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Universal Quantum Computing with Arbitrary Continuous-Variable Encoding

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Stiftung/Foundation

QUCHIP
Quantum Simulation on a Photonic Chip

EQuaM

arXiv:1605.09278 (to appear in PRL)

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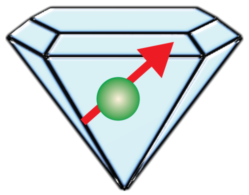
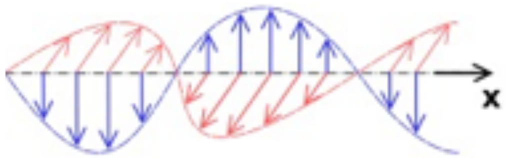
Qubits

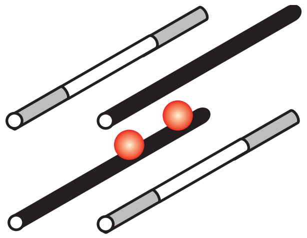
Discrete-Variable



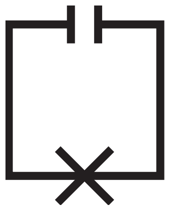
Diamond NV centers



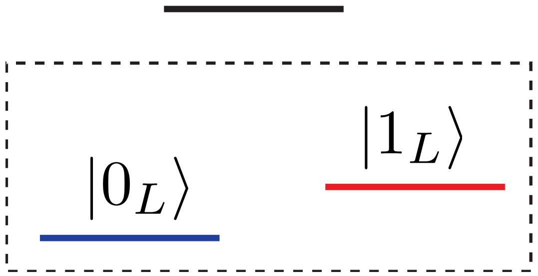
Photon Polarisation



Trapped Ion

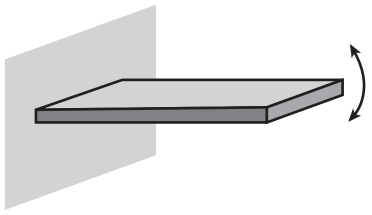


Superconducting Circuit

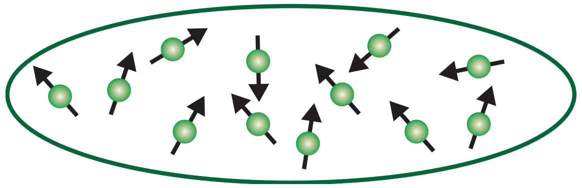


Finite individually addressable states

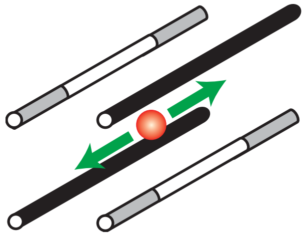
Continuous-Variable



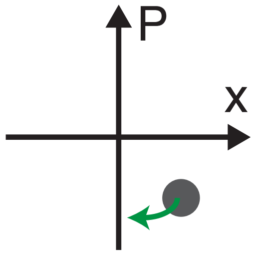
Mechanical Oscillator



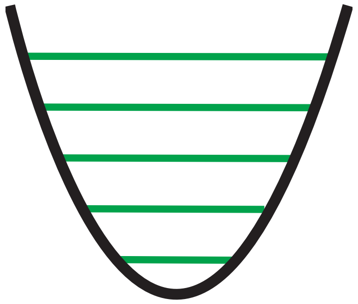
Spin Ensemble



Ion Motion



Optical Mode



Infinite non-individually addressable states

$|0_L\rangle$?

$|1_L\rangle$?

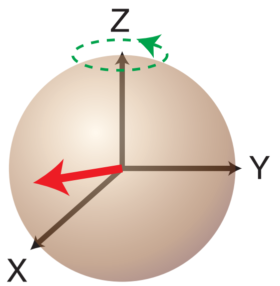
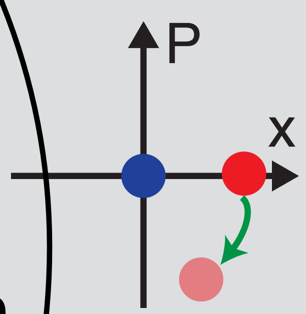
Qubit Encoding

