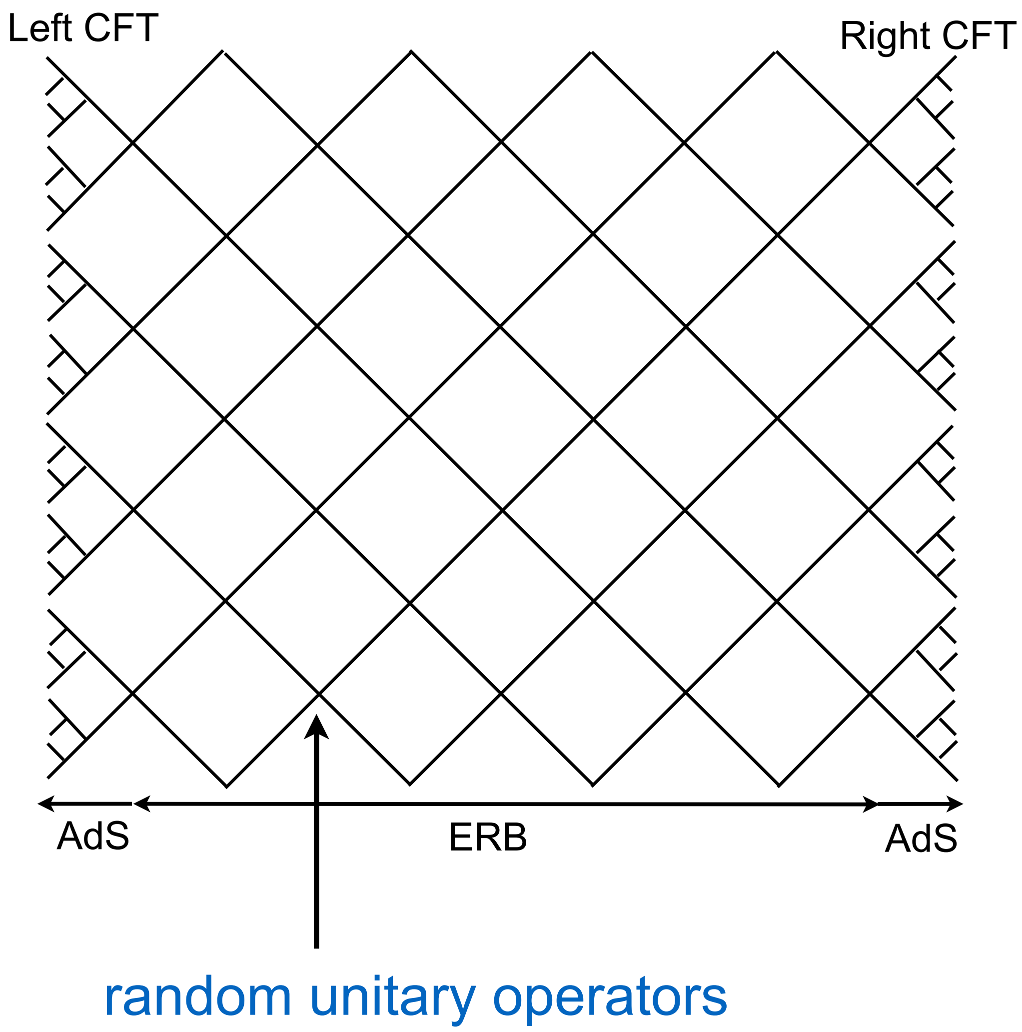
Let's construct a tensor network toy model !

[Maldacena]

Toy model of the Einstein-Rosen bridge

- Consider a network of random unitary operators, tiling the wormhole geometry.





[Hosur-Qi-Roberts-BY]

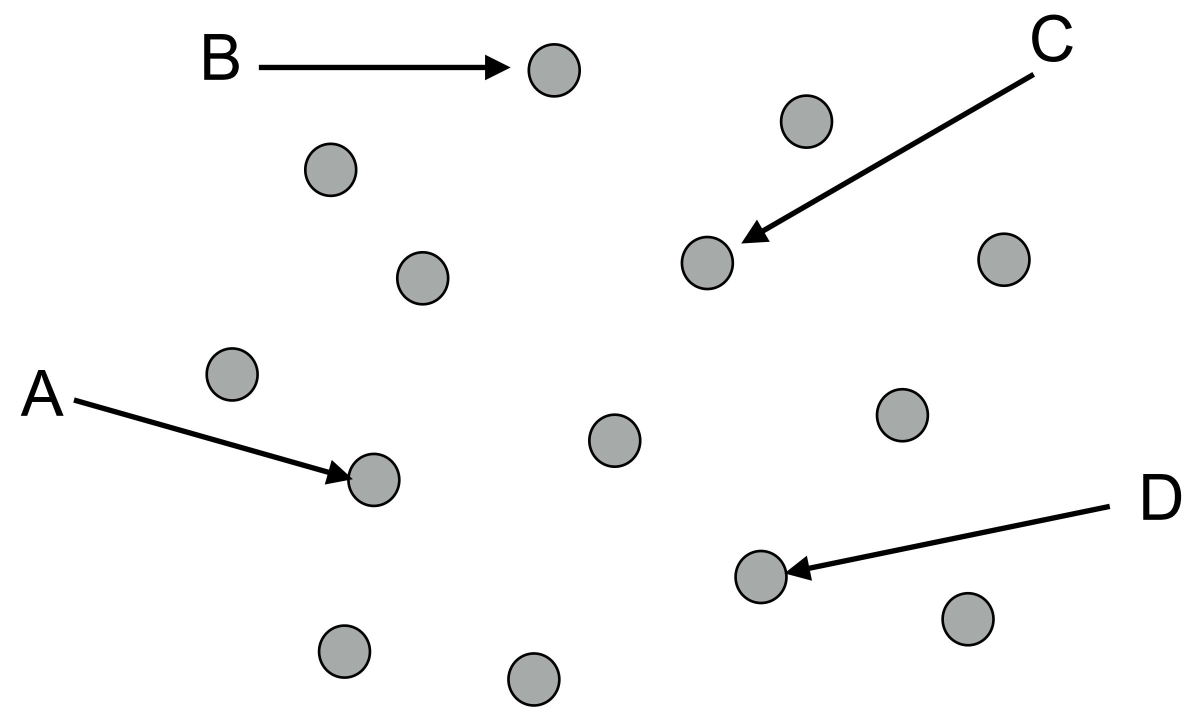
How do we probe the interior of a black hole ?

