Measurement-based quantum computation with mechanical oscillators

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QUANTUM TECHNOLOGY at QUEEN'S

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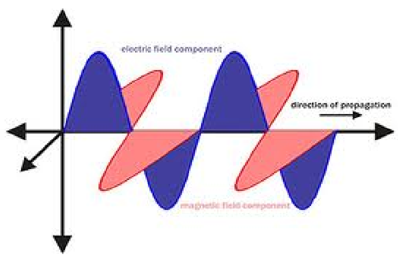
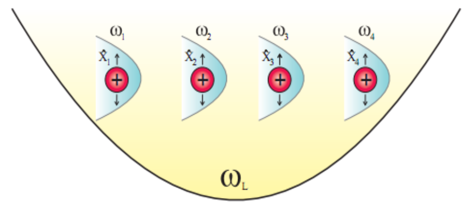
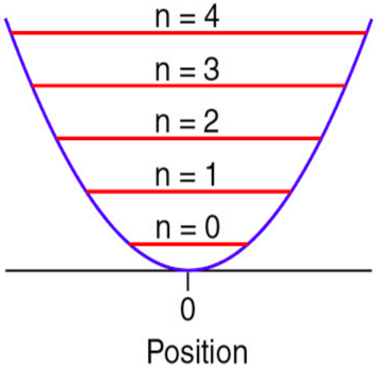
Continuous Variables (CVs)
[Distinguishable bosons, qumodes]

Light quadratures



trapped ion motion

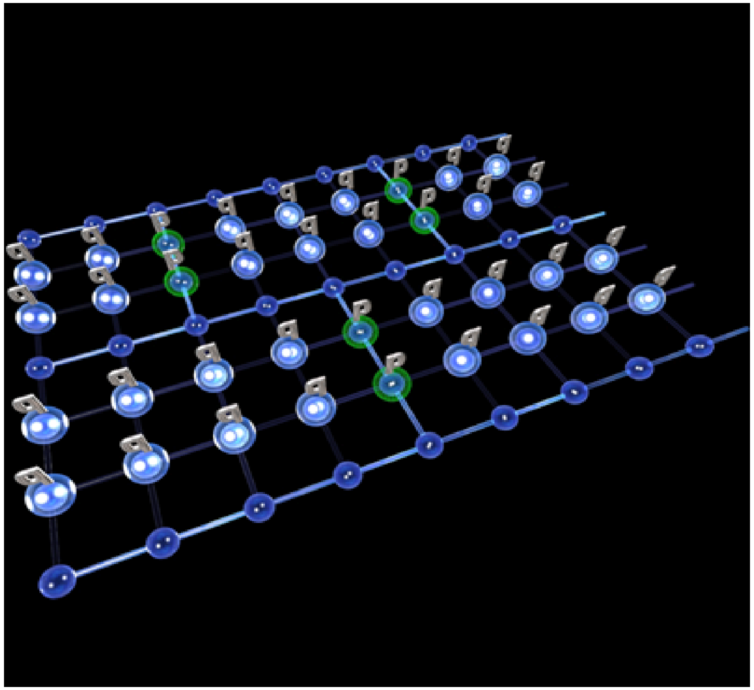




$\hat{q}\ ,\ \hat{p}$

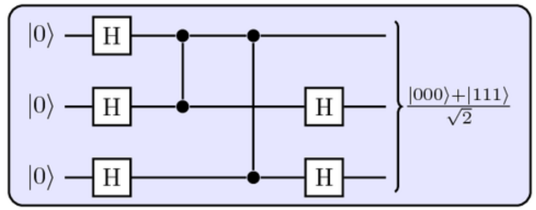
What can we do with many qumodes?

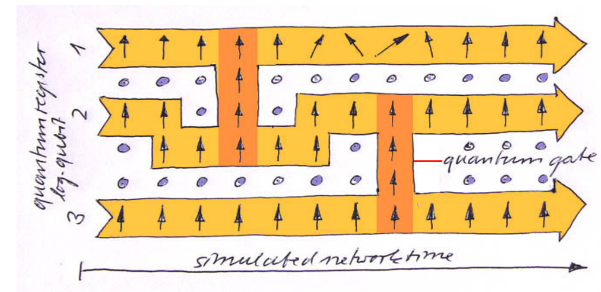
Quantum computation over CVs



Gu et al., PRA (2009)

Models of computation





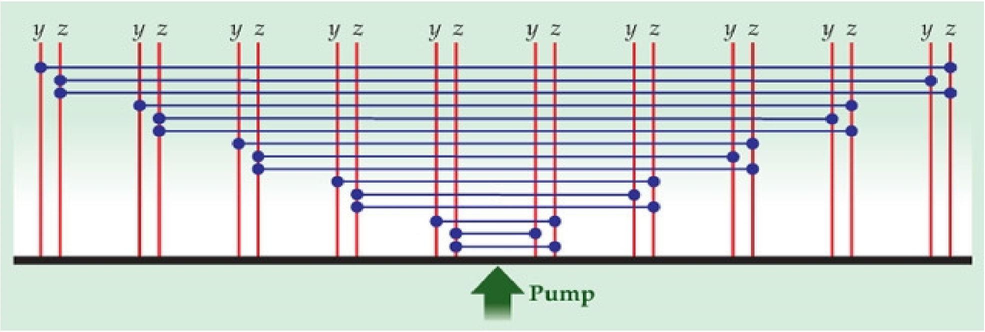
	Circuit-Based Quantum Computation	Measurement-Based Quantum Computation (MBQC)
Continuous Variables	Lloyd & Braunstein PRL (1999)	Menicucci et al. PRL (2006)
Fault tolerant (with finite energy)	Gottesman, Kitaev, Preskill PRA (2001) Lund, Ralph, Haselgrove, PRL (2008)	Menicucci PRL (2014)

MBQC resources with traveling light: recent experimental progresses

60 entangled modes

Frequency encoding

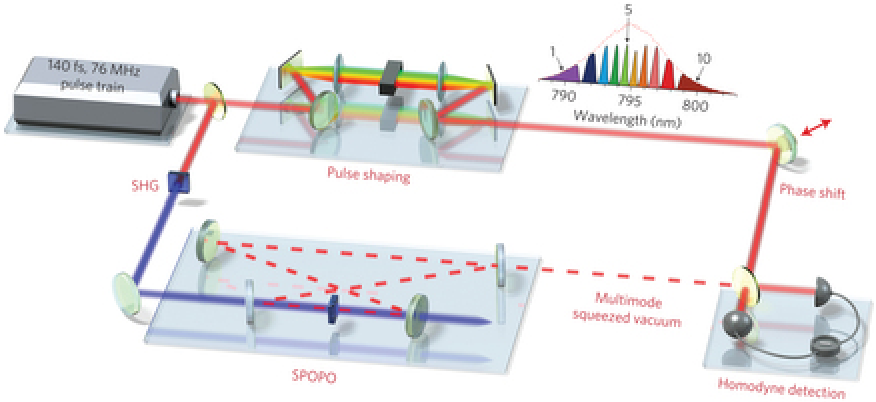
Single crystal & freq comb
[Chen et al., PRL (2014)]