	$\|0_L\rangle$	$\|1_L\rangle$	Pros	Cons
Fock	$\|0\rangle$	$\|1\rangle$	Linear optics gate	Probabilistic entanglement gate
Coherent	$\|0\rangle$	$\|\alpha\rangle$	Easy initialisation	Slow gate
Cat state	$\|C_0\rangle =$ $\|\alpha\rangle + \|-\alpha\rangle$	$\|C_1\rangle =$ $\|i\alpha\rangle + \|-i\alpha\rangle$	Loss tolerant	Slow gate
Gottesman-Kitaev-Preskill (GKP)	$\|G_0\rangle =$ $\sum_{n=-\infty}^{\infty} \|2n\rangle_q$	$\|G_1\rangle =$ $\sum_{n=-\infty}^{\infty} \|2n+1\rangle_q$	Fault tolerant, homodyne detection	Unphysical state

Infinitely squeezed state

many more…

Logic gate

	$\|0_L\rangle$	$\|1_L\rangle$	Phase-shift gate
Fock	$\|0\rangle$	$\|1\rangle$	Apply $H = \nu \hat{a}^\dagger \hat{a}$ $\|0\rangle \rightarrow \|0\rangle$ $\|1\rangle \rightarrow e^{-i\phi}\|1\rangle$ 
Coherent	$\|0\rangle$	$\|\alpha\rangle$	$\|0\rangle \rightarrow \|0\rangle$ $\|\alpha\rangle \rightarrow \|\alpha e^{-i\phi}\rangle \neq e^{-i\phi}\|\alpha\rangle$ Outside computational subspace 
Cat state	$\|C_0\rangle$	$\|C_1\rangle$	
GKP	$\|G_0\rangle$	$\|G_1\rangle$	

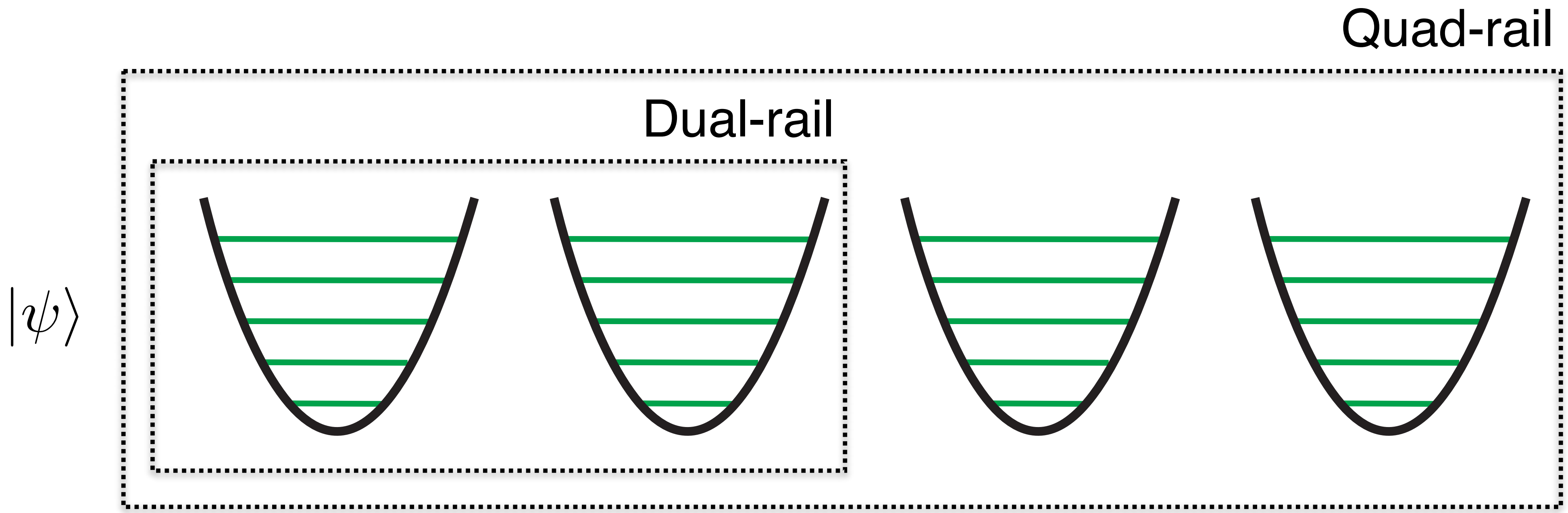
Each encoding **conventionally** needs a specific quantum computer architecture

Question: Are there unified logic gate operations that work for any encoding?

