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Dimension witnesses beyond non-classicality tests

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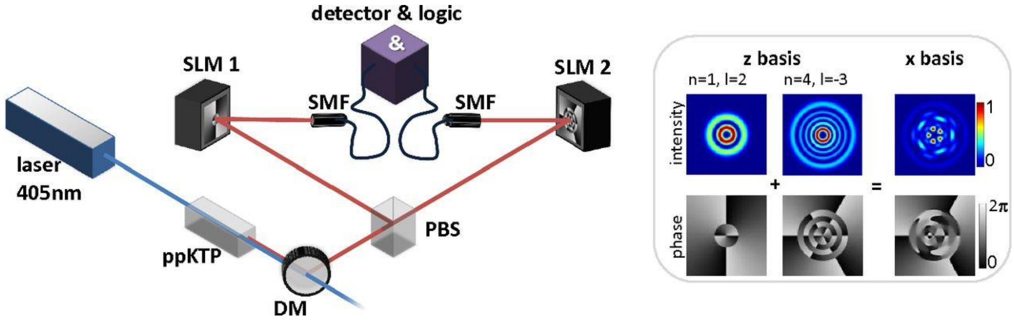
Motivation – High d Systems

Generation and confirmation of a (100 × 100)-dimensional entangled quantum system

Mario Krenn[a,b,1], Marcus Huber[c,d,e], Robert Fickler[a,b], Radek Lapkiewicz[a,b], Sven Ramelow[a,b,2], and Anton Zeilinger[a,b,1]

Author Affiliations

Contributed by Anton Zeilinger, February 24, 2014 (sent for review December 15, 2013)

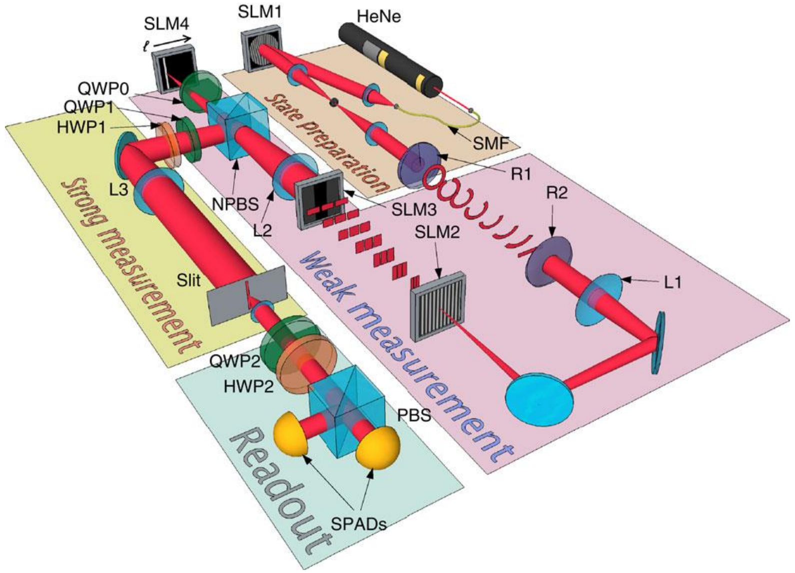


Direct measurement of a 27-dimensional orbital-angular-momentum state vector

Mehul Malik, Mohammad Mirhosseini, Martin P. J. Lavery, Jonathan Leach, Miles J. Padgett & Robert W. Boyd

Affiliations | Contributions | Corresponding author

Nature Communications **5**, Article number: 3115 | doi:10.1038/ncomms4115
Received 22 October 2013 | Accepted 16 December 2013 | Published 20 January 2014



Dimension Witnesses

Testing the Dimension of Hilbert Spaces

Nicolas Brunner, Stefano Pironio, Antonio Acin, Nicolas Gisin, André Allan Méthot, and Valerio Scarani
Phys. Rev. Lett. **100**, 210503 – Published 30 May 2008

Lower bound on the dimension of a quantum system given measured data

Stephanie Wehner, Matthias Christandl, and Andrew C. Doherty
Phys. Rev. A **78**, 062112 – Published 22 December 2008

Dimension Witnesses and Quantum State Discrimination

Nicolas Brunner, Miguel Navascués, and Tamás Vértesi
Phys. Rev. Lett. **110**, 150501 – Published 8 April 2013

Device-independent dimension tests in the prepare-and-measure scenario

Jamie Sikora, Antonios Varvitsiotis, Zhaohui Wei

(Submitted on 13 Jun 2016)

Our Result

Assuming the dimension of a physical state is d, there is a witness that distinguishes between *fully quantum* and *classical states*, including *separable partitions*.

