Symmetry-protected topologically ordered states for universal quantum computation

Hendrik Poulsen Nautrup,
Tzu-Chieh Wei

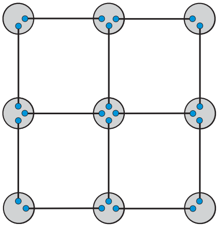
09/01/2016

Outline

Measurement-based QC in VBS formalism

TBQC

- Information processing by joint measurements
- Resource: Suitably entangled state



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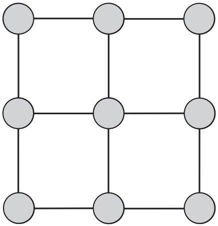

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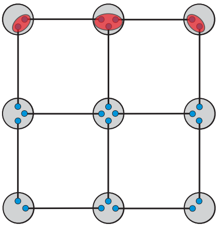

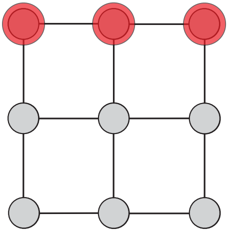

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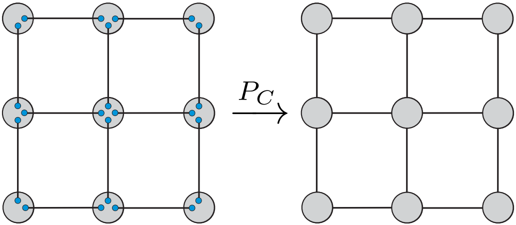
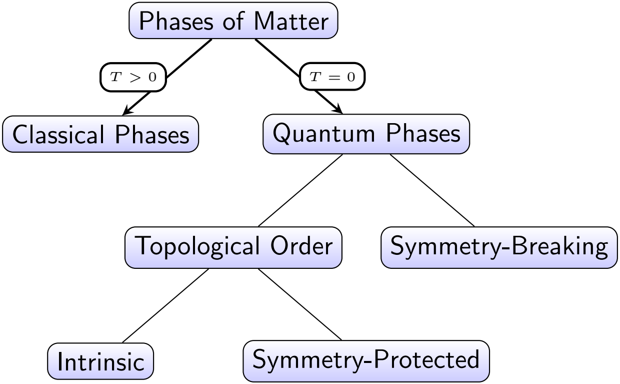


Quantum Order in Perspective



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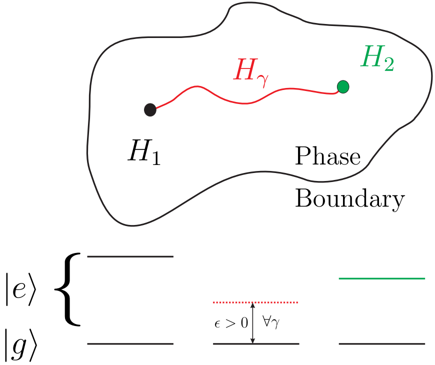
$\rightarrow$ Phase transitions are singularities in the ground state energy



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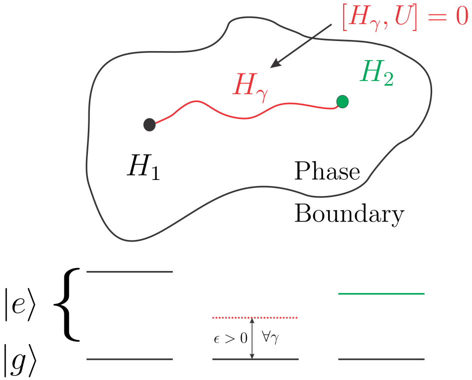

Protection of Quantum Gates in 1D SPT Phases

Theorem

The 1D SPT phase protected by S_4 symmetry allows us to efficiently implement all one-qubit unitary gates with arbitrary high gate fidelity (if some characteristic lengths are finite)

Miller et al (2015), PRL 114, 120506

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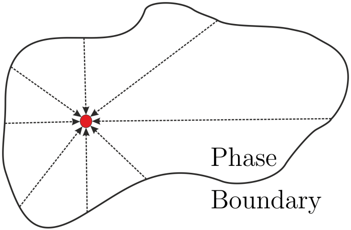

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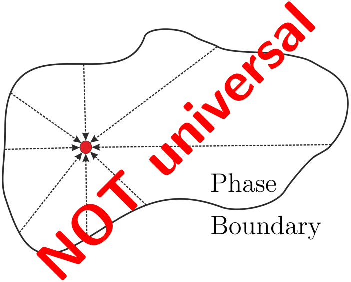

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Is there a

nontrivial SPTO state

that is

universal for QC

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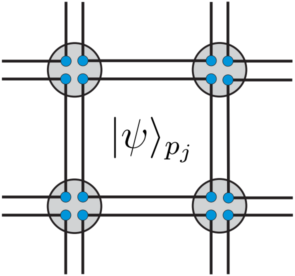
Plaquette States