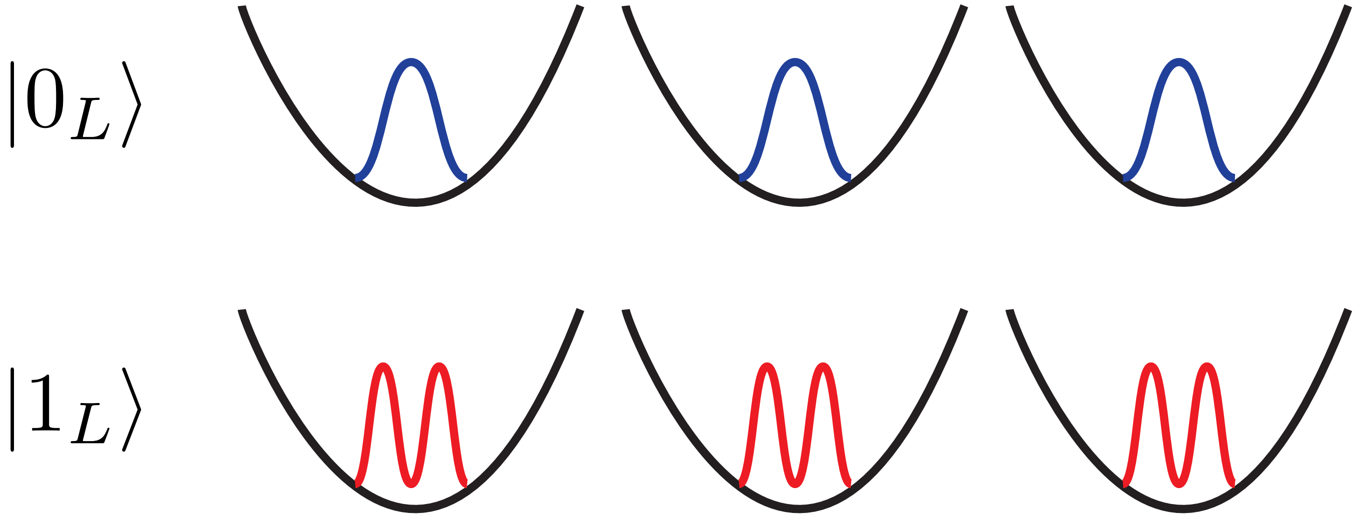
Answer: Yes*

* tricks may be applied

Trick



Encode one qubit by two or four modes

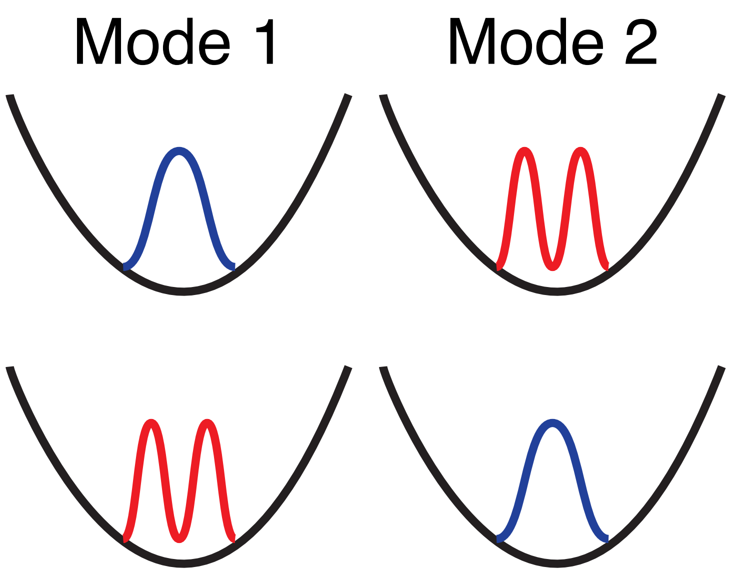


Encoding specified by preparing orthogonal basis states

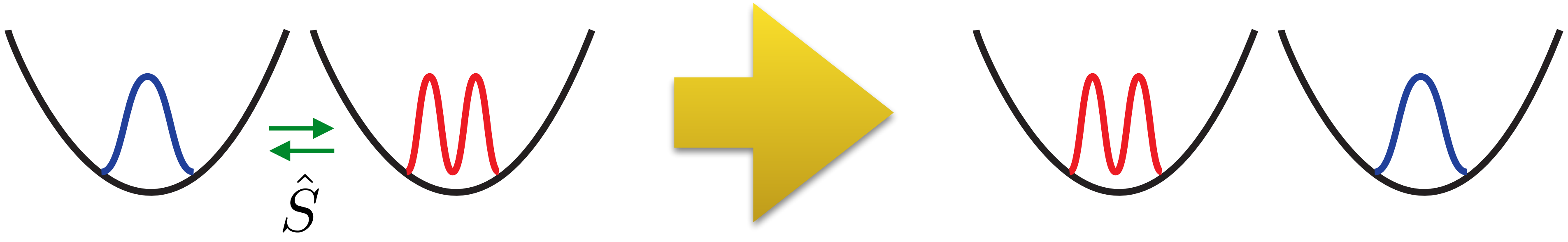
Dual-rail encoding

$$|0_D\rangle \equiv |0_L\rangle|1_L\rangle$$

$$|1_D\rangle \equiv |1_L\rangle|0_L\rangle$$



Swap operator



$$\hat{S}|0_D\rangle = |1_D\rangle \qquad \hat{S}|1_D\rangle = |0_D\rangle$$

