Phase-like transitions in quantum dots
Bayesian magnetometry

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Quantum dot spin qubits in GaAs

semiconductor band structure + litography principles

$\Downarrow$

single electron quantum dots

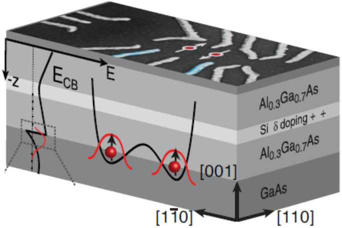
valence bound based on *s* orbitals

$\Downarrow$

decoherence from isotropic hyperfine interaction

dipol-dipol interaction negligible for $t \leqslant 10^{-7}s$

nuclear Zeeman splitting $\approx 10^3$ smaller than electron splitting



GaAs/AlGaAs heterostructure [1]

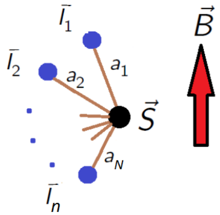
[1] T. Botzem, R. McNeil, D. Schuh, D. Bougeard, H. Bluhm, Nature Communications, Volume 7, id. 11170 (2016).

Hyperfine interaction

$$H = \Omega \hat{S}^z + \hat{D} + \hat{V},$$

$$\Omega = -g\mu_B B,\ \hat{D} = \sum_k a_k \hat{S}^z \hat{I}_k^z,\ \hat{V} = \frac{1}{2}\sum_k a_k \left(\hat{S}^+ \hat{I}_k^- + \hat{S}^- \hat{I}_k^+\right)$$

$$\hat{S}^\pm = \hat{S}^x \pm i\hat{S}^y$$



$\rho(t) = Tr_R[e^{-iHt}\rho_{tot}(0)e^{iHt}]$. Initial state $\rho_{tot}(0) = \rho(0) \otimes R(0)$
Box model: $\forall_k a_k = \mathcal{A}/n = \alpha$, $\sum_k a_k = \mathcal{A}$, valid for $t \ll \frac{n}{\mathcal{A}} \approx 10^4$ ns

Box model: $\forall_k a_k = \alpha$:

$$H = \Omega S^z + \alpha S^z I^z + \frac{\alpha}{2}(S^+ I^- + S^- I^+)$$

with $I = \sum_k I_k$ and its projection I^z,

with eigenstates $|K, m\rangle$ and associated eigenvalues $\sqrt{K(K+1)}$ and m

$$R(0) = P_{K,m}|K, m\rangle\langle K, m|,$$

$$\rho(t) = \sum_{i=1}^{4} K_i \rho(0) K_i^\dagger,$$

$$\sum_i K_i^\dagger K_i = \mathbb{I} \text{ and } K_i > 0$$

$$\rho(t) = \sum_{i=1}^{4} K_i \rho(0) K_i^\dagger,$$
$$S^z|0\rangle = \frac{1}{2}|0\rangle,\ S^z|1\rangle = -\frac{1}{2}|1\rangle,$$

$$K_1 = \sqrt{1-A}|0\rangle\langle 1|,\ K_2 = \sqrt{1-A}|1\rangle\langle 0|,$$
$$K_3 = \frac{1}{\sqrt{2}}\sqrt{A+|E|}\left[\frac{|E|}{E}|0\rangle\langle 0| + |1\rangle\langle 1|\right],\ K_4 = \frac{1}{\sqrt{2}}\sqrt{A-|E|}\left[-\frac{|E|}{E}|0\rangle\langle 0| + |1\rangle\langle 1|\right]$$

$$A(B,t,\alpha) = \sum_{Km} P_{K,m}|X_{K,m}(t)|^2,$$
$$E(B,t,\alpha) = \sum_{Km} P_{K,m} X_{K,m}(t) X_{K,m-1}(t),$$

$$\rho_B(t) = \Lambda_{B,t}(\rho^N(0)) = \sum_{i_1,\dots,i_N=1}^{4} K_{i_1} \otimes \cdots \otimes K_{i_N} \rho^N(0) K_{i_1}^\dagger \otimes \cdots \otimes K_{i_N}^\dagger$$

Definition

State ρ_{AB} on a space $\mathcal{H}_A \otimes \mathcal{H}_B$ is separable if it can be represented by $\rho_{AB} = \sum_i p_i \rho_A^i \otimes \rho_B^i$, $\sum_i p_i = 1$, $\forall_i p_i > 0$, where ρ_A^i and ρ_B^i are states on $\mathcal{H}_A$ and $\mathcal{H}_B$. Otherwise, it is entangled.