Out-of-time ordered correlation functions

- We should measure some "hidden" correlations (Kitaev 2014)

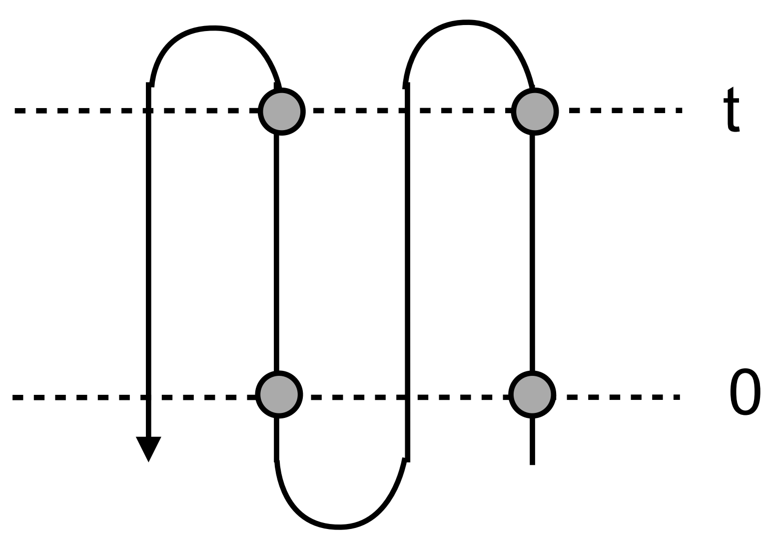
$$\text{OTO} = \langle A(0) B(t) C(0) D(t) \rangle$$

local operators



$$B(t) = e^{-iHt} B(0) e^{iHt}$$

$$D(t) = e^{-iHt} D(0) e^{iHt}$$



- Previously considered by Larkin and Ovchinikov in 1960s, and recently by Shenker and Stanford

Time evolution of operators

- OTOCs detect the growth of operators

- Consider $$\mathrm{OTO} = \langle A(0)B(t)A^{\dagger}(0)B^{\dagger}(t)\rangle$$

(commutator)

$$[A, B] = 0 \quad \text{then} \quad \mathrm{OTO} = 1$$

$$\{A, B\} = 0 \quad \text{then} \quad \mathrm{OTO} = -1$$

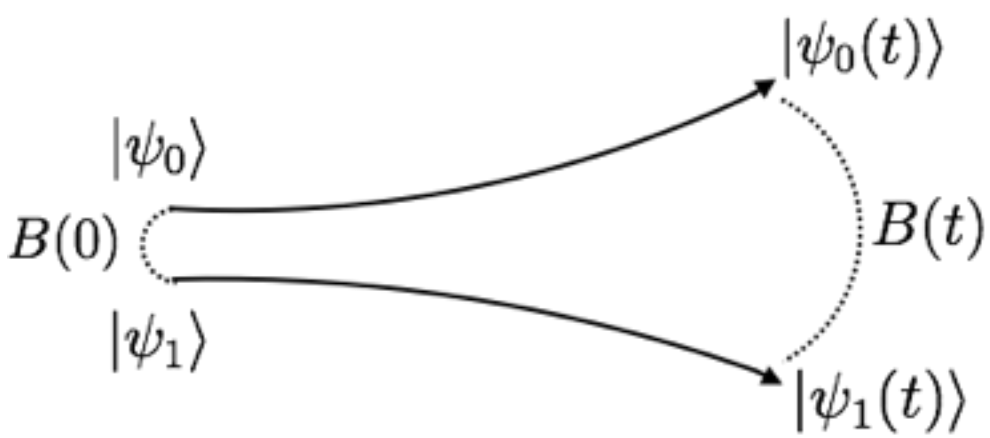
Non-commutativity between A(0) and B(t)

- Expand B(t):

$$B(t) = e^{-iHt}Be^{iHt} = \sum_j \alpha_j P_j$$

high-weight Pauli operators

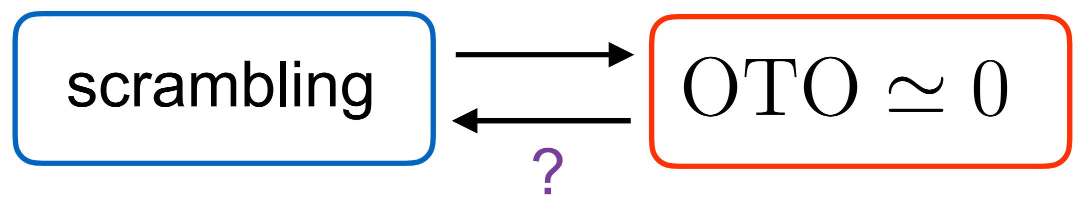
scrambling → $\mathrm{OTO} \simeq 0$



[Roberts-Stanford-Susskind]

Key Questions

- How do we define scrambling ?
- Quantum information theoretic meaning of OTO ?
- Is the converse true ?



