Quantum Spectroscopy

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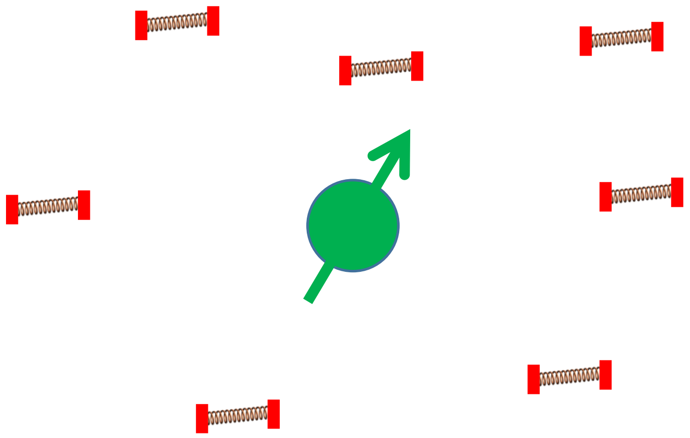
Dartmouth

1.Phys. Rev. Lett. 113, 250501 (2014)

2. Phys. Rev. Lett. 116, 150503 (2016)

3. In preparation

What can the measurable change in the quantum state of a system tell us about the "environment" it interacts with ?



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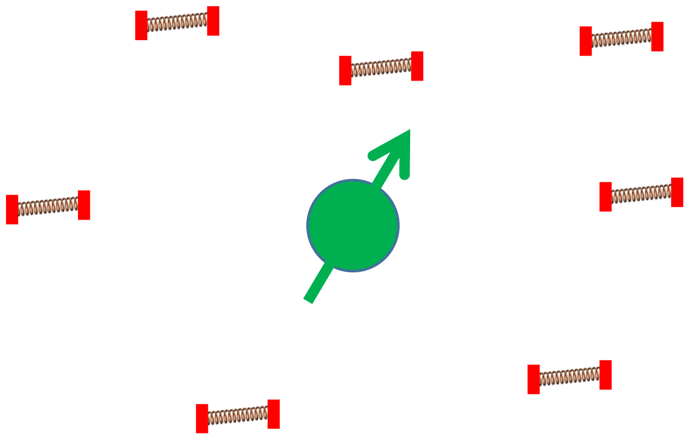

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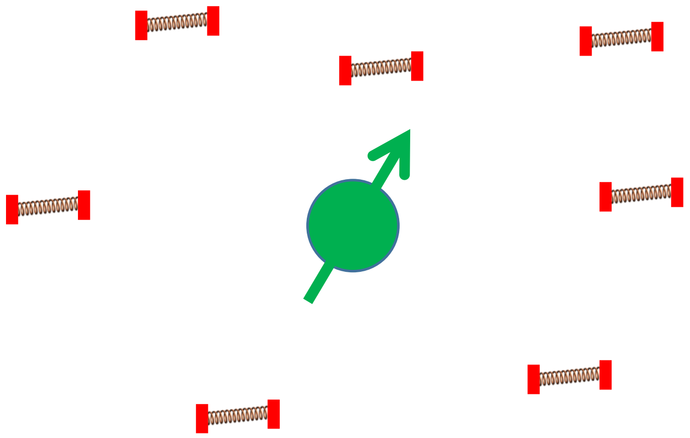

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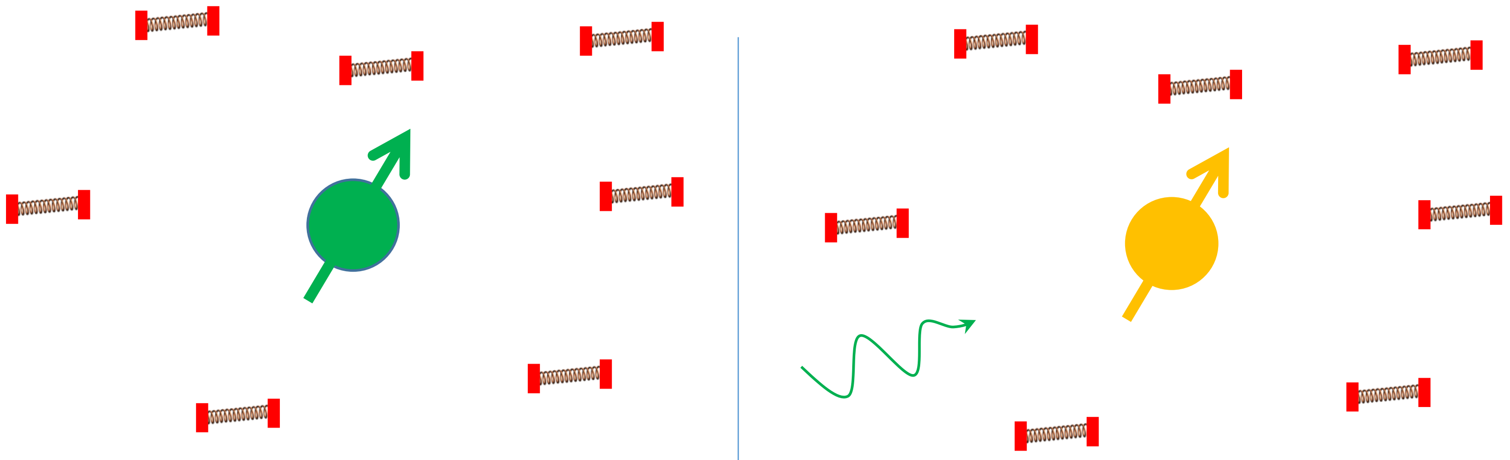

t=0: prepare a probe state on the qubit

t=0 to t =t_{final}: Qubit and "environment" evolve with joint Hamiltonian, <u>possibly in the presence of control.</u>