- Assume finite symmetry group G (e.g. Z_d)



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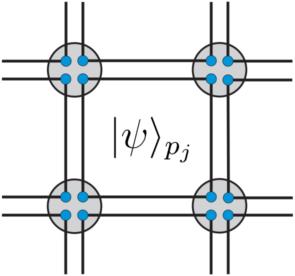

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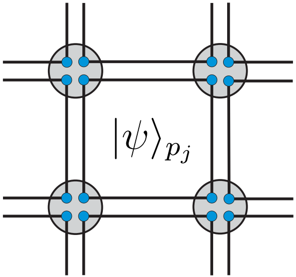
$$|\psi_{gs}\rangle = \otimes_j \sum_{g \in G} |\alpha_1 = g, \beta_2 = g, ..., \zeta_k = g\rangle_{p_j}$$



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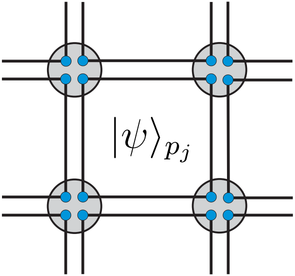

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Plaquette States exhibit nontrivial SPT order w.r.t. some symmetry fractionalization.

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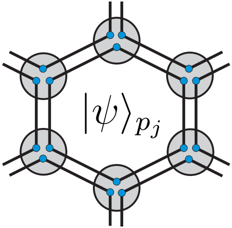

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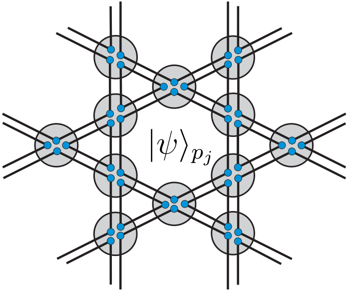

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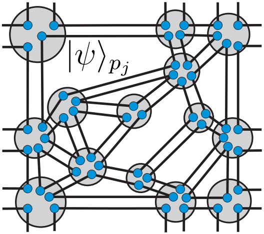

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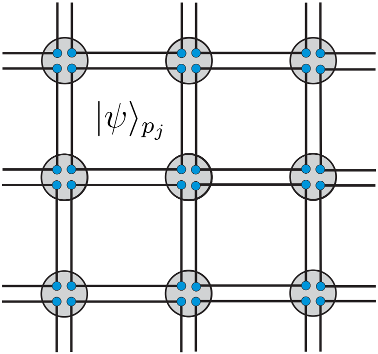
Reduction to Bond States

- Apply measurement

$$M_s(m_1, ..., m_4) = \prod_{k=1}^{4} Z_k^{m_k} \left|+\right\rangle_k \left\langle+\right|_k Z_k^{m_k}$$

with measurement outcomes
$m_1, ..., m_4 \in \{0, 1\}$



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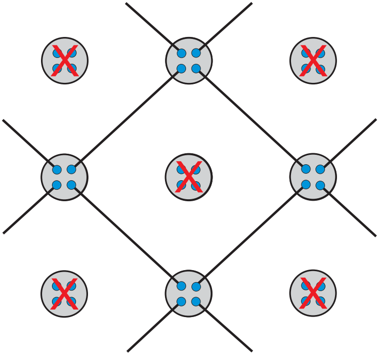
$$\left|B\right\rangle = \left|00\right\rangle + \left|11\right\rangle / \sqrt{2}$$



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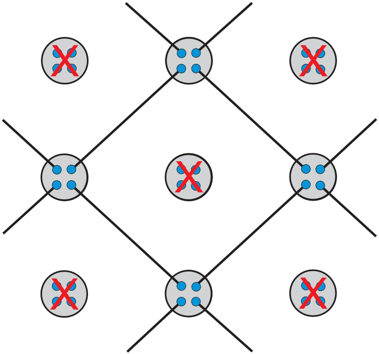

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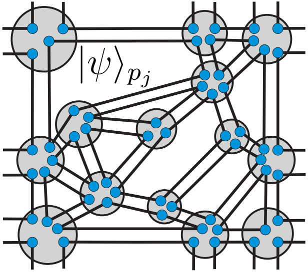

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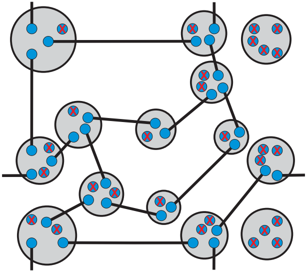

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From Plaquettes to Bonds

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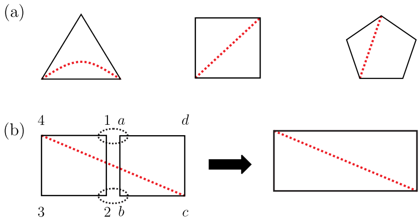