Measure of two-body entanglement: concurrence $C(\rho_{AB})$ [1]:

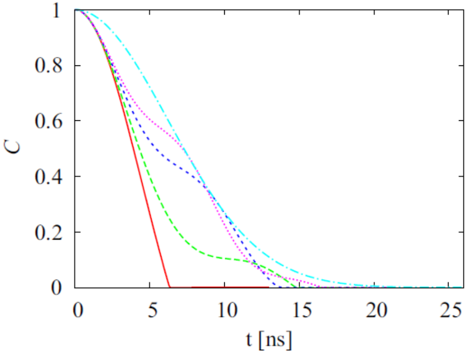
$C(\rho_{AB}) = 0 \Leftrightarrow \rho_{AB}$ is separable

$C(\rho_{AB}) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$, where $\lambda_1, \dots, \lambda_4$ are square roots of eigenvalues of $\rho_{AB}\tilde{\rho}_{AB}$, $\tilde{\rho}_{AB} = Y \otimes Y \rho_{AB}^* Y \otimes Y$.

$$|S\rangle = \frac{1}{\sqrt{2}}\Big(|01\rangle + |10\rangle\Big)$$
$$\rho(0) = |S\rangle\langle S|.$$

[1] S. Hill, W. K. Wooters, Physical Review Letters 78, 5022 (1997)

2 Dots: Entanglement evolution

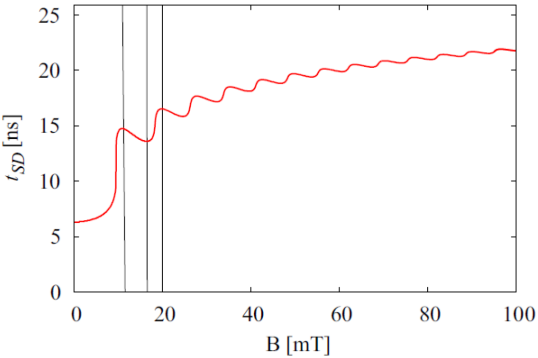


B=0 mT (red), B=11 mT (green), B=16.5 mT (blue),
B=20 mT (pink), B=1 T (lightblue)

$$C(\rho_{AB}) = 2\max\{0, |\rho_{01,10}| - \sqrt{\rho_{00,00}\rho_{11,11}}\}$$

$$g\mu_B B \gg \mathcal{A} \Rightarrow \text{dephasing}, \ |\rho_{01,10}(t)| = e^{-t^2/T_2^{*2}}, \ T_2^* = \sqrt{\frac{6}{I(I+1)}}\sqrt{n}/\mathcal{A} = 12.36 \text{ ns}$$

2 Dots: Time of suddent death vs B



Entanglement witness: $\frac{1}{2}\mathbb{I} - |S\rangle\langle S|$

bistochastic channels $\Rightarrow$ subsystems remain maximally mixed $\Rightarrow$
$\Rightarrow$ state diagonal in Bell basis

P. Mazurek, K. Roszak, R. Chhajlany, P. Horodecki, Physical Review A 89, 062318 (2014)

Bayesian metrology

$$\rho_B(t) = \sum_{i_1,\dots,i_N=1}^{4} K_{i_1} \otimes \cdots \otimes K_{i_N} \rho^N(0) K_{i_1}^\dagger \otimes \cdots \otimes K_{i_N}^\dagger$$

family of states for given t: $\{\rho_B\}$ with prior distribution $p(B) = \frac{1}{\Delta B_{prior}\sqrt{2\pi}} e^{-\frac{-(B-B_0)^2}{2\Delta^2 B_{prior}}}$

POVM measurements $\{\Pi_{\tilde{B}}\}$, with outcomes $\tilde{B}$.

$$\Delta^2 B_{est} = \int \mathrm{d}B\mathrm{d}\tilde{B}\, p(B,\tilde{B})(B-\tilde{B})^2$$
$$p(B,\tilde{B}) = p(B)\mathrm{Tr}(\rho_\mathrm{B}\Pi_{\tilde{\mathrm{B}}}).$$

von Neumann measurement: $L = \sum_{\tilde{B}} \tilde{B}\Pi_{\tilde{B}}$.

$$\Delta^2 B_{prior} - \Delta^2 B_{est} = \mathrm{Tr}(\bar{\rho}L^2) - B_0^2$$

where $\bar{\rho} = \int \mathrm{d}B p(B)\rho_B$ and

$\frac{1}{2}(L\bar{\rho} + \bar{\rho}L) = \bar{\rho}'$, with $\bar{\rho}' = \int \mathrm{d}B p(B)\rho_B B$.