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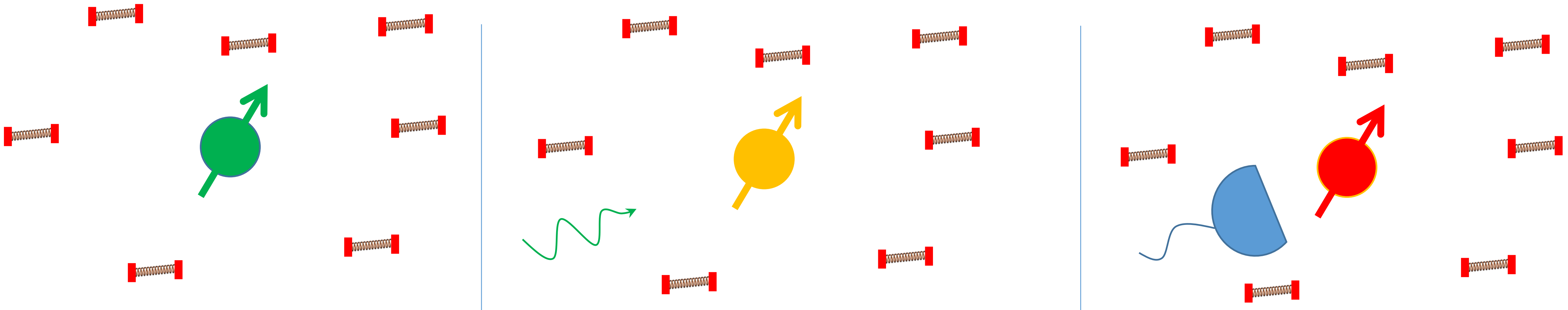

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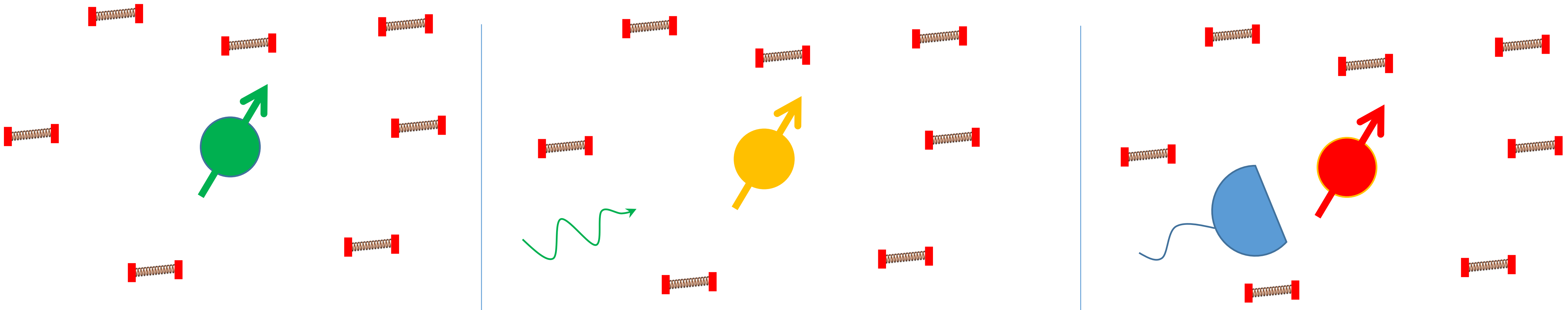

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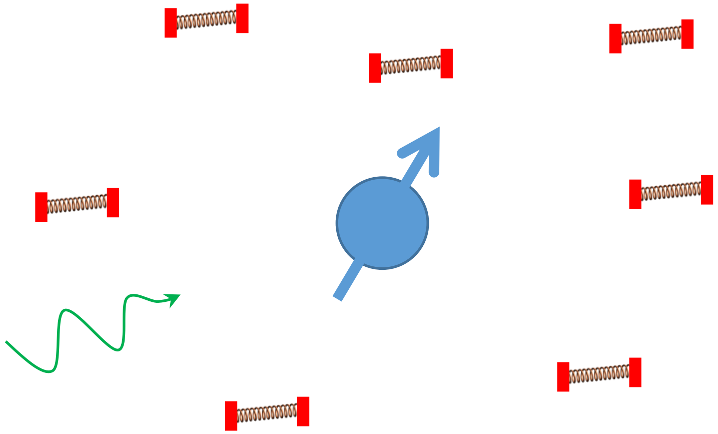
Simplest scenario → Magnetometry: $H = Z \times \Omega$

Lets start with an example…



$$H(t) = Z \otimes \underbrace{\sum_k (g^k a_k + (g^k)^* a_k^\dagger)}_{= B} + \sum_k \omega_k a_k^\dagger a_k$$

$$\rho(0) = \rho_S(0) \otimes \rho_B(0) = \rho_S(0) \otimes \frac{e^{-\beta H_B}}{\mathrm{Tr}[e^{-\beta H_B}]}$$

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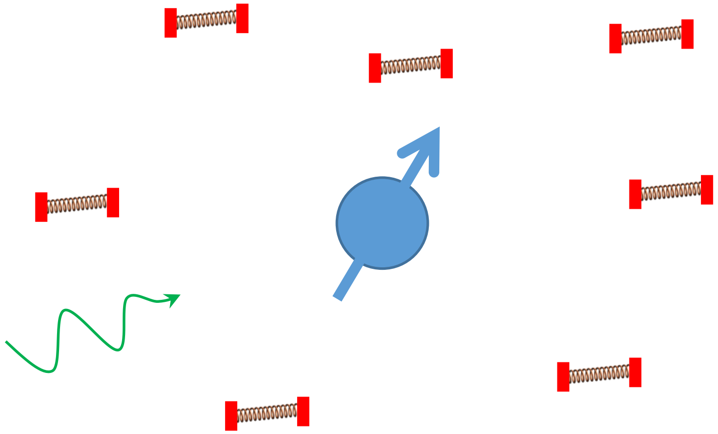

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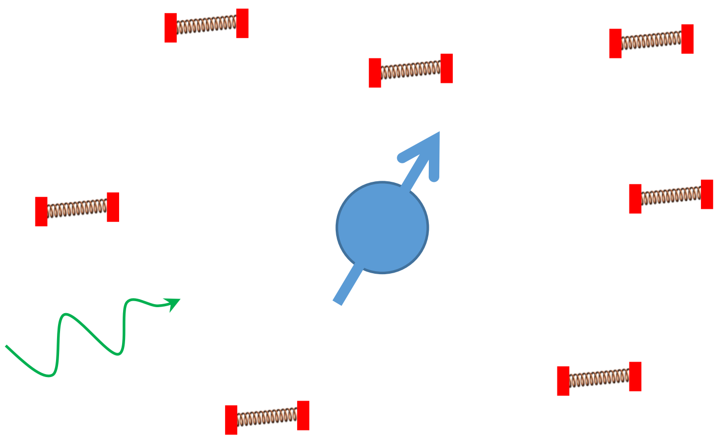