500+ entangled partitions

Frequency encoding

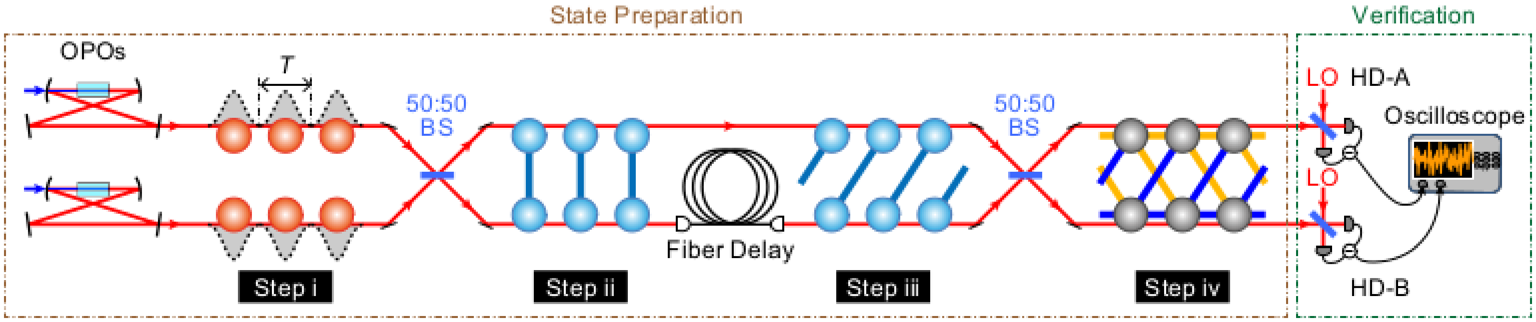
Single crystal & freq comb
[Roslund et al., Nat. Photonics (2014)]



10^6 entangled modes

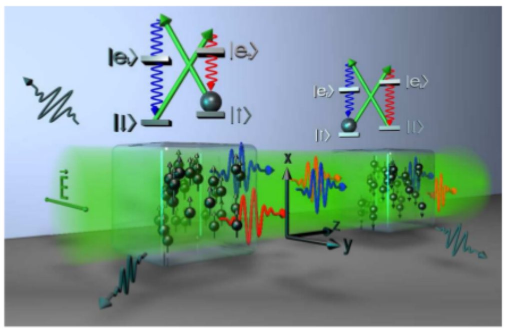
Temporal encoding

Pulsed squeezed states
[Yoshikawa et al., arXiv 1606.06688]