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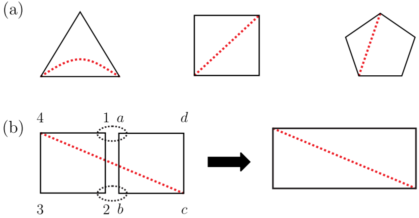
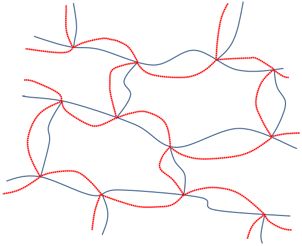
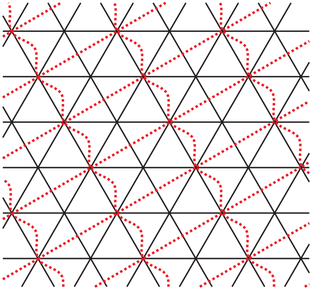


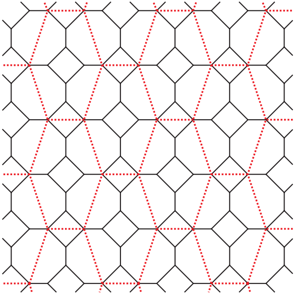
- Within supercritical phase of percolation, ∃ minor graph that is topologically equivalent to honeycomb lattice



Browne et al (2008), NJP 10, 023010

Examples





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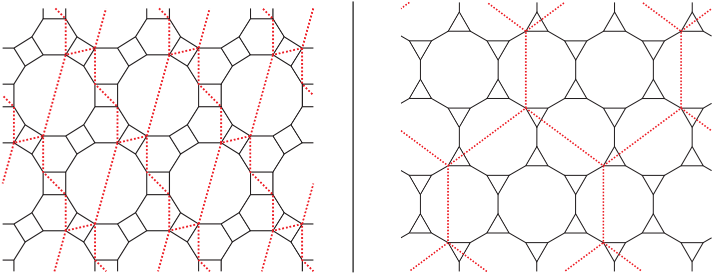

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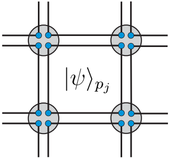
- Introduced  - states on arbitrary lattices

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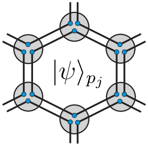
- Introduced  - states on arbitrary lattices

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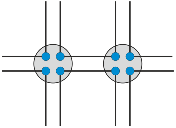
- Introduced  - states on arbitrary lattices

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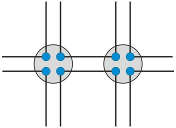
- Introduced  - states on arbitrary lattices
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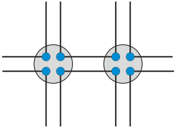
- Introduced  - states on arbitrary lattices
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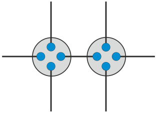
- Reducible to  - states

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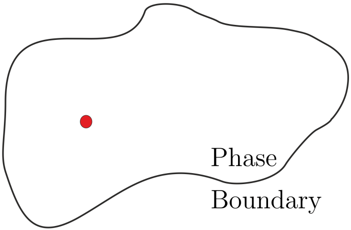

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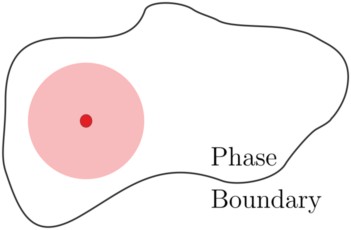

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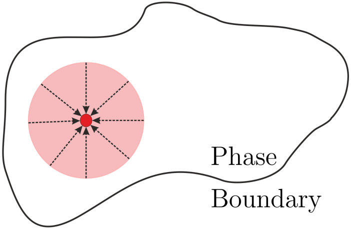

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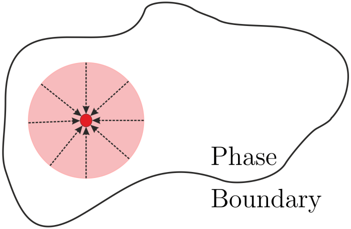

Yoshida (2016), arXiv:1509.03626

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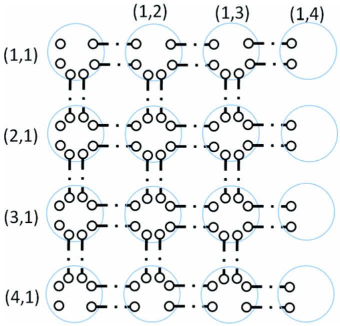

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Chiu et al. (2013), PRA 87, 012305

Thank You