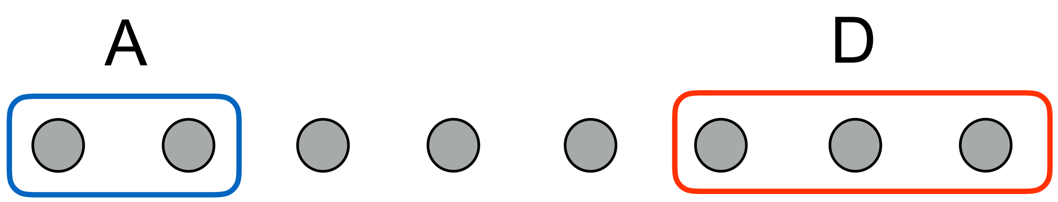
I will relate OTOCs to entanglement entropies (joint with Hosur, Qi and Roberts)

[Theorem] Average value of OTO

- Average of OTO over local operators A and D at T=infty

$$\Big|\langle A(0)D(t)A^{\dagger}(0)D^{\dagger}(t)\rangle\Big|$$

average over A, D



[Hosur-Qi-Roberts-BY]

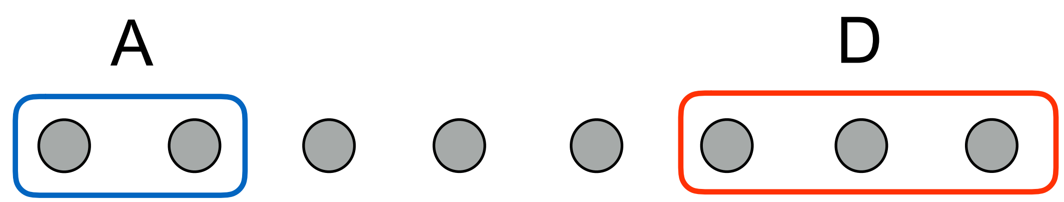
[Theorem] Average value of OTO

- Average of OTO over local operators A and D at T=infty

$$\left|\langle A(0)D(t)A^{\dagger}(0)D^{\dagger}(t)\rangle\right| = \frac{1}{4^{a+d}} \sum_{A,D} \langle A(0)D(t)A^{\dagger}(0)D^{\dagger}(t)\rangle$$

average over A, D

A,D: Pauli operators (unitary 1-design)



[Hosur-Qi-Roberts-BY]

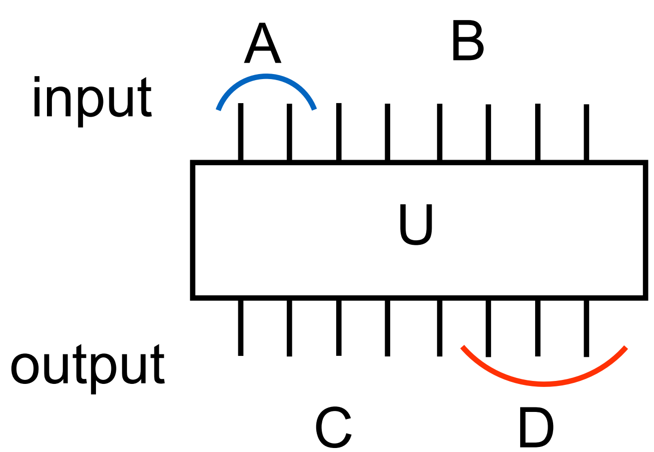
[Theorem] Average value of OTO

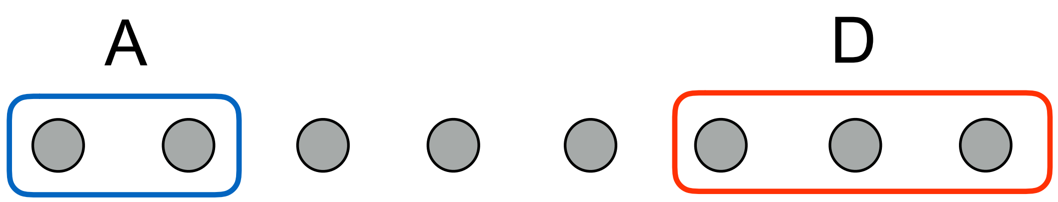
- Average of OTO over local operators A and D at T=infty

$$\left|\langle A(0)D(t)A^{\dagger}(0)D^{\dagger}(t)\rangle\right| = \frac{1}{4^{a+d}}\sum_{A,D}\langle A(0)D(t)A^{\dagger}(0)D^{\dagger}(t)\rangle$$

average over A, D

Pauli operators (unitary 1-design)





[Hosur-Qi-Roberts-BY]

[Theorem] Average value of OTO

- Average of OTO over local operators A and D at T=infty

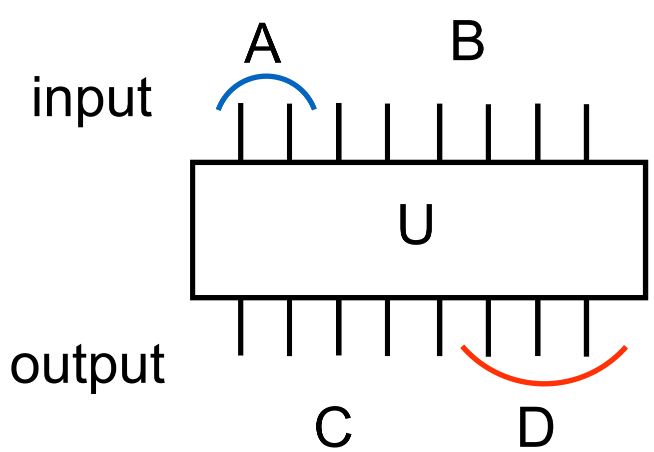
$$\left|\langle A(0)D(t)A^\dagger(0)D^\dagger(t)\rangle\right| = \frac{1}{4^{a+d}}\sum_{A,D}\langle A(0)D(t)A^\dagger(0)D^\dagger(t)\rangle$$