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What does the qubit feel ??

$$\rho_S = \frac{I+X}{2} \Rightarrow \qquad E[X(t)] = e^{-1/2 \int \frac{d\omega}{2\pi} |F^{(1)}(\omega,t)|^2 \langle B(\omega) B(-\omega)\rangle_q} E[X(0)]$$

$$\begin{cases} S(\omega) = \langle B(\omega) B(-\omega)\rangle_q = \mathrm{Tr}[B(\omega)B(-\omega)\rho_B] \\ |F(\omega,t)|^2 = |\int_0^T ds\, e^{i\omega s} y(s)|^2 \end{cases}$$

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What do we want?

$$H_P(t) = H_{\mathrm{ctrl}_Q}(t) + H_Q + H_{Q,B} + H_B$$

$$H(t) = \sum_{a,a'} y_{a,a'}(t) O_a \otimes B_{a'}(t)$$

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$$\Rightarrow$$

$$\langle B_{a_1}(\omega_1) \cdots B_{a_k}(\omega_k) \rangle_c$$

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But…why should we care?

The possibilities (i)…

- DECOHERENCE SUPRESSION (optimal control)

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Minimize decoherence applying two pulses at times t_1, t_2 in $[0, T = 1]$.

Find t_1, t_2 such that $1 - e^{-\chi}$ is minimized, with

$$\chi = 1/2 \int \frac{d\omega}{2\pi} |F^{(1)}(\omega, t)|^2 \langle B(\omega) B(-\omega) \rangle_q$$

$$S(\omega) = \langle B(\omega) B(-\omega) \rangle_q = e^{-\left(\frac{\omega - \omega_0}{\omega_c}\right)^2}$$

Constraints: Time resolution

$$\delta\tau = \frac{1}{32} T \rightarrow t_i = k\, \delta\tau$$

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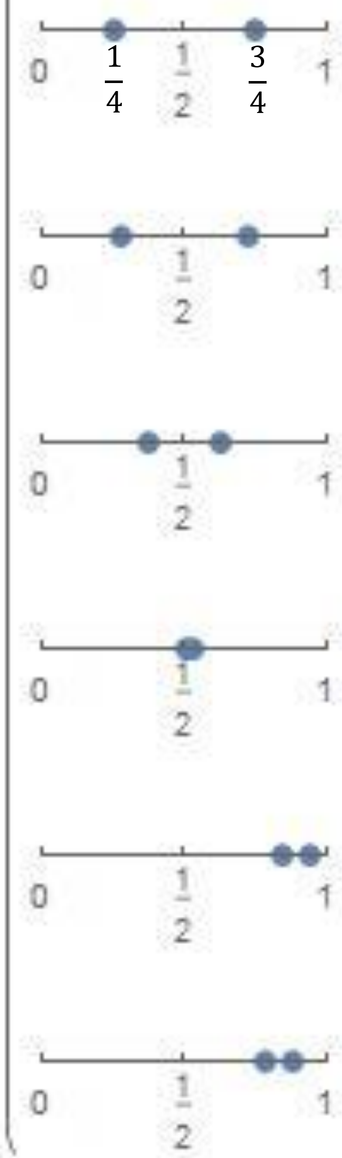
Optimal Timings



	$e^{-\chi}$ Optimal	$e^{-\chi}$ CDD_2
$\omega_0 = 0$	0.999399	0.999399
$\omega_0 = 2$	0.986826	0.961935
$\omega_0 = 4$	0.96393	0.757127
$\omega_0 = 6$	0.974726	0.520491
$\omega_0 = 8$	0.992534	0.495142
$\omega_0 = 10$	0.994877	0.714215

The possibilities(ii)…

- METROLOGY

Example: Magnetometry

Example: Thermometry

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$$S^+(\omega) = J(|\omega|) \coth(\beta|\omega|/2)$$

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Higher (non-Gaussian) correlations → more possibilities!

The necessity…

Simplified argument: Imagine all noise in your quantum computer comes from a bosonic thermal environment

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How can you verify this if the bath is usually inaccessible?

→ Noise Spectroscopy with multiple qubits!

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Ok... so how do we do it?

Before we start…Noise and correlations

Generalized cumulants [Kubo 62]

$$
\begin{aligned}
C^{(1)}(B(\omega)) &= \langle B(\omega)\rangle_q; \\
C^{(2)}(B(\omega_1)B(\omega_2)) &= \langle B(\omega_1)B(\omega_2)\rangle_q - \langle B(\omega_1)\rangle_q\langle B(\omega_2)\rangle_q \\
C^{(2)}(B(\omega_2)B(\omega_1)) &= \langle B(\omega_2)B(\omega_1)\rangle_q - \langle B(\omega_2)\rangle_q\langle B(\omega_1)\rangle_q \\
C^{(k>2)}(B(\omega_1)\cdots) &= \cdots
\end{aligned}
$$

$$
\langle\cdot\rangle_q \equiv \overbrace{\langle \underbrace{\mathrm{Tr}[\,\cdot\,\rho_B]}_{\text{Quantum}} \rangle}^{\text{Classical}}
$$

In general… all cumulants are non-vanishing.