E.g. let $d = ab$,
then the witness should distinguish between

$$Q_{ab}\ , Q_a Q_b\ , C_a Q_b\ , C_b Q_a\ , C_{ab}$$

Random Access Codes (RACs)

- Input for Alice

$$\boldsymbol{x} = x_0 x_1 \dots x_{n-1} \quad ; \quad x_i \in \{0,1,\dots,d-1\}$$

- Input for Bob

$$y \in \{0,1,\dots,n-1\}$$

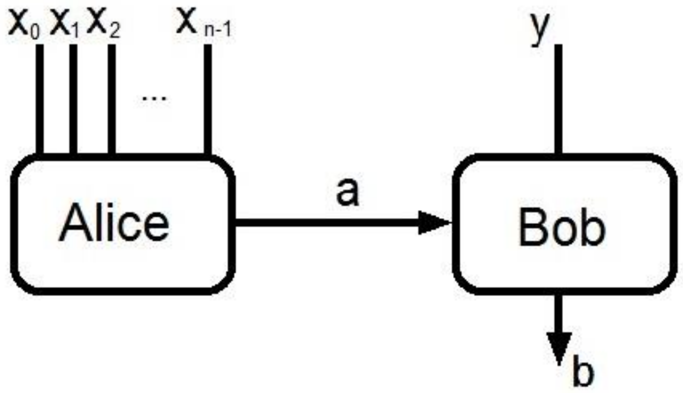
- Rules (Prepare & Measure)
  Alice prepares and sends a d-dimensional state to Bob.
  Bob measure the state and outputs $b \in \{0,1,\dots,d-1\}$.

- Success Condition

$$b = x_y$$

- Figure of Merit
  Average Success Probability

$$\bar{p} = \frac{1}{nd^n} \sum_{\boldsymbol{x},y} p(\, b = x_y | X = \boldsymbol{x}, Y = y) = \frac{1}{n} \sum_{y} p(\text{Correctly guessing } x_y)$$



$2^2 \rightarrow 1$ QRAC ; An Example

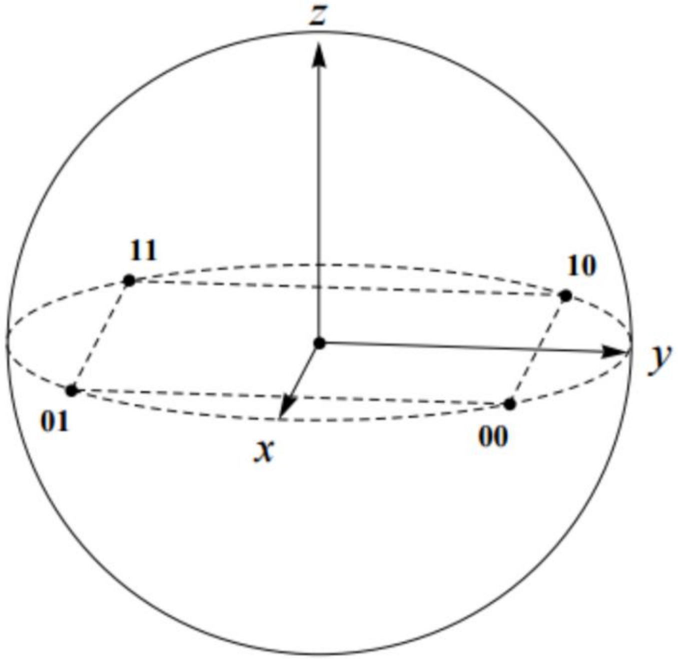


Figure 5: Bloch sphere representation of encoding for $2 \mapsto 1$ quantum random access code.

In general, for $2^d \rightarrow 1$ RACS:

$$\bar{p}_q = \frac{1}{2}\left(1 + \frac{1}{\sqrt{d}}\right)$$

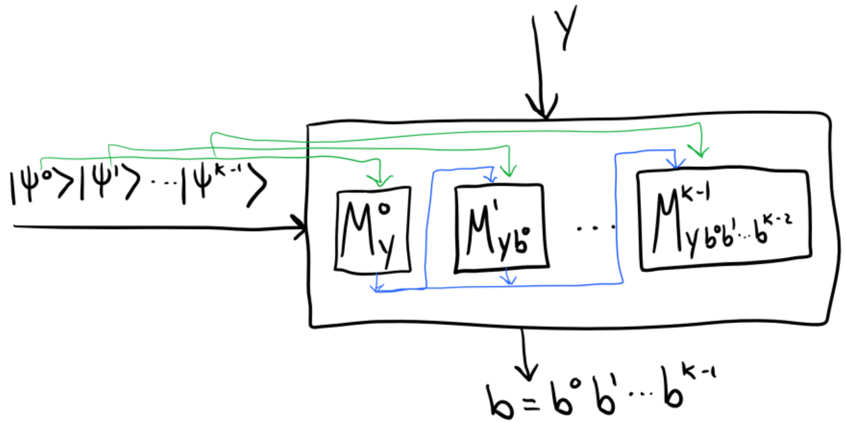
$$\bar{p}_c = \frac{1}{2}\left(1 + \frac{1}{d}\right)$$