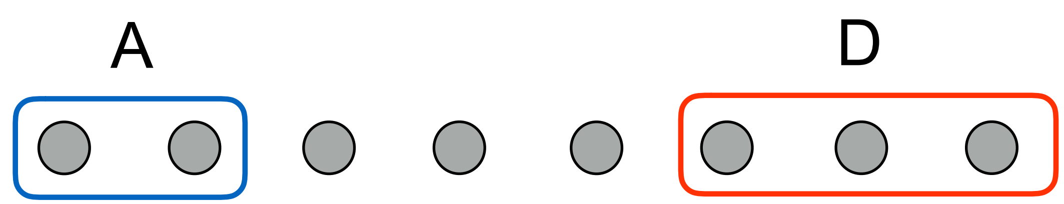
average over A, D

A,D: Pauli operators (unitary 1-design)

$$= 2^{n-a-d-S^{(2)}_{BD}}$$

$S^{(2)}_{BD}$: Renyi-2 entropy





[Hosur-Qi-Roberts-BY]

[Theorem] Average value of OTO

- Average of OTO over local operators A and D at T=infty

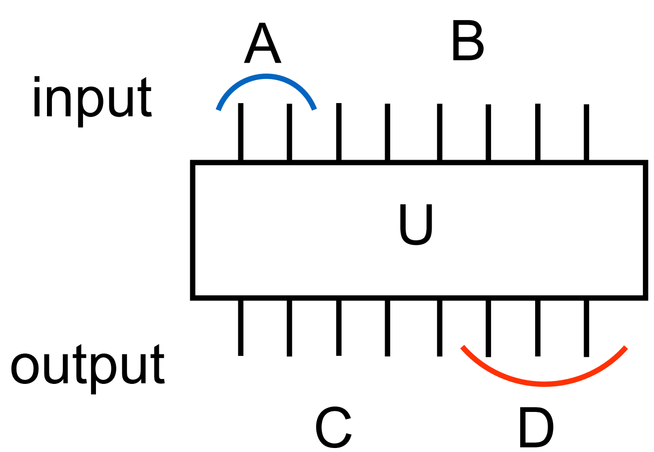
$$\left|\langle A(0)D(t)A^\dagger(0)D^\dagger(t)\rangle\right| = \frac{1}{4^{a+d}}\sum_{A,D}\langle A(0)D(t)A^\dagger(0)D^\dagger(t)\rangle$$

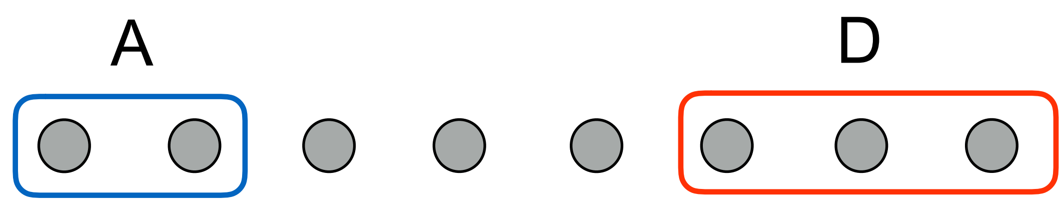
average over A, D

Pauli operators (unitary 1-design)

$$= 2^{n-a-d-S^{(2)}_{BD}}$$

Renyi-2 entropy





- If $\mathrm{OTO} \simeq 0$ then, $S^{(2)}_{BD}$ is large

This implies the mutual information $I^{(2)}_{BD} = S^{(2)}_B + S^{(2)}_D - S^{(2)}_{BD}$ is small

B and D are not correlated, so the system is scrambling.

[Hosur-Qi-Roberts-BY]

[Theorem] Average value of OTO

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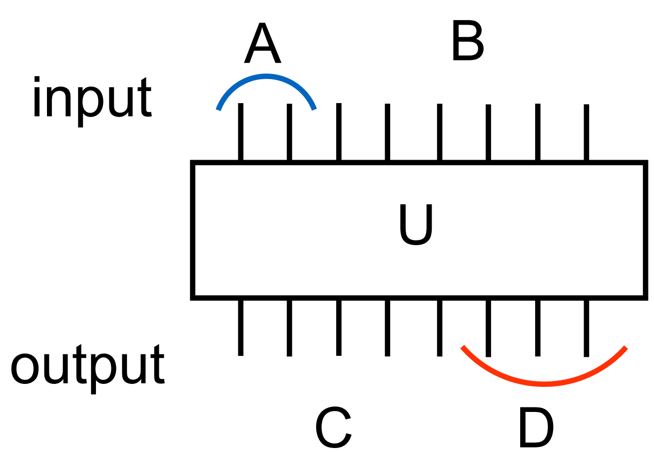
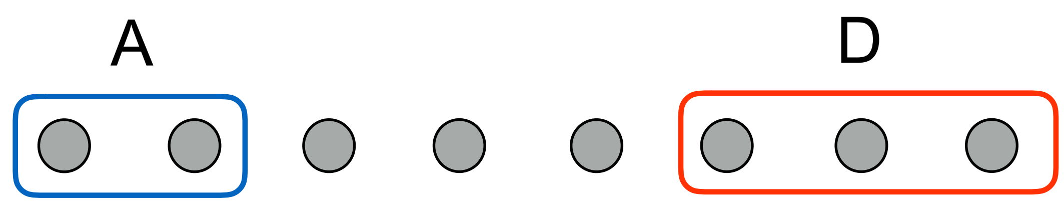
average over A, D

A,D: Pauli operators (unitary 1-design)

$S^{(2)}_{BD}$: Renyi-2 entropy





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  B and D are not correlated, so the system is scrambling.

- For finite T, we will consider the so-called Thermofield double state.