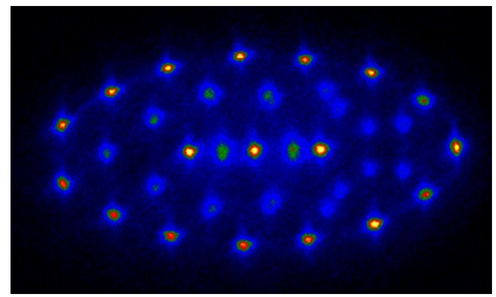


Also interesting alternative platforms: confined/massive continuous variables

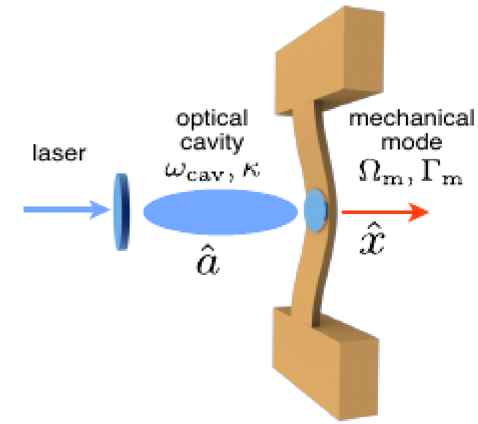
Atomic ensembles



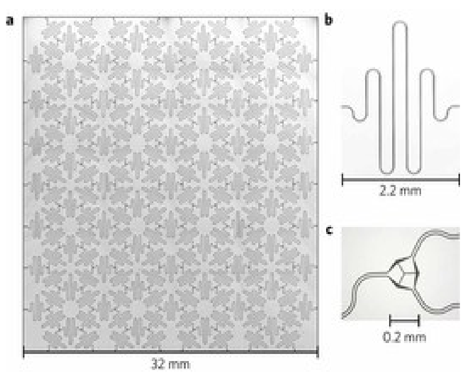
Trapped Ions



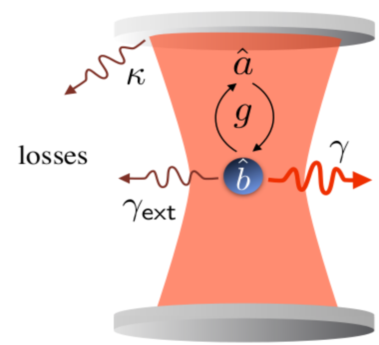
Optomechanics



Circuit-QED



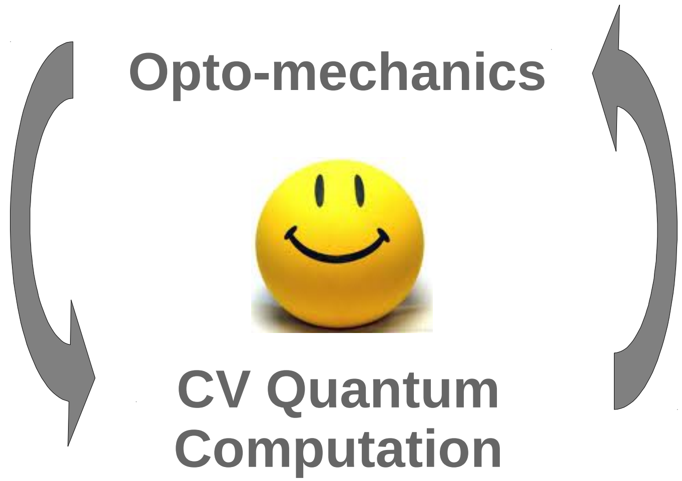
Cavity-QED

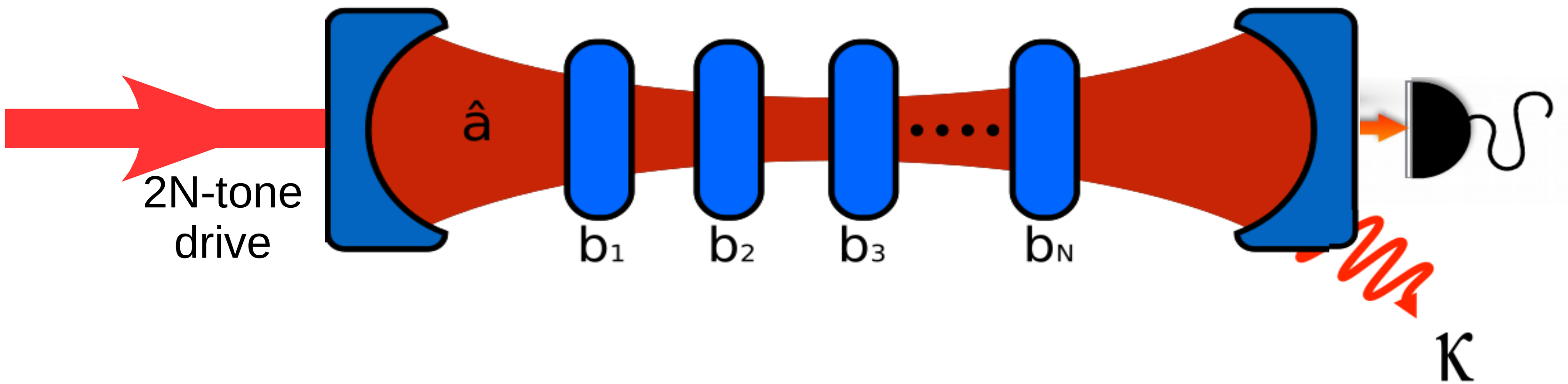


Why interesting?

Confined systems could be integrated easily

Take-home message





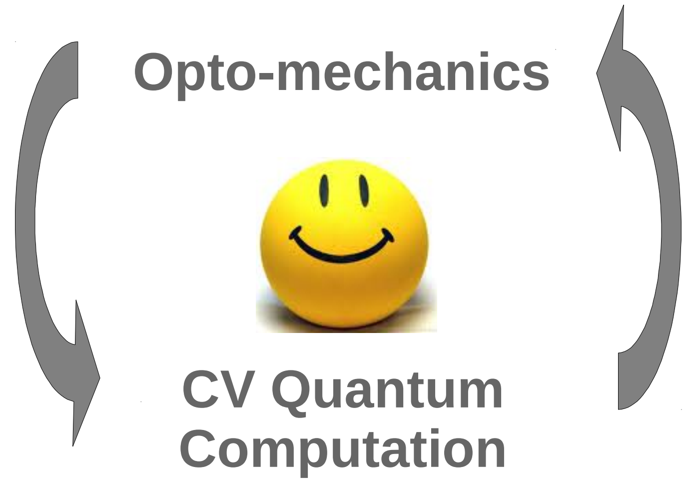
A cavity-optomechanics setup
with multiple mechanical oscillators allows for:

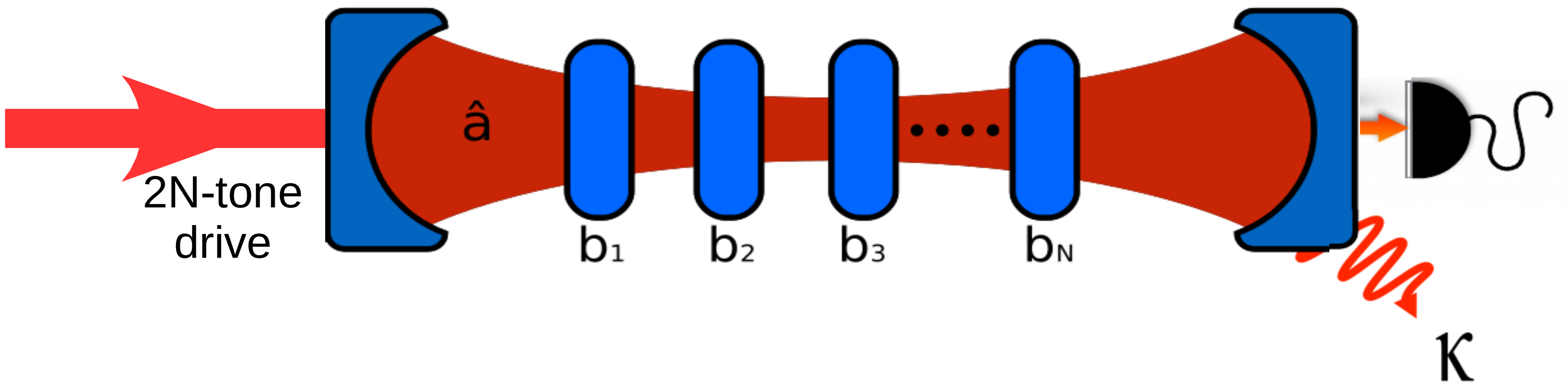
1) Generation of universal resources for computation

2) Quantum tomography of the resource

3) Arbitrary Gaussian computation

Take-home message





A cavity-optomechanics setup
with multiple mechanical oscillators allows for:

Now 1) Generation of universal resources for computation

Saturday 2) Quantum tomography of the resource

Now 3) Arbitrary Gaussian computation

Outline

- Measurement-based quantum computation with CVs
- Generation of universal resources for CV quantum computation
- Arbitrary Gaussian computation

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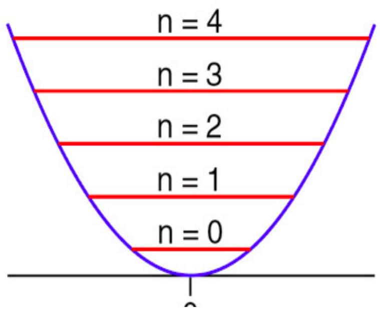
Continuous Variables



Position and momentum operators

$$q_j = \frac{1}{\sqrt{2}}(b_j + b_j^\dagger) \qquad p_j = \frac{1}{i\sqrt{2}}(b_j - b_j^\dagger) \qquad [q_j, p_k] = i\delta_{j,k}$$