Special case: Gaussian noise → Only $C^{(1)}$ and $C^{(2)}$ are non-vanishing
(but they can have any functional form)

Stationary noise: $C^{(k)}(B(\omega_1)\cdots B(\omega_k)) = \delta(\omega_1+\cdots+\omega_k)\times\mathcal{F}(\omega_1,\cdots,\omega_k)$

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Paz & Viola PRL 2014

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$$E(O(T)) = \mathcal{F}(\{\textstyle\int d\vec{\omega}\, G^{(k)}_{\vec{a},\vec{b}}(\vec{\omega},T) S^{(k)}_{\vec{b}}(\vec{\omega})\})$$

Can be formally written as a Cumulant-like power series of convolutions of…

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Generalized filter Functions:

$$G^{(\alpha)}_{\vec{a}\vec{b}}(\vec{\omega},T) = \mathcal{F}(F^{(\alpha)}_{\vec{a}\vec{b}}(\vec{\omega},T))$$

Fundamental filter Functions:

$$F^{(k)}_{\vec{a},\vec{b}}(\vec{\omega}) = \int_0^t ds_1 \int_0^{s_1} ds_2 \cdots \int_0^{s_{k-1}} ds_k \prod_j \Big(y_{a_j b_j}(s_j)\Big) e^{i\vec{\omega}\cdot\vec{s}}$$

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Power Poly spectra:

$$S^{(k)}_{b_1,\cdots,b_k} = \langle B_{b_1}(\omega_1) \cdots B_{b_k}(\omega_k) \rangle_c$$

Paz & Viola PRL 2014

Step 2: How to deconvolve the integrals?

- Isolate, Truncate and Discretize

$$\Psi = \int_{-\Omega}^{\Omega} d\vec{\omega} G(\vec{\omega}, T) S(\vec{\omega}) \rightarrow \Psi \sim \sum_{r \in \Lambda} G_r S_r$$

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Solve linear system of equations to obtain $\{S_r\}$

→ 'Reconstruct' $S(\omega)$ in the chosen basis!

Simplest case: one qubit dephasing Gaussian noise

$$\rho_S = \frac{I+X}{2} \Rightarrow \quad E[X(t)] = e^{-1/2\int \frac{d\omega}{2\pi}|F^{(1)}(\omega,t)|^2 S(\omega)} E[X(0)]$$

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- Control repetition: $y_{a,a'}(t) = y_{a,a'}(t+T_p)$

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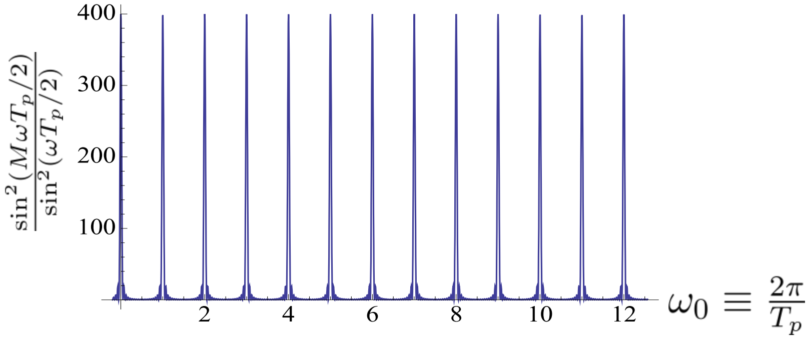

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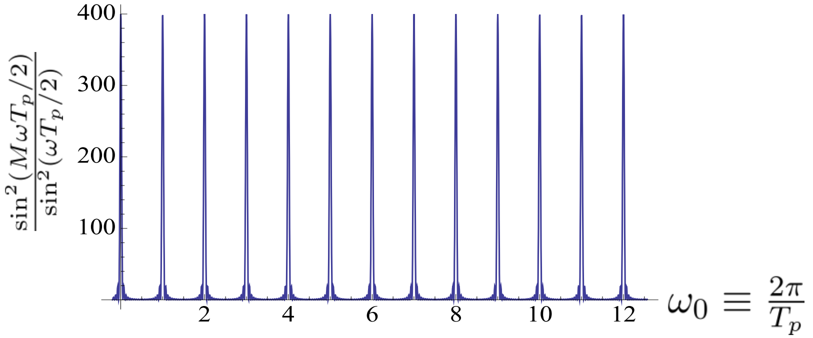

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[Alvarez & Suter PRL 09]: Fixed Sequence, variable cycle time: $y(t)\colon \{CDD_2\left(T_p = \frac{T_{max}}{k} \geq 4\tau_0\right)\}$

[Norris, Paz-Silva, Viola PRL 16]: Variable sequence, variable cycle time

→ more reconstruction power: limited only by time resolution

Moving on: what about beyond Gaussian noise?

$$H(t) = y(t)/2Z \otimes \underbrace{B(t)}_{\text{Classical or bosonic}} \qquad E[X(t)] = \langle \mathrm{Tr}[e^{-iZ\int d\omega F^{(1)}(\omega,t)B(\omega)}\rho_S\, X \otimes \rho_B]\rangle \qquad \rho_S = \frac{I+X}{2}$$

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Generalized cumulants [Kubo 62]

$$C^{(1)}(B(\omega)) = \langle B(\omega) \rangle_q;$$
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A higher dimensional frequency comb ??

Once more unto the breach…

$y(t)$ periodic w/ period T_p → Repeat a 'composed' sequence

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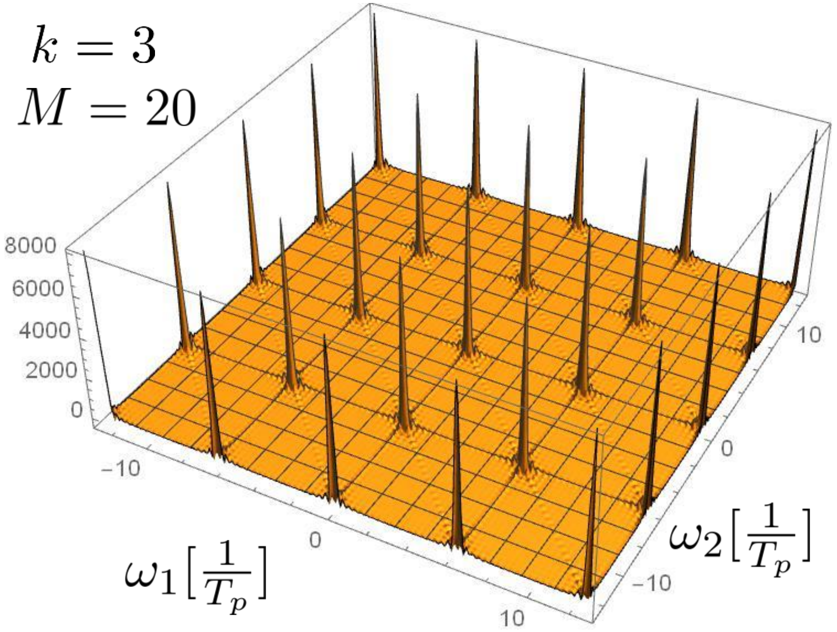
$k = 3$