[Hosur-Qi-Roberts-BY]

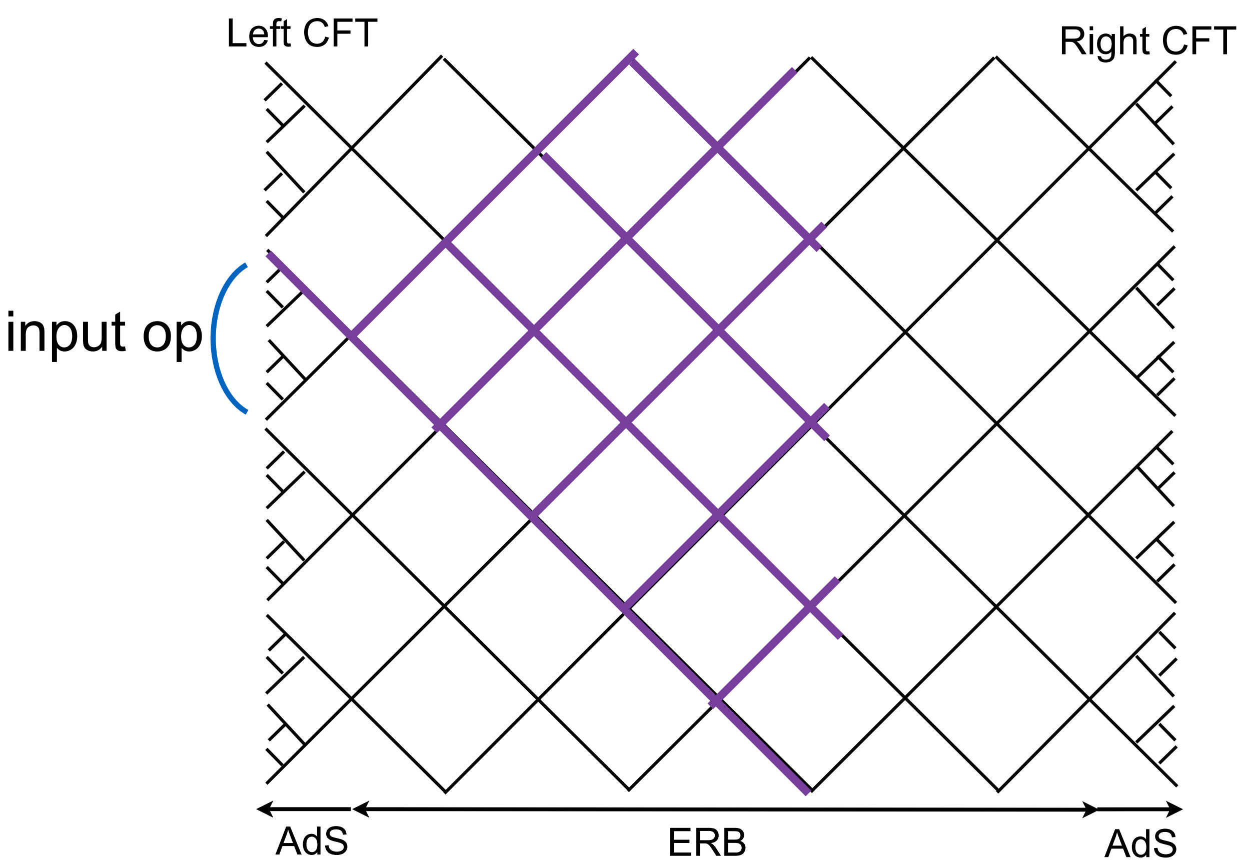
Scrambling phenomena in a black hole

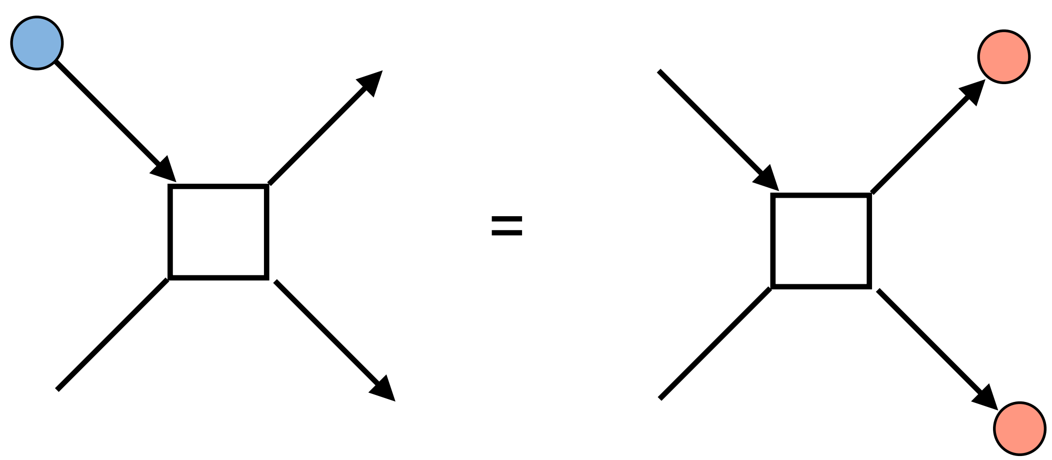
- This captures key properties of scrambling for "large-N" theories.

  Eg) Ballistic propagations of entanglement, RT formula in a wormhole geometry…

- Operator size grows linearly, OTO will pick it up.