C.W. Helstrom, *Quantum detection and estimation theory, (1976)*

K. Macieszczak, M. Fraas, and R. Demkowicz- Dobrzański, New Journal of Physics 16, 113002 (2014)

Optimizing over L and $\rho^N(0)$ for given t

finding optimal L for given ρ:

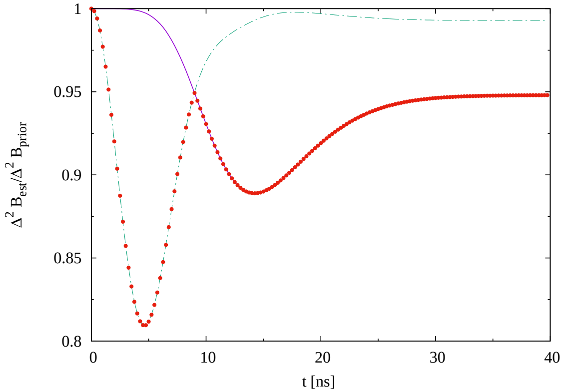
$$\Delta^2 B_{est} = \Delta^2 B_{prior} + B_0^2 + \mathrm{Tr}\Big[\int \mathrm{d}B p(B) \Lambda_B(\rho)(L^2 - 2BL)\Big]$$

finding optimal ρ for given L:

$$\Delta^2 B_{est} = \Delta^2 B_{prior} + B_0^2 + \mathrm{Tr}\Big[\rho \underbrace{\int \mathrm{d}B p(B) \Lambda_B^*(L^2 - 2BL)}_{O}\Big]$$

K. Macieszczak, M. Fraas, and R. Demkowicz- Dobrzański, New Journal of Physics 16, 113002 (2014)

N=1



'Phase ∥' (green) vs. 'population ⊥' (purple) strategy.
$B_0 = 7$ mT and $\Delta B = 4$ mT. Revival of coherences due to memory effects, no $e^{ig\mu_B Bt}$ dependency in coherences

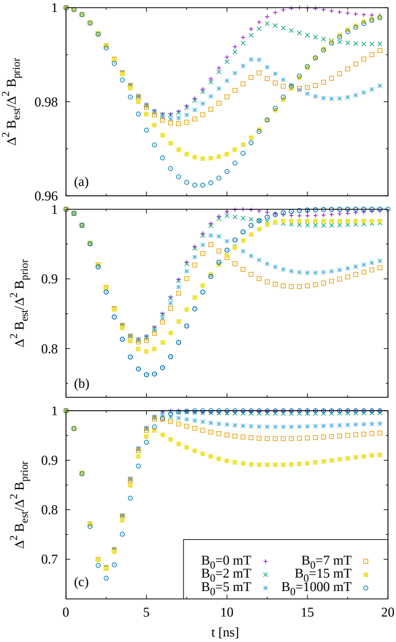
N=1

(a) $\Delta B_{prior} = 1$ mT

(b) $\Delta B_{prior} = 4$ mT

(c) $\Delta B_{prior} = 10$ mT

- high magnetic fields ($B > \frac{N\alpha}{g\mu_B}$):
  $A(t) = 1,\ E(t) = \exp(ig\mu_B Bt)\exp(-(\frac{t}{T_2^*})^2)$
  optimal Bloch vector $\perp \vec{B}$
  optimal observable $\perp \vec{B}$ and Bloch vector
- medium magnetic fields $\frac{N\alpha}{g\mu_B} > B_0 > \frac{\sqrt{N}\alpha}{g\mu_B}$:
  phase regime vs. population regime
  optimal Bloch vector $\parallel \vec{B}$
  optimal observable $\parallel \vec{B}$
- small magnetic fields $\frac{\sqrt{N}\alpha}{g\mu_B} > B_0$:
  only phase regime; $A(B, t, \alpha) = A(-B, t, \alpha)$



N=2

(a): GHZ(2) (green), g=0.1 (blue),
$|0+\rangle$ (black dashed), $|00\rangle$ (black dot-dashed)