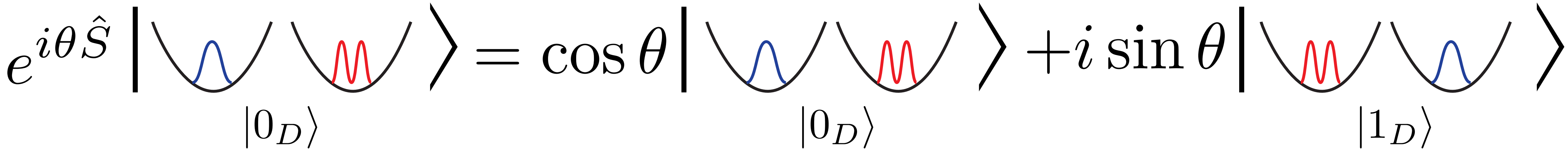
$\hat{S} = \hat{X}_D$ Pauli *X* operator

Swap test = *X*-basis measurement

Exponential-swap

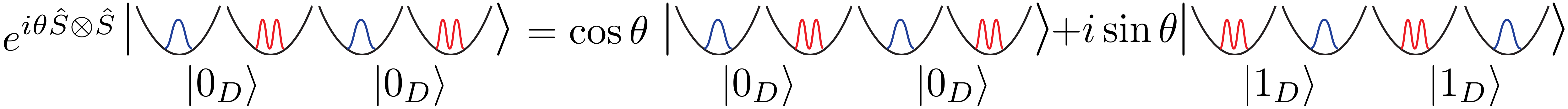
X-axis rotation

$$e^{i\theta\hat{S}} = \cos\theta\hat{\mathbb{I}} + i\sin\theta\hat{S}$$

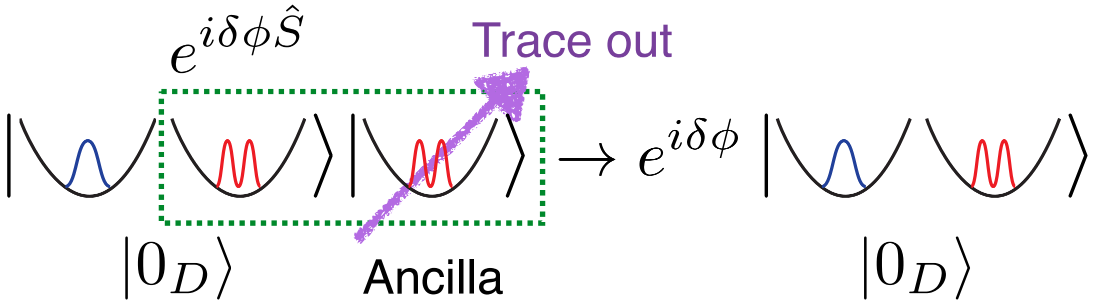


Entanglement gate

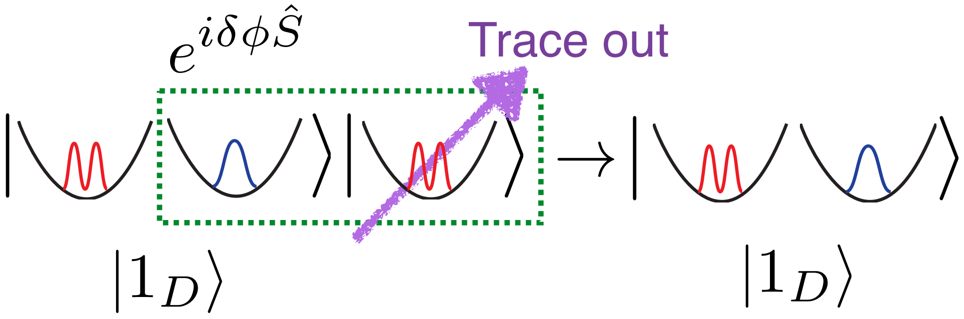
$$e^{i\theta\hat{S}\otimes\hat{S}} = \cos\theta\hat{\mathbb{I}} + i\sin\theta\hat{S}\otimes\hat{S}$$