Computational basis

$$|v\rangle_q \qquad (\ q|v\rangle_q = v|v\rangle_q\ , \quad v \in \mathbb{R})$$

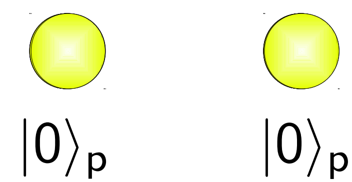
Entangling gate

$$CZ_{jk} \equiv \exp[iq_j \otimes q_k]$$

Ideal measurement-based quantum computation

CV cluster state: the universal resource for computation

- Prepare each node in zero-momentum eigenstate



[Zhang and Braunstein, PRA (2006); Menicucci et al., PRL (2006)]

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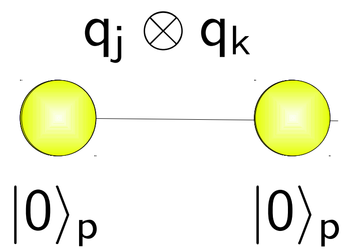

- Prepare each node in zero-momentum eigenstate

- Entangle connected nodes with

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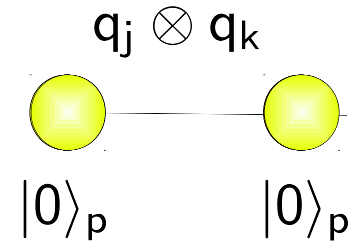
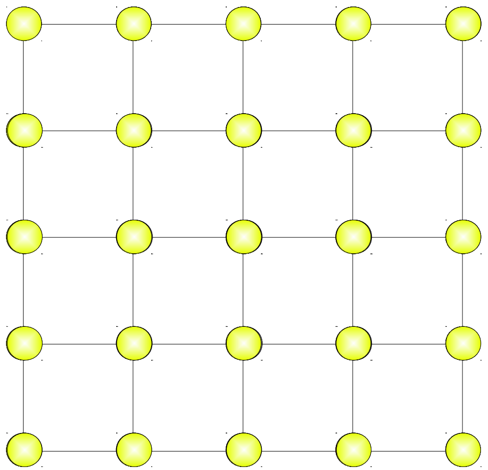


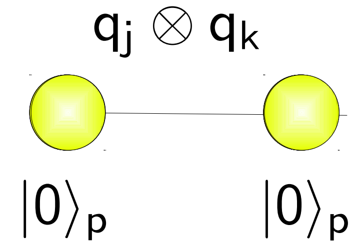


CV cluster state

[Zhang and Braunstein, PRA (2006); Menicucci et al., PRL (2006)]

Ideal measurement-based quantum computation

CV cluster state: the universal resource for computation



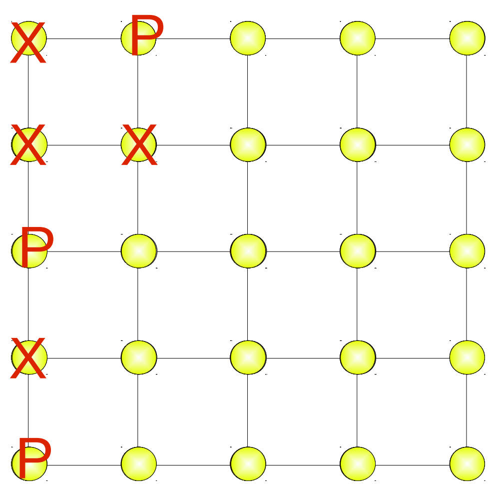
- Prepare each node in zero-momentum eigenstate

- Entangle connected nodes with

$$CZ_{jk} \equiv \exp[i q_j \otimes q_k]$$

- Measure each node locally

- Quadrature measurements → Gaussian computation
  Non-Gaussian measurements → universal



CV cluster state

[Zhang and Braunstein, PRA (2006); Menicucci et al., PRL (2006)]

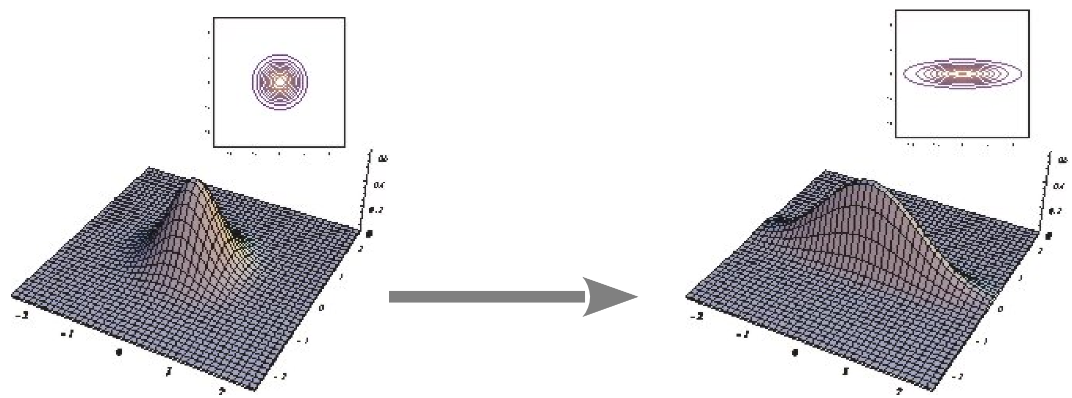
Finite energy: finitely squeezed states

In realistic settings momentum eigenstates are substituted by squeezed states

Squeezing operator T(r)

$$T^\dagger(r)qT(r) = \sqrt{\frac{1+r}{1-r}}q$$

$$T^\dagger(r)pT(r) = \sqrt{\frac{1-r}{1+r}}p$$



Position and momentum basis are infinitely squeezed (0<r<1):

$$\lim_{r\to 1} T(r)|0\rangle \equiv |0\rangle_p$$

Fault tolerance is guaranteed for large enough squeezing

Gaussian states

Restricting to quadratic operations (CZ)
and finite energy (squeezed states)

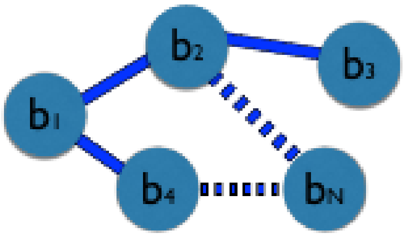
	Full quantum mechanics	Gaussian world
States	Density operator ρ	First and second moments $R = (q_1, \ldots, q_N, p_1, \ldots, p_N)$ $\langle R \rangle$ $[V]_{kl} = \langle R_k R_l + R_l R_k \rangle / 2$
Closed Dynamics	Unitaries W $\rho' = W \rho W^\dagger$	Symplectic S $V' = S V S^T$