Regimes:

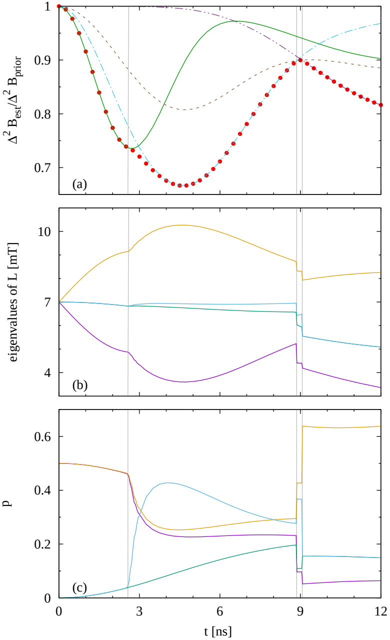
$$\overbrace{GHZ(N) \rightarrow gGHZ(N) + (1-g)|+\rangle^{\otimes N}}^{ent.}$$
$$\rightarrow |0+\rangle \rightarrow |00\rangle$$

L for small t:

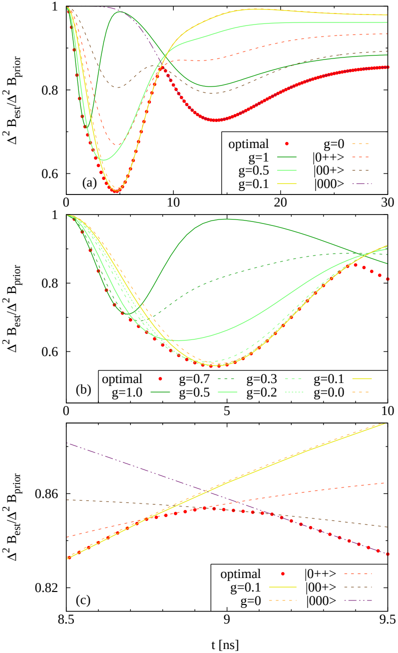
$$e^{-2ig\mu_B B_0 S^z t} e^{\pm \frac{i\pi}{4}\left(S^z \otimes \mathbb{I} + \mathbb{I} \otimes S^z\right)} GHZ(2)$$

$|01\rangle$ and $|10\rangle$

$$GHZ(N) = \frac{1}{\sqrt{2}}\left(|0\rangle^{\otimes N} + |1\rangle^{\otimes N}\right)$$
$$|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$



N=3

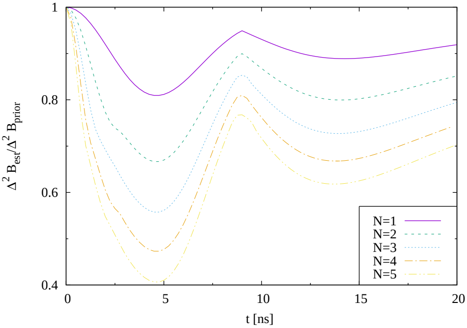


(a) measurement strategies for $N = 3$

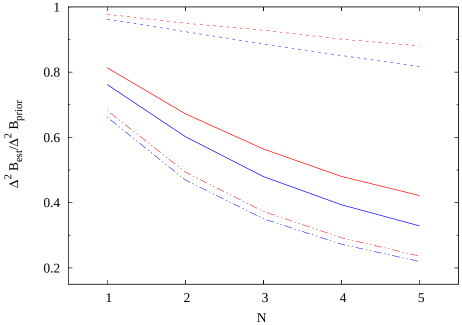
(b) smooth optimal state transitions

(c) sharp optimal state transitions

Comparision for different N



$B_0 = 7$ mT, $\Delta B = 4$ mT



Global minima for
$\Delta B = 1$ mT (dashed)
$\Delta B = 4$ mT (solid)
$\Delta B = 10$ mT (dot-dashed)

$B_0 = 0$ (red) and $B_0 = 1$ T (blue)

Open questions

- Global minimum for noisy system always worse that the one for unitary evolution? Effects of anisotropy of hyperfine interaction and polarization of environment?
- Exploitation of non-Markovianity of evolution in parameter sensing
- Role of non-GHZ entanglement
- Scalling behaviour for large N

Sharp transitions in low-number quantum dots Bayesian magnetometry, P.M., M. Horodecki, Ł. Czekaj, P. Horodecki, *arXiv:1605.04279*