$M = 20$



$$\sim 2\pi M\prod_{s=1}^{k-1}\left(\frac{2\pi}{T_p}\delta(\omega_s - r_s\omega_0)\right)\delta(\omega_1+\cdots+\omega_k)$$

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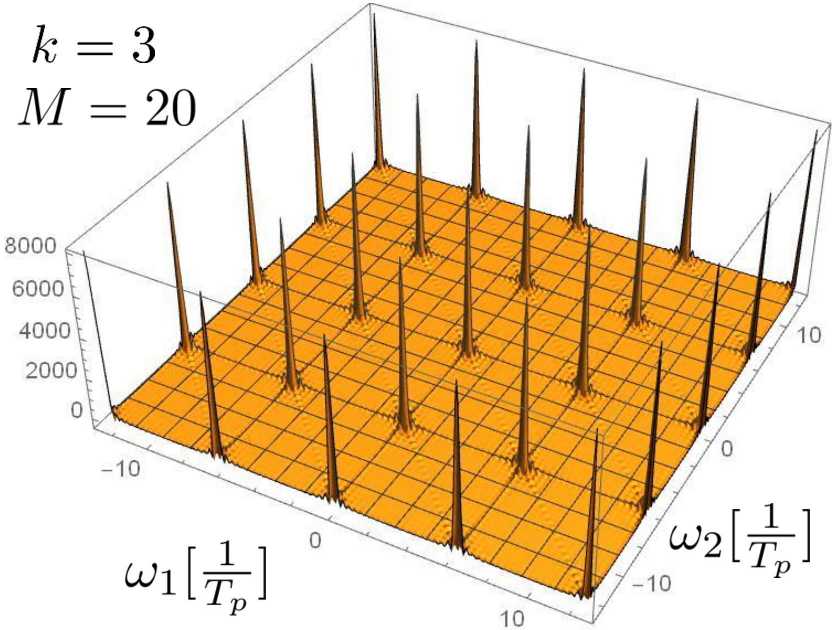

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Frequency comb in multiple dimensions.

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One can now reconstruct $S^{(k<K)}(\vec{r}\omega_0)$!!

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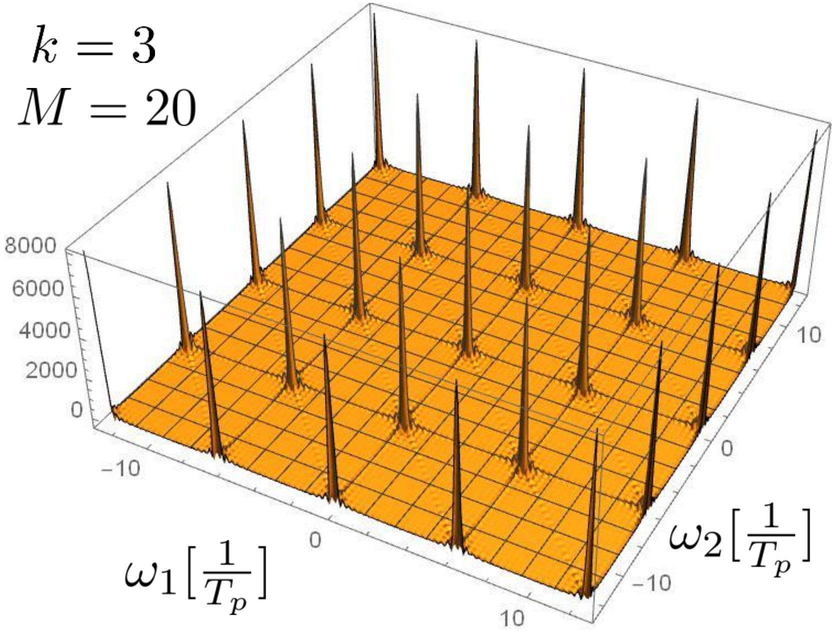

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→ Many variables, so one must be careful with 'numerical' issues (condition number, etc.).

Take a breath slide…

- We have shown: single qubit spectroscopy of dephasing Gaussian and non-Gaussian, classical & quantum (bosonic), noise.

What about the noise affecting multiple qubits?

Is there an advantage of using more than one probe?

Multiqubit spectroscopy

Multiqubit spectroscopy

General dephasing model: $H(t) = \sum_{a,b} y_{a,b}(t) Z_a \otimes B_b(t)$

plus widely Gaussian, zero-mean, stationary noise : $Z_a O = f_a^O O Z_a;\qquad f_a^O = \pm 1$

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with:

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Multiqubit spectroscopy (ii)

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- Control repetition: $$y_{a,a'}(t) = y_{a,a'}(t+T_p)$$
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- Mirror (anti) symmetry: $$y_{a,a'}(\frac{T_p}{2}-t)y_{b,b'}(\frac{T_p}{2}-t') = \pm y_{a,a'}(\frac{T_p}{2}+t)y_{b,b'}(\frac{T_p}{2}+t')$$

Multiqubit spectroscopy (iii)

Need a larger set of initial states and observables

State	Observables
$\rho_\ell^{(\vec{s})} = \frac{\mathbf{1}_\ell + X_\ell}{2} \otimes_{r\neq\ell} \frac{\mathbf{1}_r + s_r Z_r}{2}$	X_ℓ, Y_ℓ
$\rho_{\ell,\ell'}^{(\vec{s})} = \frac{\mathbf{1}_\ell + X_\ell}{2} \otimes \frac{\mathbf{1}_2 + X_2}{2} \otimes_{r\neq\ell,\ell'} \frac{\mathbf{1}_r + s_r Z_r}{2}$	$X_\ell X_{\ell'}, Y_\ell Y_{\ell'}$ $Y_\ell X_{\ell'}, X_\ell Y_{\ell'}$