Finite energy CV graph states are Gaussian

Consider the union

$$G_{(\mathcal{V},\mathcal{E})} \equiv \{\mathcal{V}, \mathcal{E}\}$$

of vertices $\mathcal{V}$ and edges $\mathcal{E}$



Associated **finite-energy** graph state:

$$|\mathsf{G_r}\rangle \equiv \mathsf{CZ} \bigotimes_{\mathsf{j} \in \mathcal{V}} \mathsf{T_j(r)} |\mathbf{0}\rangle$$

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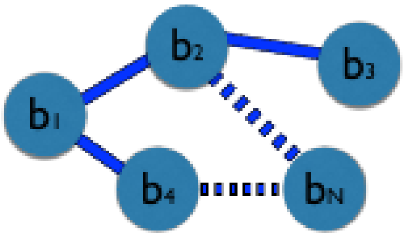

Associated **finite-energy** graph state:

$$|G_r\rangle \equiv CZ \bigotimes_{j\in\mathcal{V}} T_j(r)|\mathbf{0}\rangle$$

$$\begin{pmatrix} \frac{1+r}{1-r} I_N & 0_N \\ 0_N & \frac{1-r}{1+r} I_N \end{pmatrix}$$

Finite energy CV graph states are Gaussian

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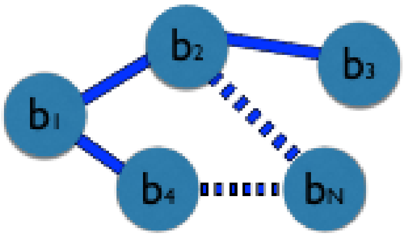

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Associated **finite-energy** graph state:

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$$CZ = \prod_{\{j,k\}\in\mathcal{E}} CZ_{jk}$$

$$\frac{1}{2} S^T \begin{pmatrix} \frac{1+r}{1-r} I_N & 0_N \\ 0_N & \frac{1-r}{1+r} I_N \end{pmatrix} S$$

Finite energy CV graph states are Gaussian

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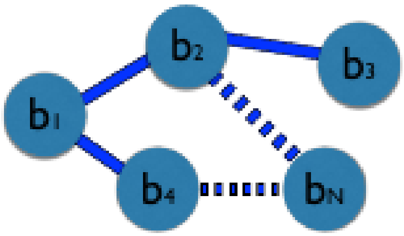

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Associated **finite-energy** graph state:

$$|\mathsf{G_r}\rangle \equiv \mathsf{CZ} \bigotimes_{\mathsf{j}\in\mathcal{V}} \mathsf{T_j(r)}|\mathbf{0}\rangle$$

$$\mathsf{V} = \frac{1}{2}\mathsf{S}^\mathsf{T} \begin{pmatrix} \frac{1+\mathsf{r}}{1-\mathsf{r}}\,\mathsf{I_N} & \mathsf{0_N} \\ \mathsf{0_N} & \frac{1-\mathsf{r}}{1+\mathsf{r}}\,\mathsf{I_N} \end{pmatrix} \mathsf{S}$$

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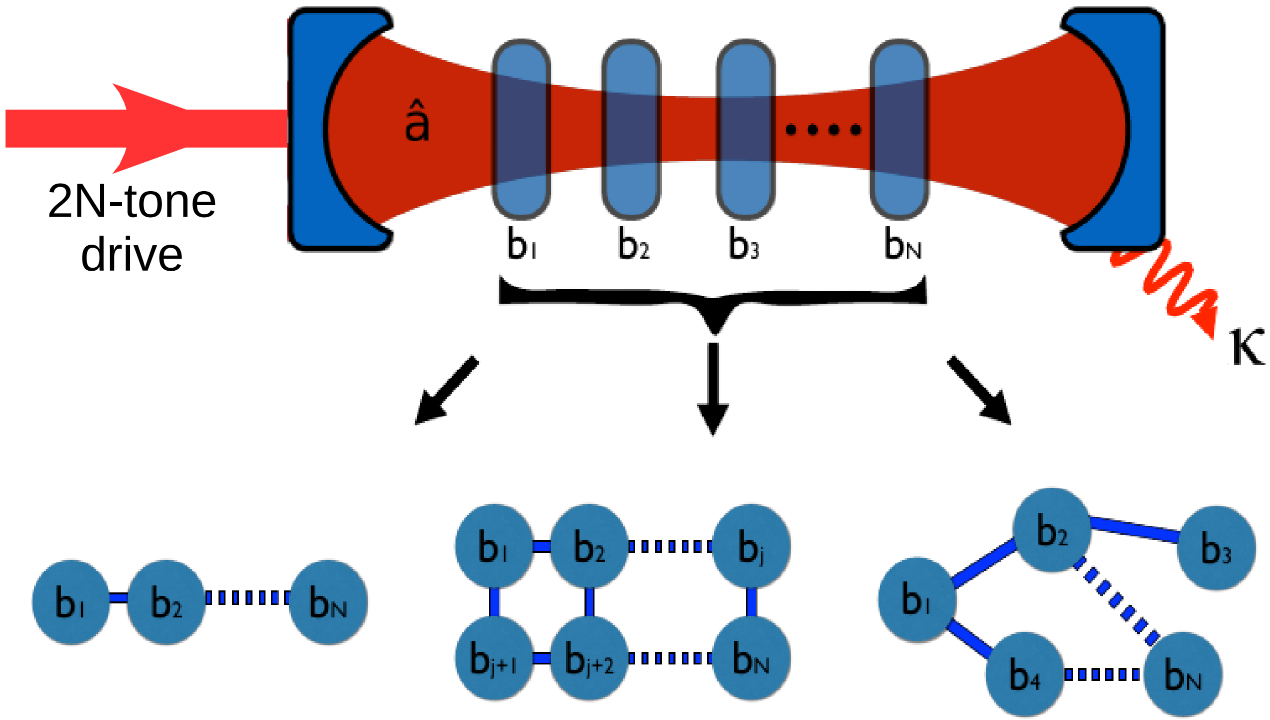
Generate arbitrary graph states of mechanical oscillators exploiting the dissipative dynamics of optomechanical systems



Dissipation-driven steady state

Generic graph state

[Houhou, Aissaoui, AF, PRA '15]

Exploiting the dissipative dynamics (1 mode)

Assume the two-mode Hamiltonian system

$$H = \beta a^\dagger (c + r\, c^\dagger) + \mathrm{H.c.}$$

with losses on mode a only

$$\frac{\mathrm{d}\,\rho}{\mathrm{d}\,t} = -i[H, \rho] + \kappa(a\rho a^\dagger - \frac{1}{2} a^\dagger a \rho - \frac{1}{2}\rho a^\dagger a)$$