- For a non-local random quantum circuit, the scrambling time is log(n) [Cleve et al 2006]

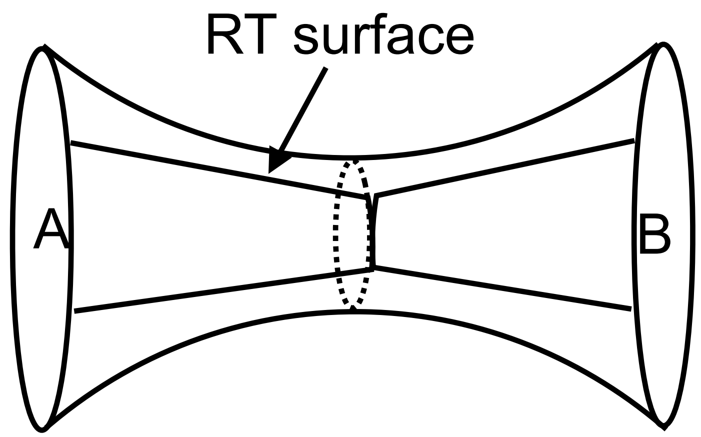




[Hosur-Qi-Roberts-BY]

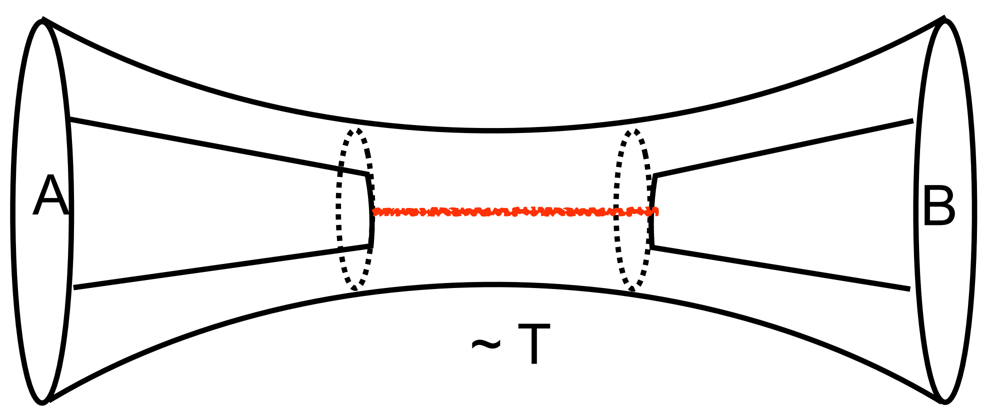
Scrambling in AdS black hole

TFD(0)



mutual information $I(A,B)$ is large

TFD(T)



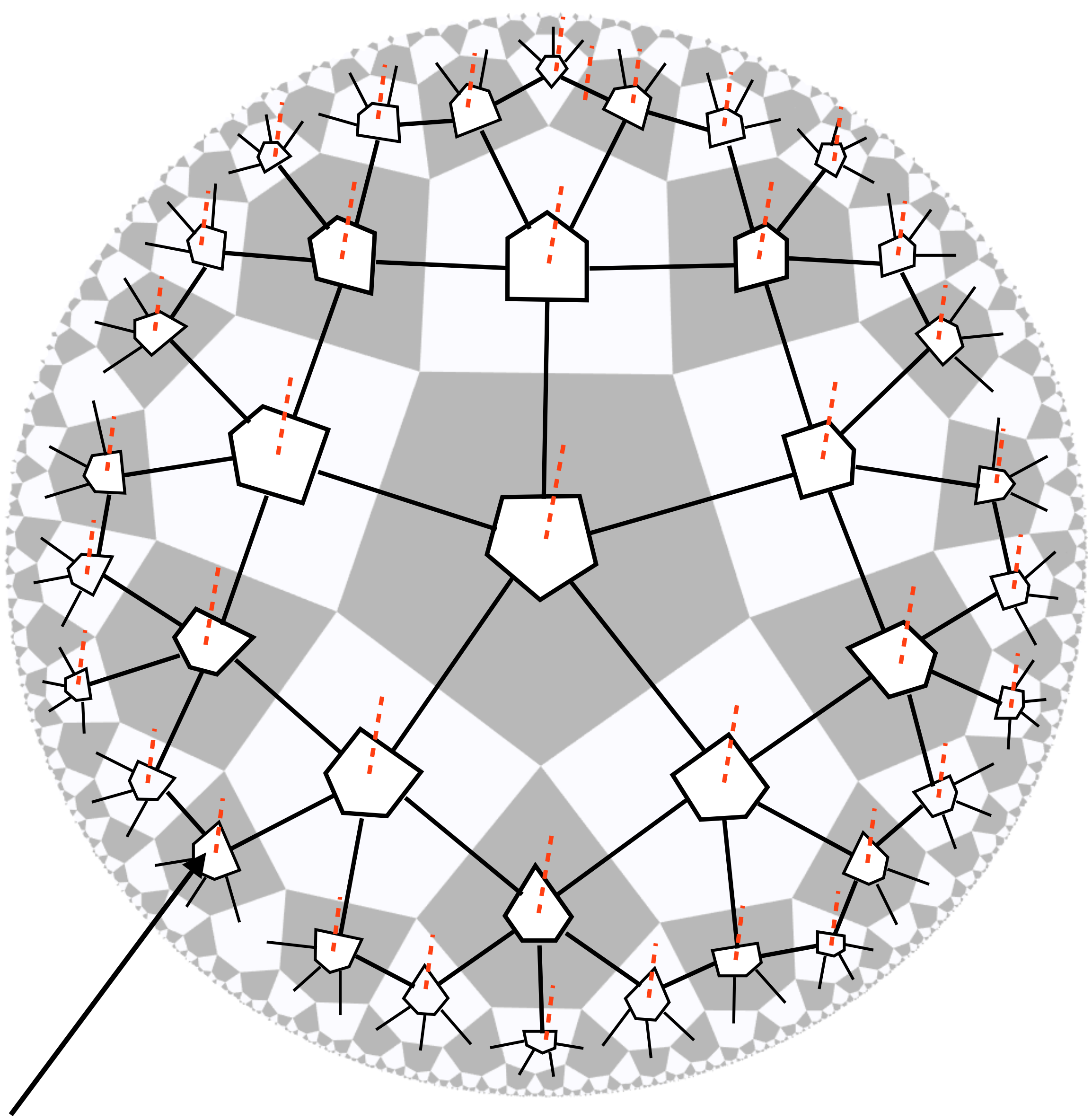
OTO correlators will be small.

mutual information $I(A,B)$ is (almost) zero

What did we learn ?