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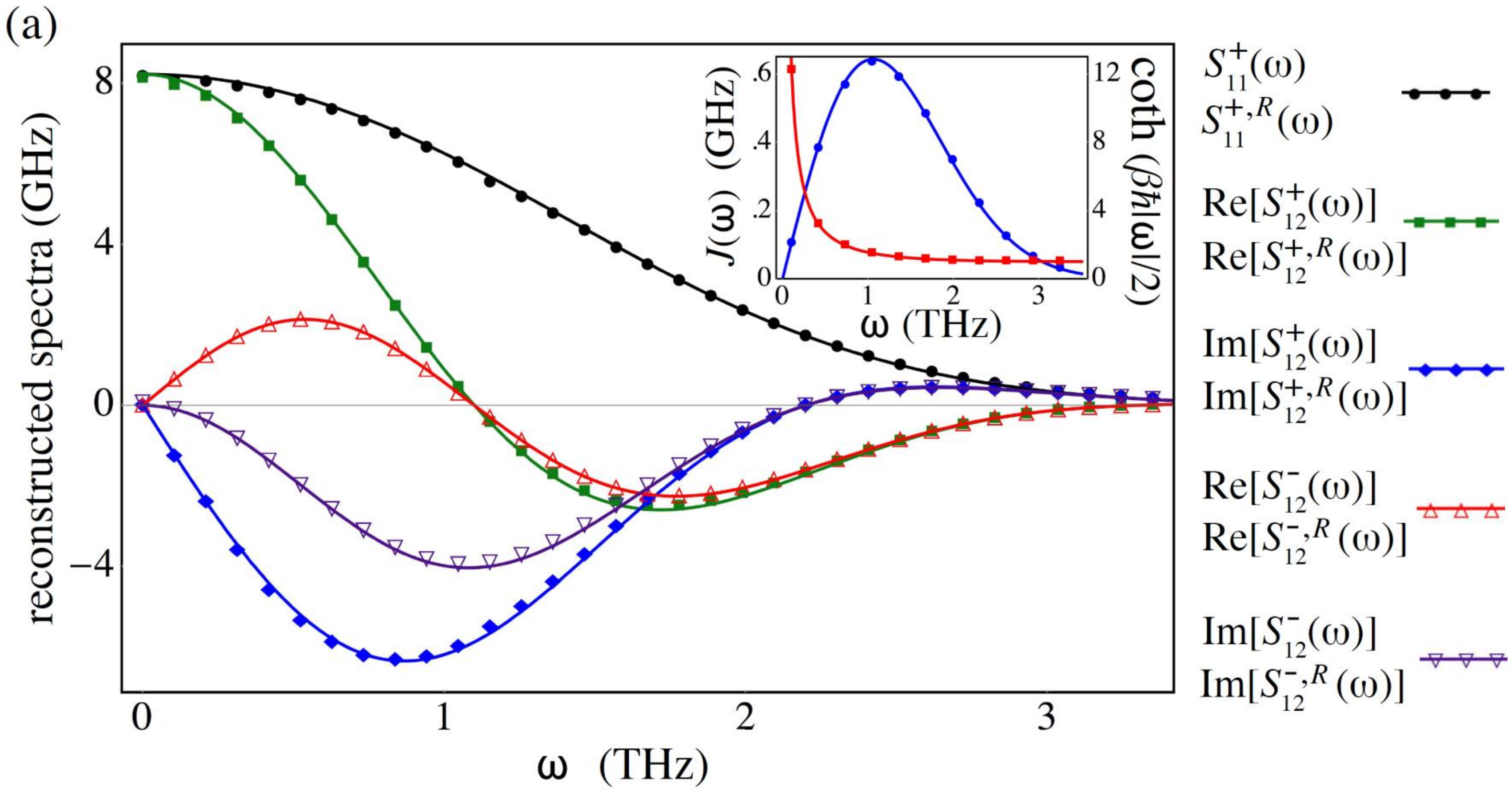
Numerical Experiment

Two excitons in a bosonic thermal environment



→ thermometry, verify decay of correlations with distance, etc.

Conclusions and Outlook

Characterizing correlations in the bath is

- Possible (dephasing multiqubit)
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- The ultimate goal: Self-calibrating quantum devices.

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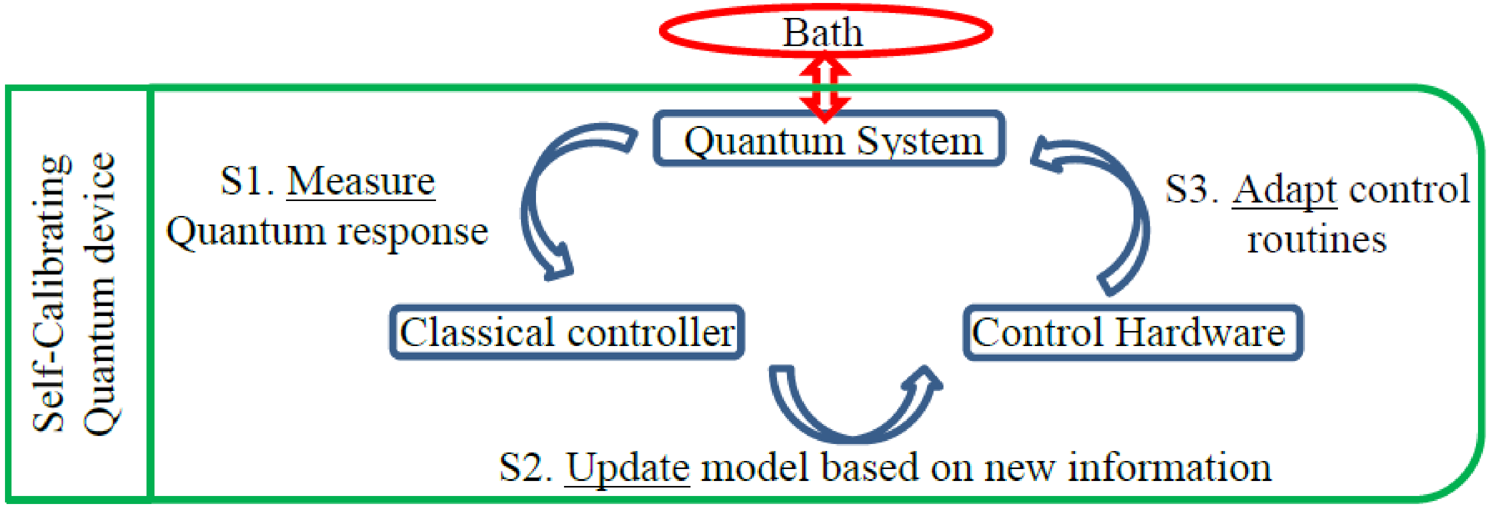