The system is **dissipatively driven to a unique and squeezed steady state**

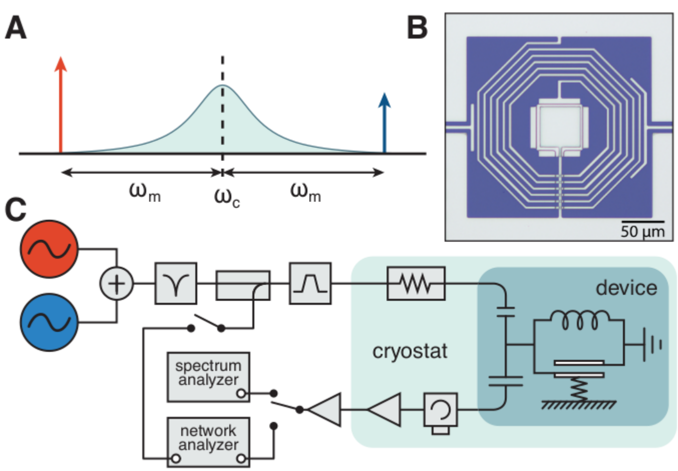
$$V_{ss} = \frac{1}{2} I_2 \oplus \begin{pmatrix} \frac{1+r}{1-r} & 0 \\ 0 & \frac{1-r}{1+r} \end{pmatrix}$$

[A. Kronwald et al., PRA (2013)]

Electro-mechanical implementation

Driving the mechanical sidebands with two tones

$$\mathrm{H} = \beta a^\dagger (c + r\, c^\dagger) + \mathrm{H.c.}$$

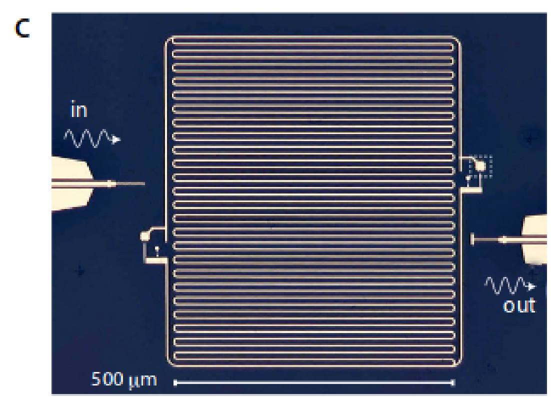


[Woolman et al.,

Science 349, 952 (2015)]

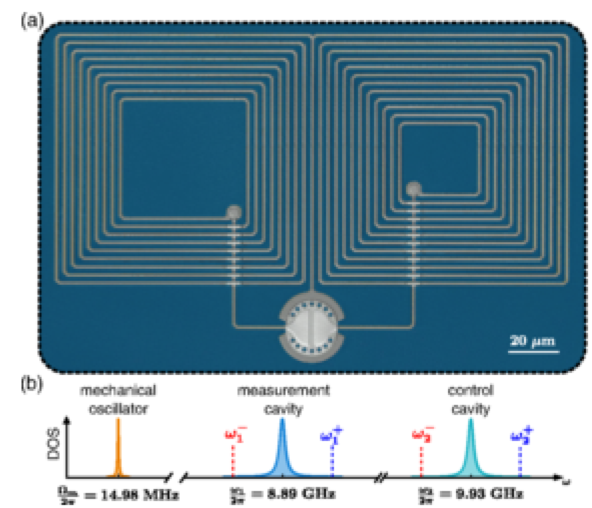
[Lei et al.,

arXiv:1605.08148]



[Pirkkallainen et al.,

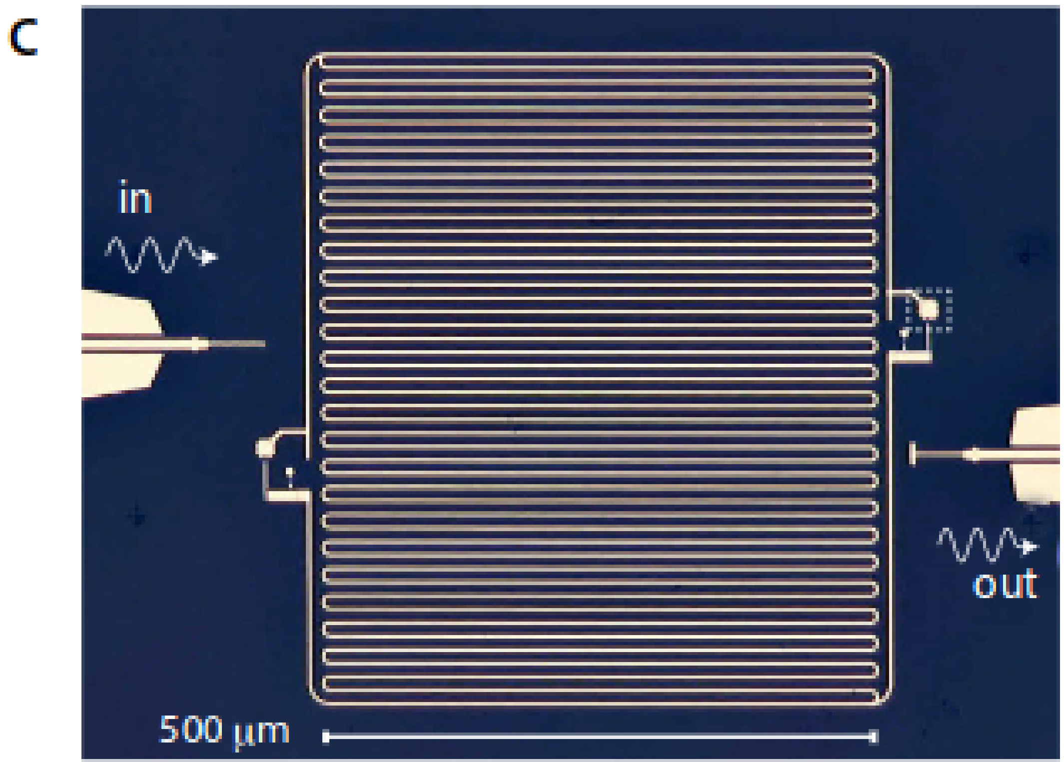
PRL 115, 243601 (2015)]



[Lecocq et al.,

PRX 5, 041037 (2015)]

Electro-mechanical implementation



[Pirkkallainen et al.,

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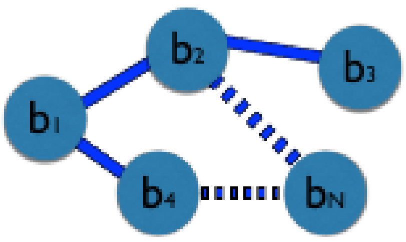
Exploiting the dissipative dynamics (graph)