Lesson 1

The AdS/CFT correspondence is a quantum error-correcting code.

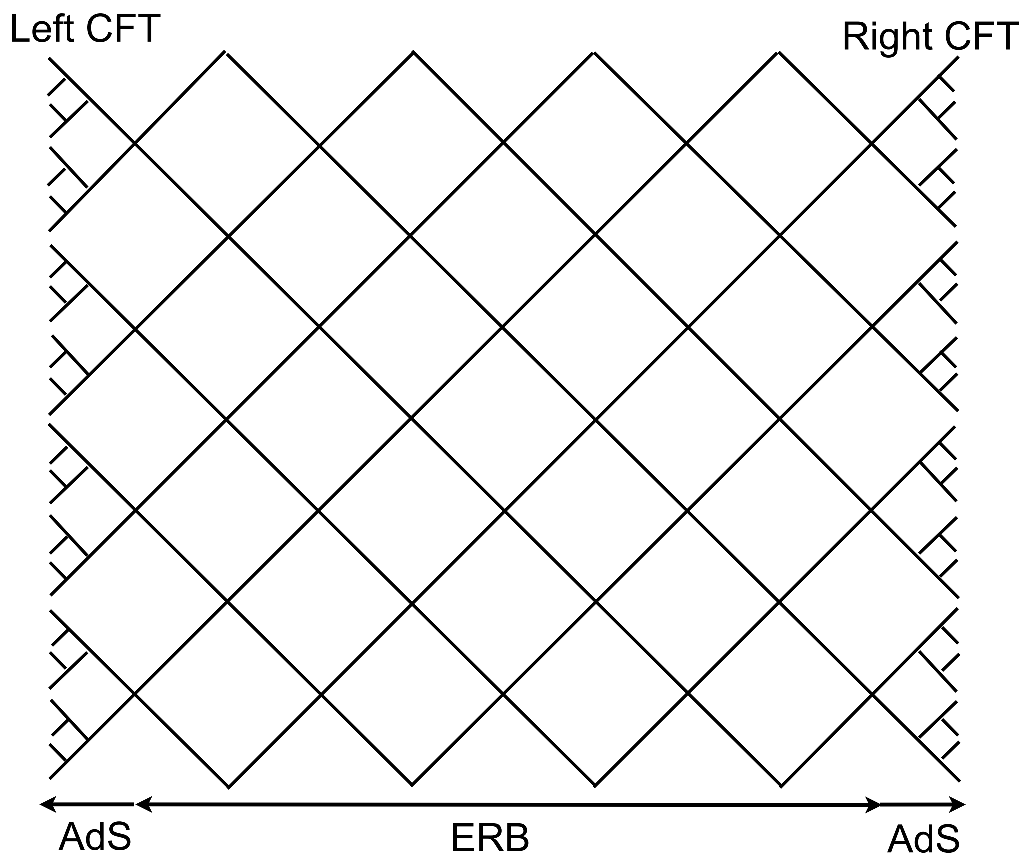


Bulk quantum information is encoded in boundary like a hologram.

very entangled tensor
(eg random tensor)