For d=2,
$\bar{p}_c = 0.75$, $\bar{p}_q \approx 0.854$

Ambainis et al., *Quantum Random Access Codes with Shared Randomnes*, q-ph: 0810.2937

Our Framework and non-Adaptive Strategies

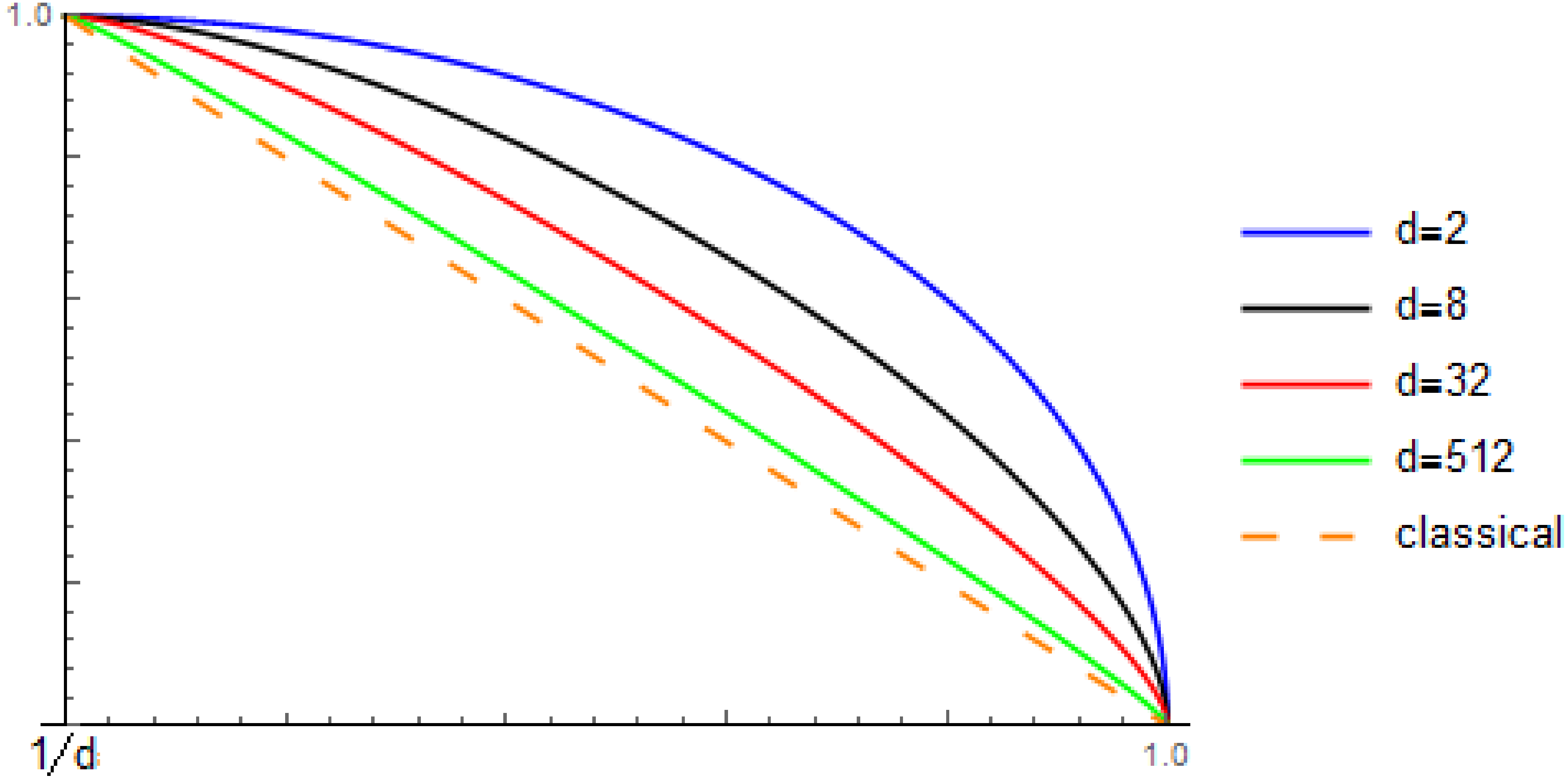
- Let $d = d^0 d^1 \cdots d^{k-1}$, and suppose we want to know the maximum $\bar{p}$, for $Q_{d^0} Q_{d^1} \cdots Q_{d^{k-1}}$
- Alice sends $|\psi\rangle = |\psi^0\rangle|\psi^1\rangle \cdots |\psi^{k-1}\rangle$ to Bob.
- Bob measures each qudit with projective measurements.
- In general, the choice of the jth measurement basis could depend on all previous measurement outcomes. We call this an Adaptive Strategy.