Consider an arbitrary N-mode graph state (with finite squeezing)

$$V = \frac{1}{2} S^T \begin{pmatrix} \frac{1+r}{1-r} I_N & 0_N \\ 0_N & \frac{1-r}{1+r} I_N \end{pmatrix} S$$



$$S \longleftrightarrow W \qquad \Longrightarrow \qquad c = W\, b$$

$b = (b_1, \ldots, b_N)$ **local**

$c = (c_1, \ldots, c_N)$ **collective**

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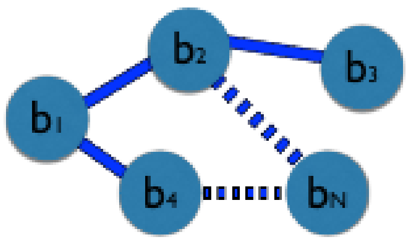

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If we squeeze the collective modes,
the local modes will automatically be in the desired graph state!

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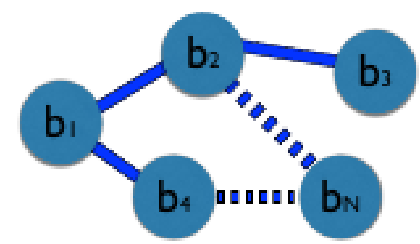

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Hamiltonian switching: - N temporal steps

$$H^{(k)} = \beta a^\dagger (c_k + r\, c_k^\dagger) + \mathrm{H.c.}$$

$$(k = 1, \ldots, N)$$

- In each step one collective mode is coupled and squeezed

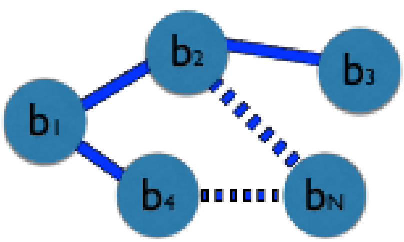
[Li, Ke, and Ficek, PRA (2009); Ikeda & Yamamoto, PRA (2013)]

How can we implement the Hamiltonian switch?

Consider the set of Hamiltonians with free parameters $\alpha_j^{\pm}$, $\phi_j^{\pm}$:

$$H = a^\dagger \sum_{j=1}^{N} g_j \left(\alpha_j^+ e^{i\phi_j^+} b_j^\dagger + \alpha_j^- e^{i\phi_j^-} b_j \right) + \mathrm{H.c.}$$

arbitrary graph



$\longleftrightarrow$ S , W

$b = (b_1, \ldots, b_N)$ **local**

$c \equiv Wb = (c_1, \ldots, c_N)$ **collective**

At each step k set the
free parameters as follows

$$\alpha_j^- = \frac{\beta}{g_j} |W_{kj}|$$

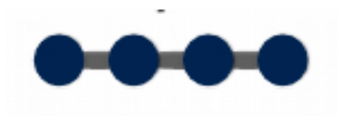
$$\alpha_j^+ = r\alpha_j^-$$

$$\phi_j^- = -\phi_j^+ = \arg(W_{kj})$$

$$\Downarrow$$

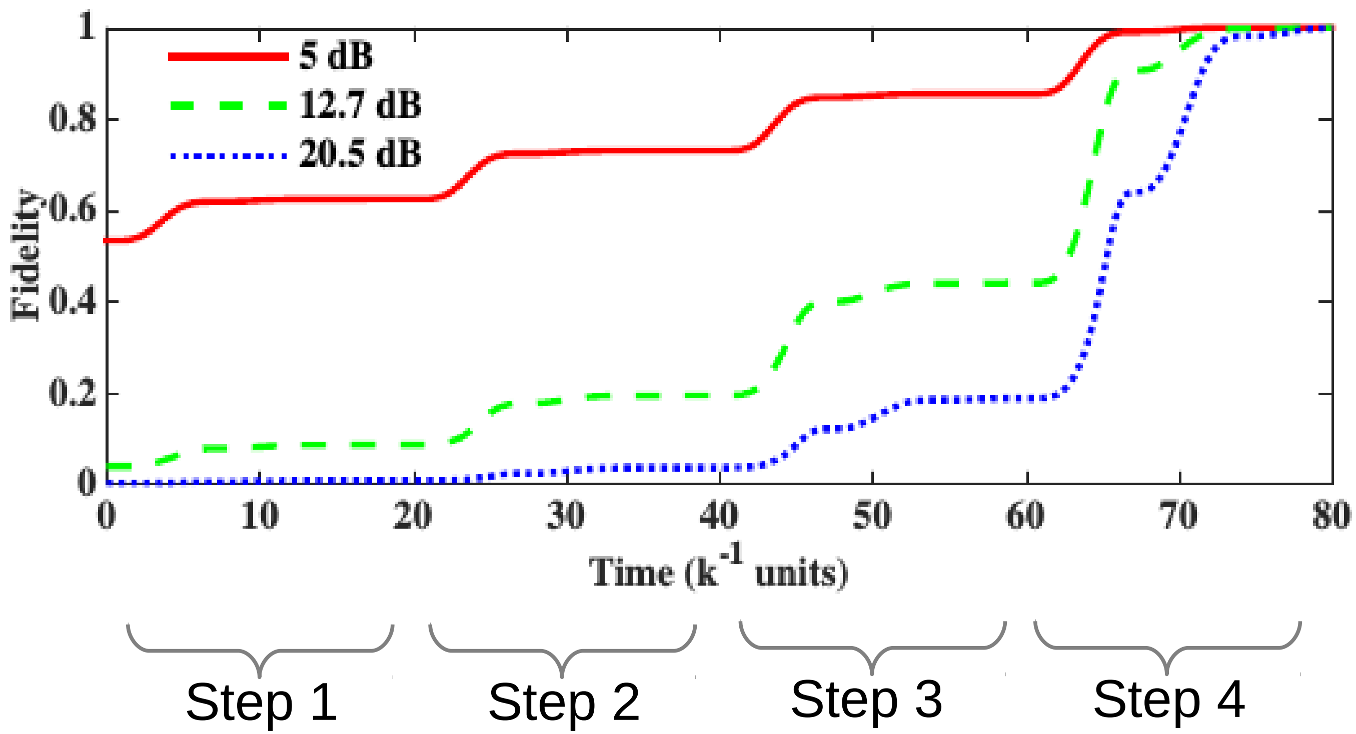
$$H \equiv H^{(k)} = \beta a^\dagger (c_k + r c_k^\dagger) + \mathrm{H.c.}$$