Quantum Machine learning



Z-axis rotation



CV Machine Learning (Lau *et. al.* arXiv/quant-ph:1603.06222)

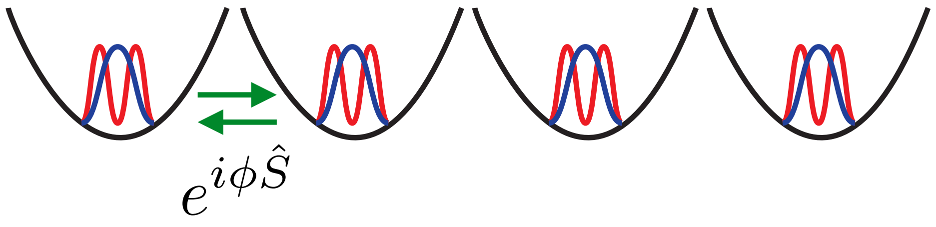
Quad-rail encoding

$$|0_Q\rangle = (|0_L 1_L\rangle + |1_L 0_L\rangle)(|0_L 1_L\rangle - |1_L 0_L\rangle)$$

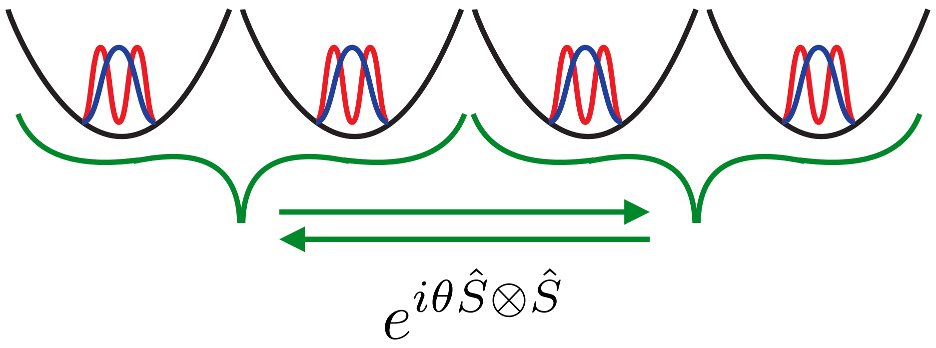
$$|1_Q\rangle = (|0_L 1_L\rangle - |1_L 0_L\rangle)(|0_L 1_L\rangle + |1_L 0_L\rangle)$$

No auxiliary mode is needed!