Some numerical ‘experiments’…

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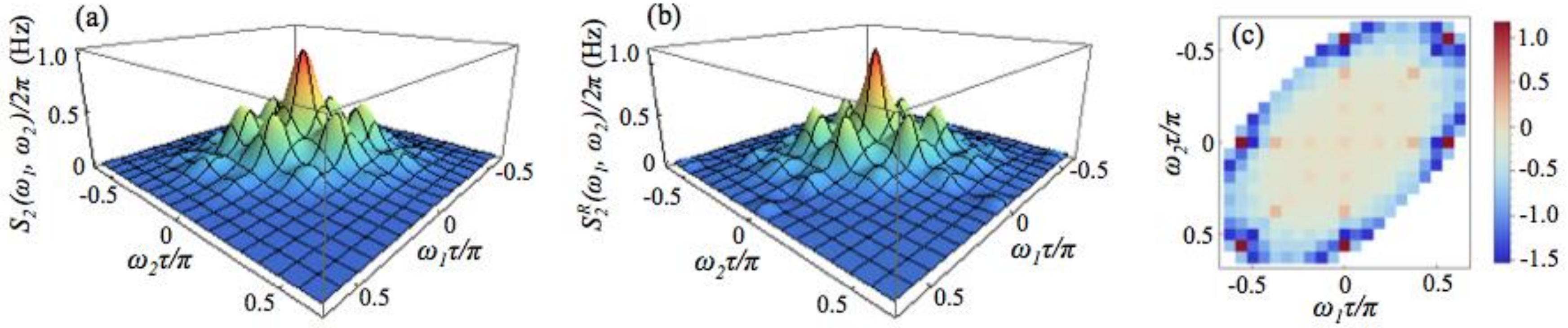

Fig. (Actual, Reconstructed, Relative Error) $\beta(t)^2$ classical noise , $M = 40$, 325 points in the reconstruction of the bispectrum. Sequences chosen from the CDD_{0-5}

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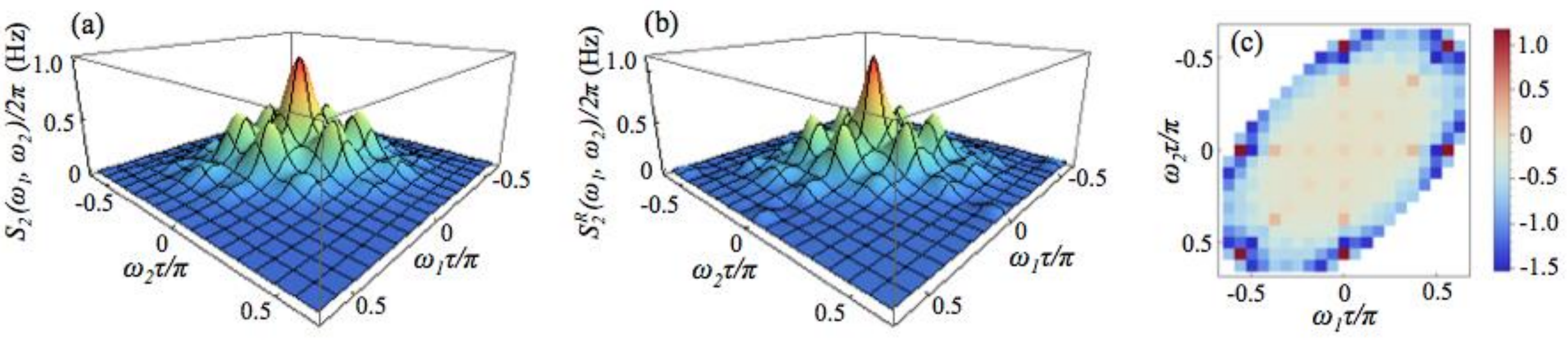

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QUANTUM

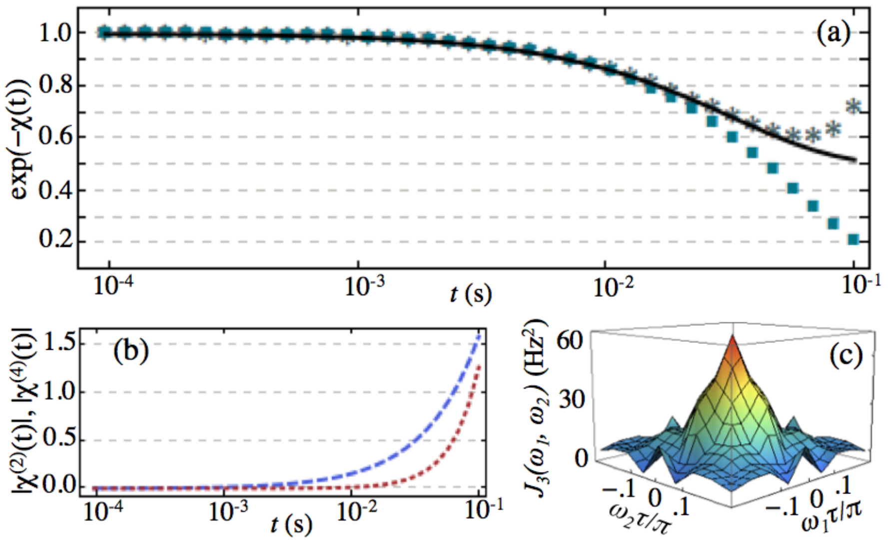


Fig. Bosonic bath, $\rho_B(0) = \frac{1}{2}(\rho_\beta + \rho_{\beta'})$

$$S_3(\vec{\omega}) \propto \delta(\omega_1 + \omega_2) J_3(\omega_1, \omega_3) + \delta(\omega_2 + \omega_3) J_3(\omega_1, \omega_2) + \delta(\omega_1 + \omega_3) J_3(\omega_2, \omega_3)$$