Lesson 2

OTO correlator is the probe of space-time

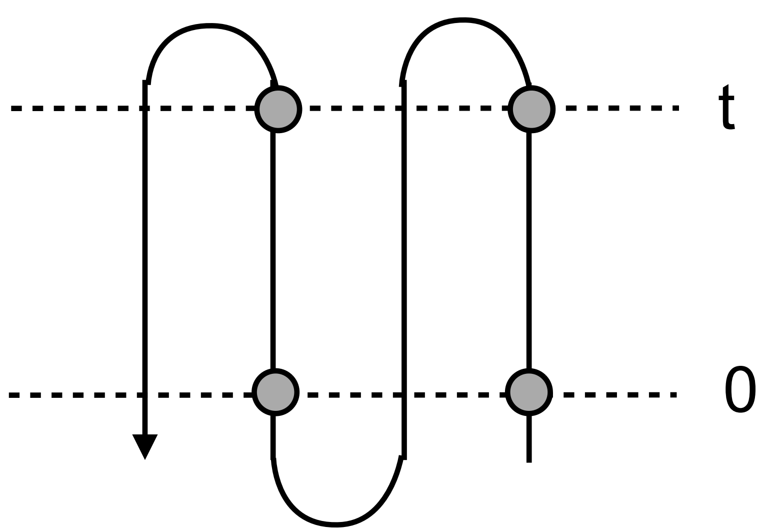


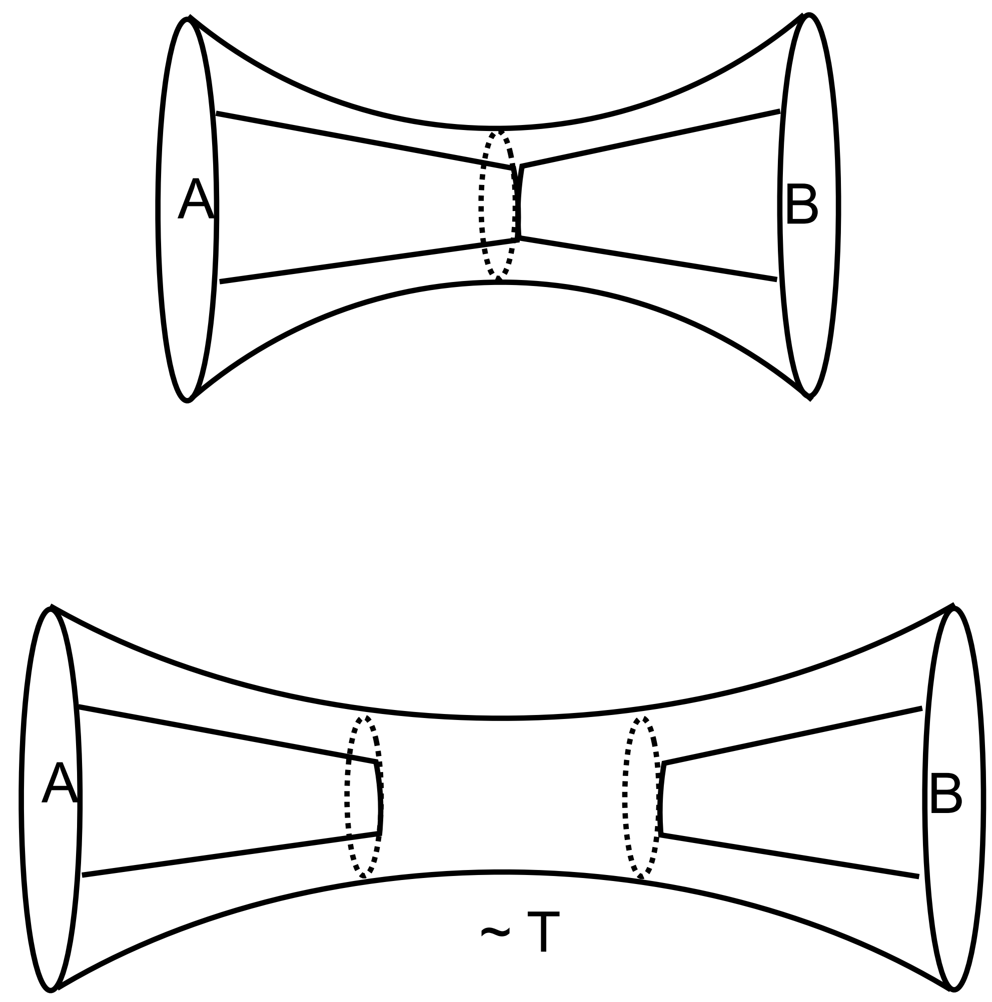
OTO correlator detects scrambling/ chaos

Lesson 3

OTO correlators are the probes of space-time (seeing the interior of a black hole)

$$\mathrm{OTO} = \langle A(0)B(t)C(0)D(t)\rangle$$



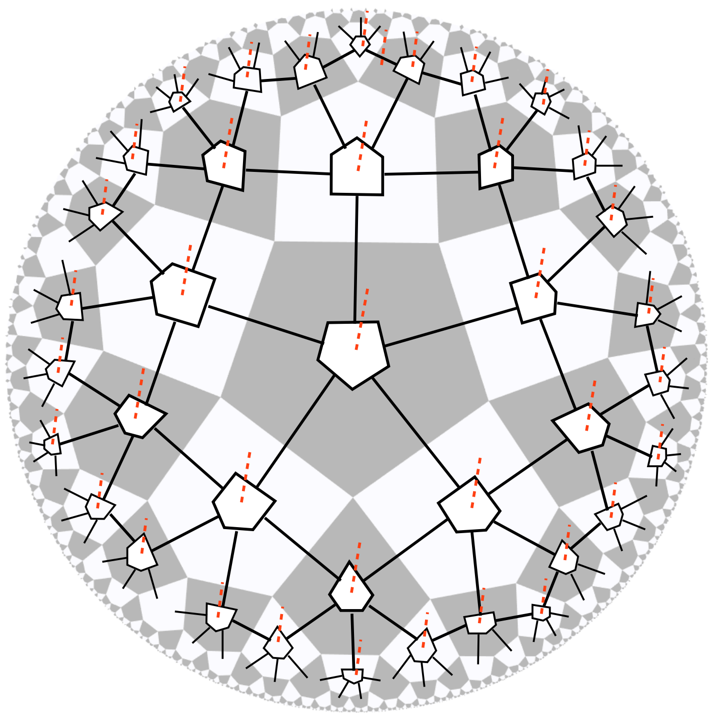


1970s

Black hole =

2010s

Black hole =



[1] Holographic quantum error-correcting codes

[2] Chaos in quantum channel,

[3] Complexity by design, arXiv:1609:xxxxx

Thank you !

String

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QI

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QI

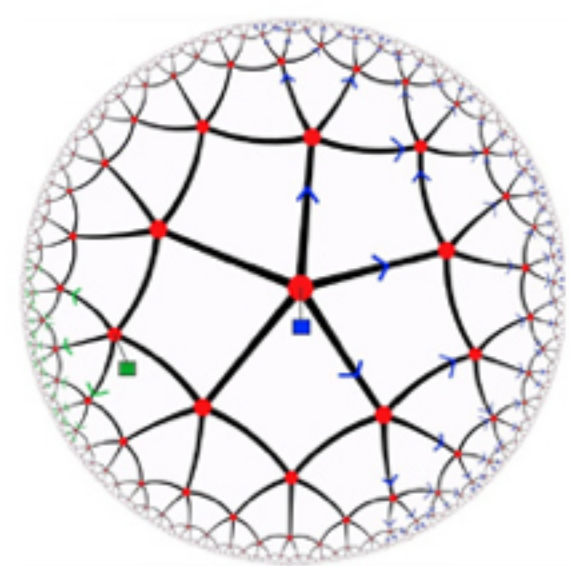
John Preskill (Caltech)

CMT

Xiao-liang Qi (Stanford)

String

Daniel Roberts (MIT→IAS)



Simons collaborations