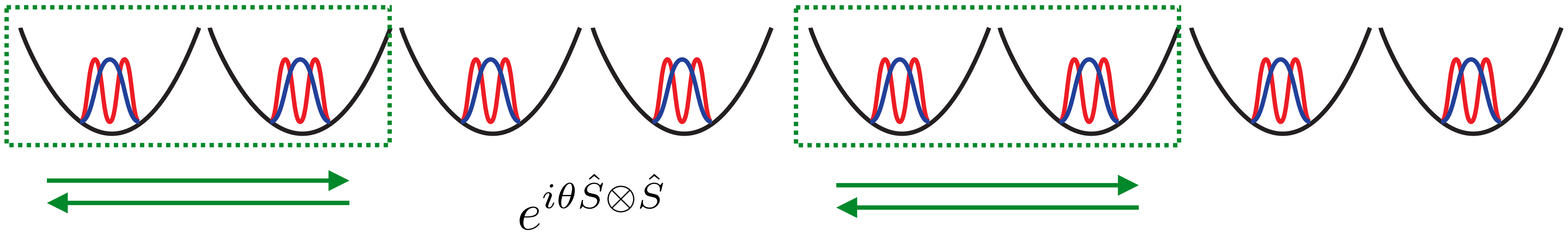
Z-axis rotation



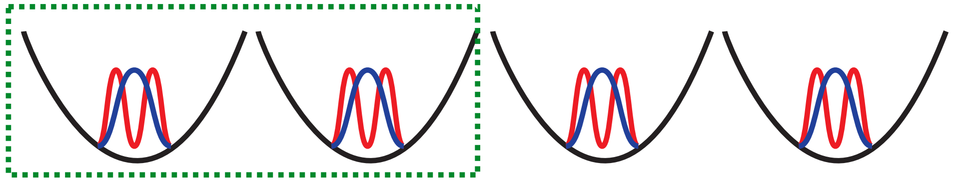
X-axis rotation



Entanglement gate



Z-axis measurement

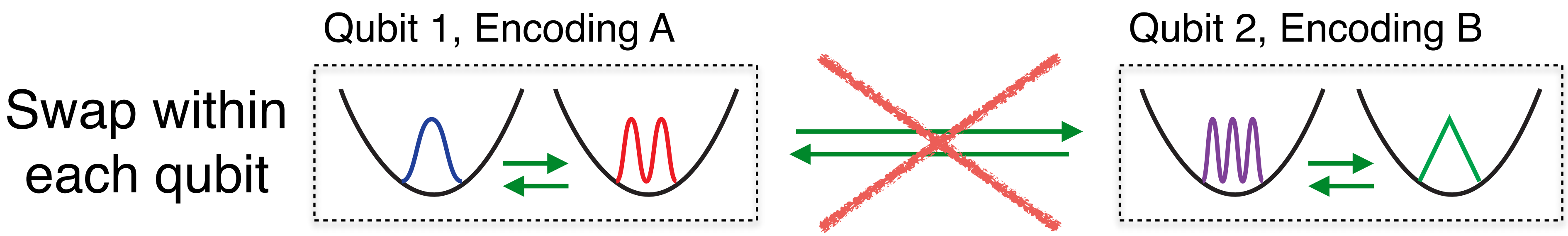


Benefit 1: Encoding-Independent gates

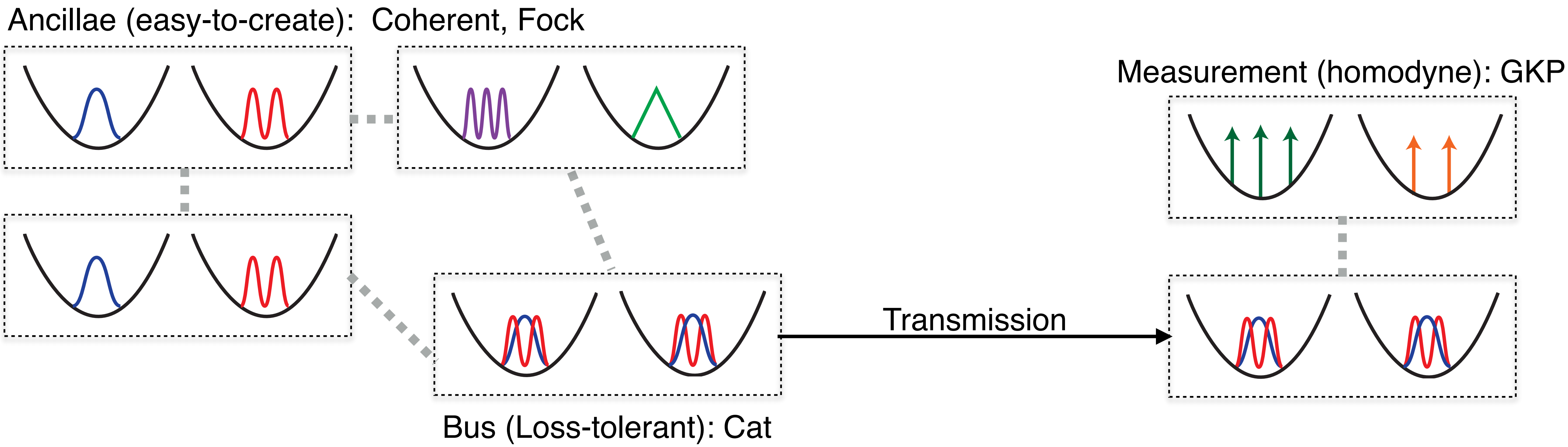
	$\vert 0_D\rangle$	$\vert 1_D\rangle$	X-axis rotation
Fock	$\vert 01\rangle$	$\vert 10\rangle$	$e^{i\theta\hat{S}}$
Coherent	$\vert 0\alpha\rangle$	$\vert \alpha 0\rangle$	$e^{i\theta\hat{S}}$
Cat	$\vert C_0 C_1\rangle$	$\vert C_1 C_0\rangle$	$e^{i\theta\hat{S}}$
GKP	$\vert G_0 G_1\rangle$	$\vert G_1 G_0\rangle$	$e^{i\theta\hat{S}}$