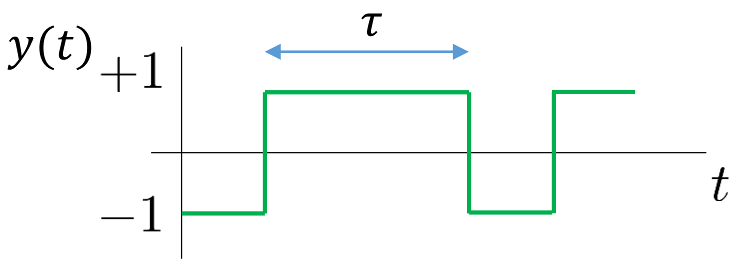
(a) Predicted dynamics with Gaussian, first non-Gaussian contribution (solid is theory); (b) relative magnitude of higher order contributions; (c) reconstructed

Simplest case (ii): one qubit dephasing Gaussian noise

[Norris, Paz-Silva, Viola PRL 16]

$$y(t)\colon \{CDD_{\delta_1}(T_1) \oplus \cdots \oplus CDD_{\delta_n}(T_n)\}$$

$$T_1 + \cdots + T_n = T_p \;\; ; \;\; T_s \geq 2^{\delta_s}\tau_0$$



$$\tau = k\delta\tau \geq \tau_0$$

Time resolution & minimum switching time constraints

- Maximum ‘sampling’ frequency:

$$\frac{2\pi}{\delta\tau} > \frac{\pi}{2\tau_0}$$

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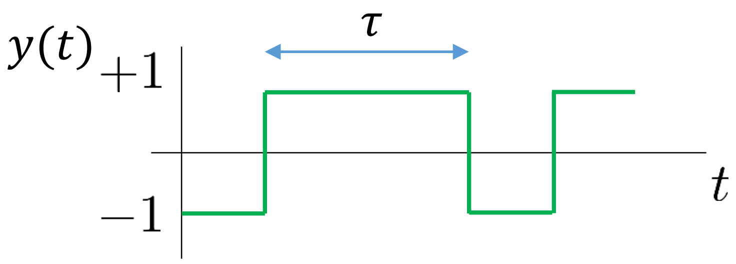

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Fig. $\beta(t)$ classical noise: •: Our sampling; ◊: A&S sampling; — : theoretical spectrum

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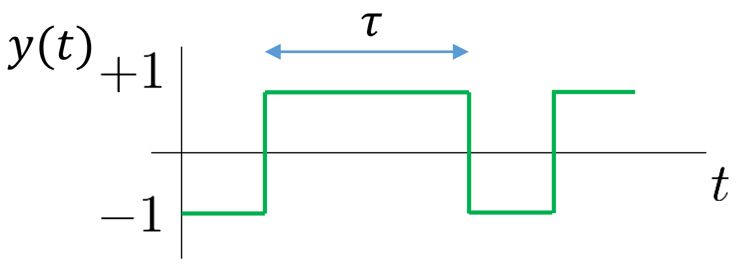
$$y(t): \{CDD_{\delta_1}(T_1) \oplus \cdots \oplus CDD_{\delta_n}(T_n)\}$$

$$T_1 + \cdots + T_n = T_p \;\; ; \;\; T_s \geq 2^{\delta_s} \tau_0$$



$\tau = k\delta\tau \geq \tau_0$

Time resolution & minimum switching time constraints

- Maximum ‘sampling’ frequency:

$$\frac{2\pi}{\delta\tau} > \frac{\pi}{2\tau_0}$$

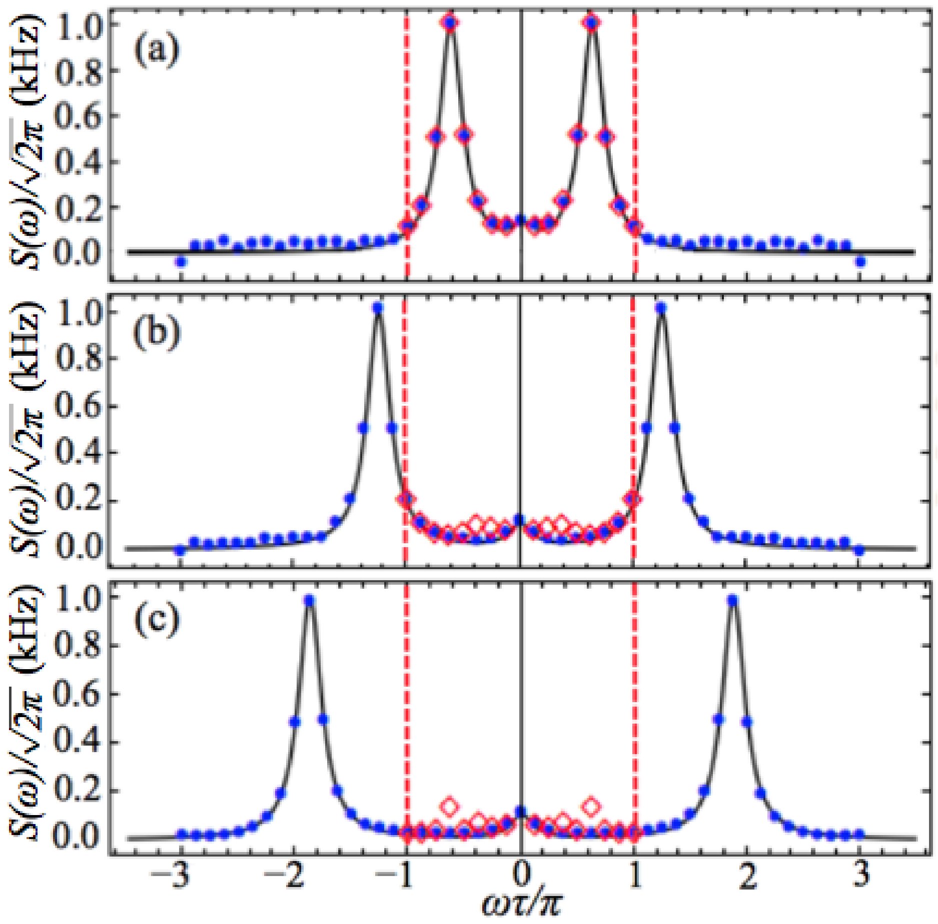


Fig. $\beta(t)$ classical noise: •: Our sampling; ◊: A&S sampling; — : theoretical spectrum