- ***Lemma:*** *For an Adaptive Strategy there exists a non-Adaptive Strategy which has at least the same average success probability.*

Magic Curves for $2^d \rightarrow 1$ QRACs

- $m_d(x) = \cos\left(\cos^{-1}\left(\frac{1}{\sqrt{d}}\right) - \cos^{-1}(\sqrt{x})\right)^2 \quad = \quad m_d(x) = 1 - \left(\frac{d-1}{d}\right)\left(\sqrt{x} - \sqrt{\frac{1-x}{d-1}}\right)^2$

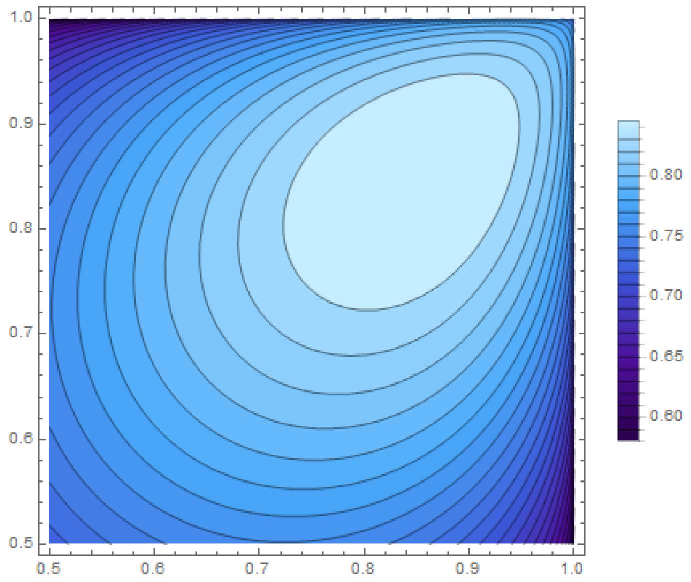


The Magic…

- $\bar{p} = \frac{1}{2}(p(\text{correctly guessing 0th dit}) + p(\text{correctly guessing 1st dit}))$

- Now, e.g., let $d = ab$, then write:
  $x_0 = x_0^0 x_0^1 \quad , x_1 = x_1^0 x_1^1$ with $x_i^0 \in \{0,1,\ldots,a-1\} \quad , x_i^1 \in \{0,1,\ldots,b-1\}$

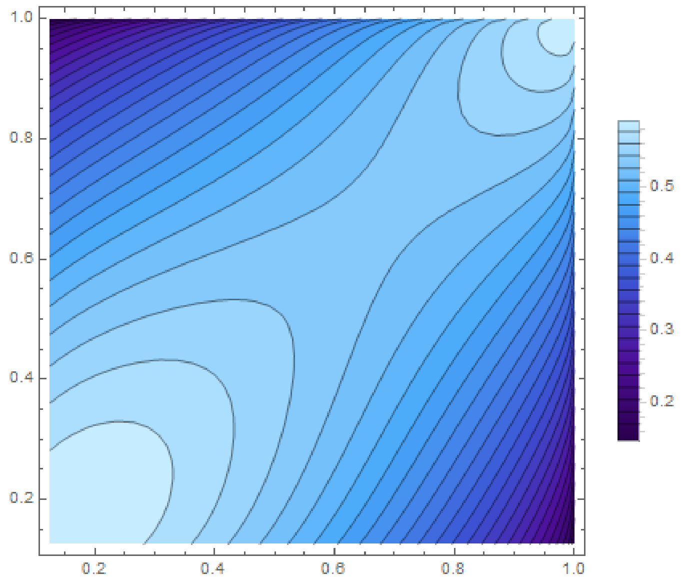
- $u = p(\text{correctly guessing FIRST part of 0th dit})$
  $v = p(\text{correctly guessing SECOND part of 0th dit})$



$$\bar{p} = \frac{1}{2}\max_{u,v}(uv + m_a(u)m_b(v))$$

d=2*2

d=8*8



Our Result Applied to Examples

d=4	
Q_4	0.75
Q_2Q_2	0.728553
C_2Q_2	0.654508
C_4	0.625

d=6	
Q_6	0.704124
Q_2Q_3	0.673176
C_2Q_3	0.614357
C_3Q_2	0.596856
C_6	0.583333

d=8	
Q_8	0.676777
Q_2Q_4	0.640165
$Q_2Q_2Q_2$	0.621859
C_2Q_4	0.591506
$C_2Q_2Q_2$	0.582955
C_4Q_2	0.570164
C_8	0.5625

Questions?