Unified architecture for every (existing & future) encoding

Benefit 2: Different Encoding per Qubit

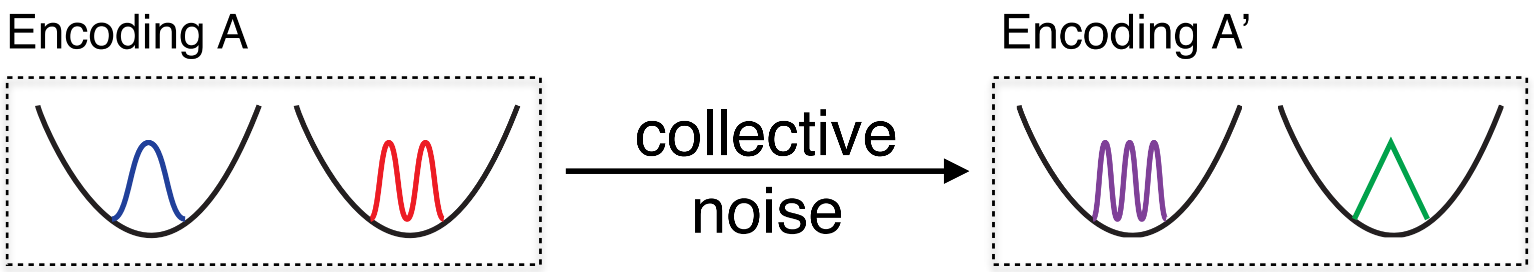


Encoding preserved after entanglement

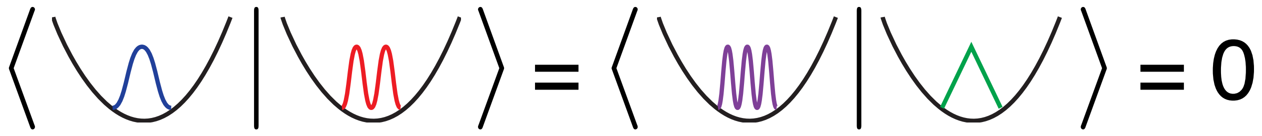


Utilise advantages of different encodings

Benefit 3: Decoherence-Free Subsystem



1. Orthogonality:



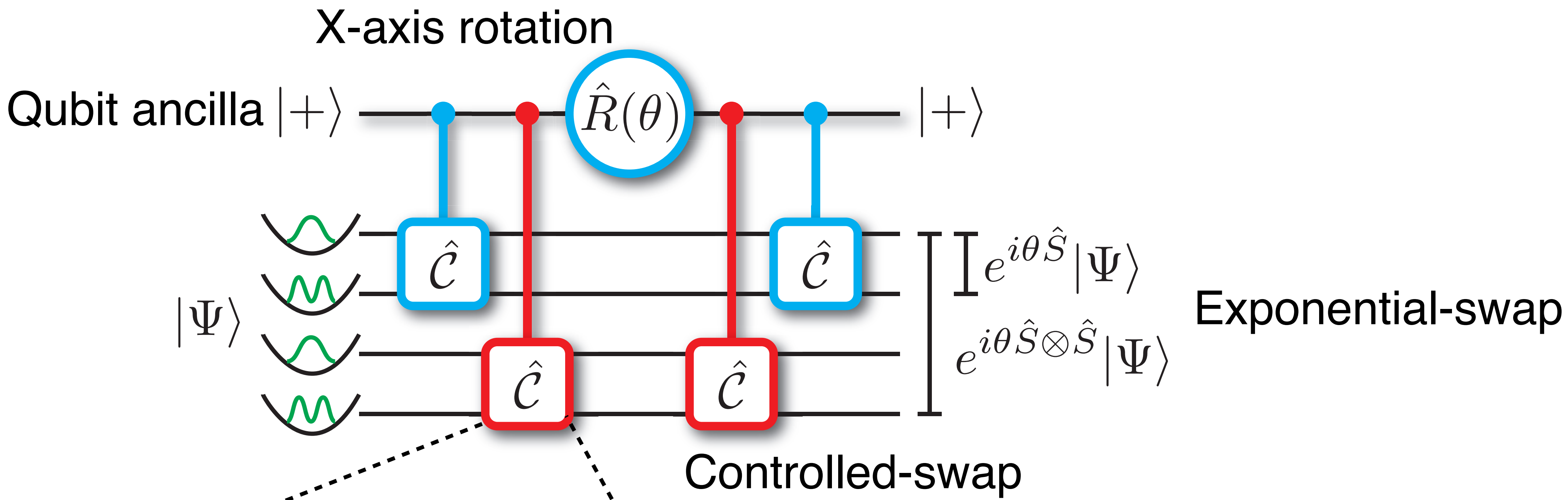
2: Logic gates are independent of encoding

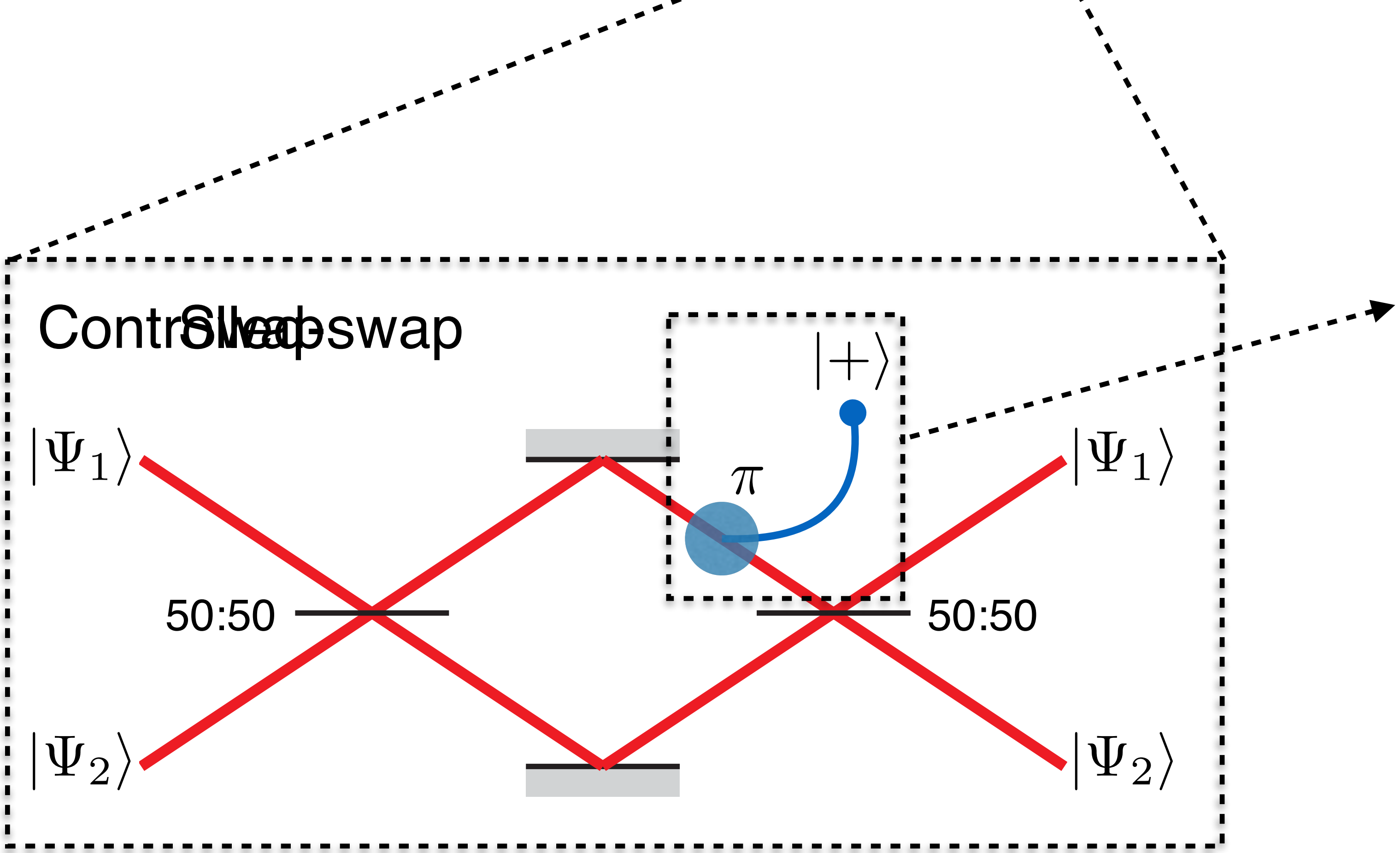
Collective noise commutes with logical (swap) operator

$$[\hat{U}_{\text{noise}} \otimes \hat{U}_{\text{noise}}, \hat{S}] = 0$$

Resilient to any collective noise

Implementation





Second order hybrid Hamiltonian

$$H_{\text{hybrid}} = \lambda \hat{Z}_A \hat{a}^\dagger \hat{a}$$

Example:
superconducting qubit + cavity (dispersive coupling),
NV center + mechanical oscillator (2nd order magnetic field gradient)

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Take home message:

1. Unified logic gates: Exponential-swap
2. Utilise different encoding in same computation
3. Decoherence-free-subsystem for any collective noise

Experimentalist: Exponential-swap is the best nonlinear operation

Theorist: No worry about logic gates implementation

One more thing: Quantum computation with highly-mixed states (arXiv:1608.03213)

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