Example: 4-mode linear graph



$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \longrightarrow -(i I_N + A) = R W$$

Real time evolution of the fidelity:

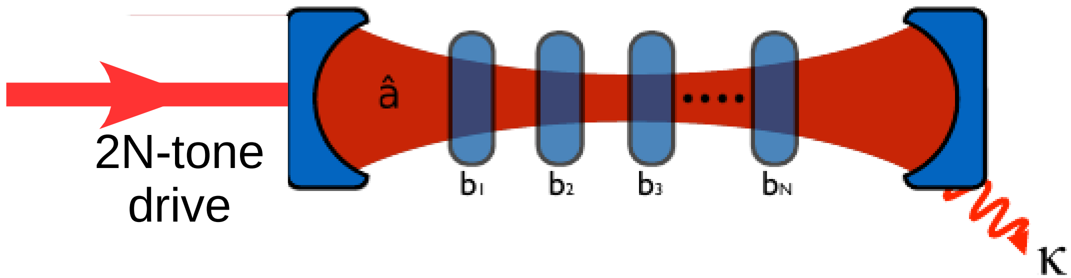


Finite-time evolution is enough to reach the target state

Hamiltonian engineering in optomechanics

Inspired by 1- and 2-mode schemes [Clerk, Hartmann, Marquardt, Meystre, Vitali,...]

$$\epsilon(t) = \sum_{k=1}^{2N} \epsilon_k e^{-i\omega_k t} e^{i\Phi_k}$$



$$\mathcal{H} = \omega_c a^\dagger a + \sum_{j=1}^{N} \left[\Omega_j b_j^\dagger b_j + g_j a^\dagger a (b_j^\dagger + b_j)\right] + \epsilon(t) a^\dagger + \epsilon^*(t) a$$

$$g_j \ll \omega_c, \Omega_j$$
$$a \to a + \alpha$$
$$b_j \to b_j + \beta_j$$

Two drives per mechanical mode

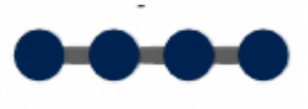
$$\omega_j^\pm = \omega_c \pm \Omega_j$$

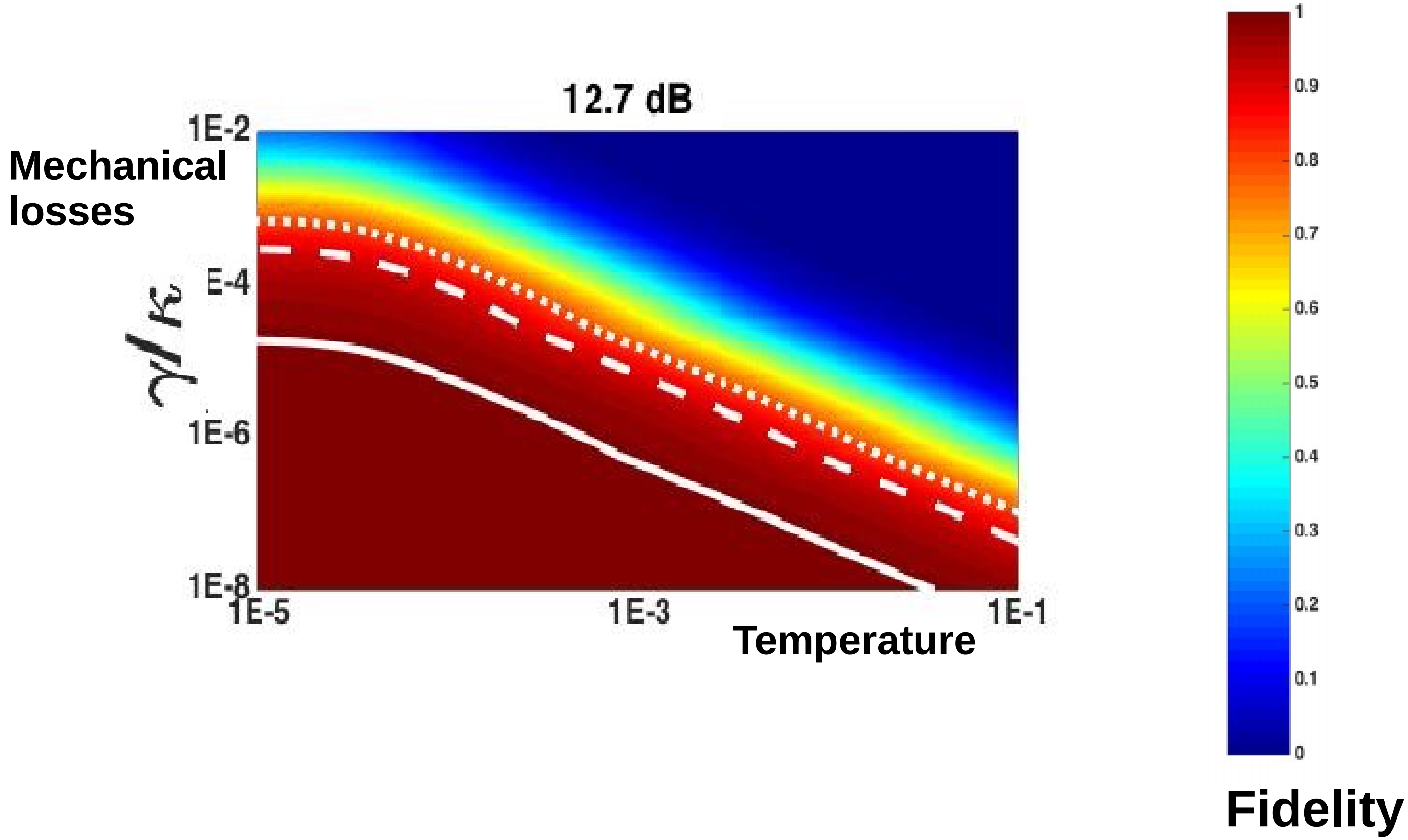
- Linearizing
- Non-overlapping mechanical frequencies $\alpha_j^\pm g_j \ll \Omega_j \Rightarrow \kappa \ll \Omega_j$
- Rotating wave approximation
- Resolved sideband regime

$$H = a^\dagger \sum_{j=1}^{N} g_j \left(\alpha_j^+ e^{i\phi_j^+} b_j^\dagger + \alpha_j^- e^{i\phi_j^-} b_j\right) + \mathrm{H.c.}$$

[Houhou, Aissaoui, AF, PRA '15]

Effects of mechanical noise: examples





Working regime:

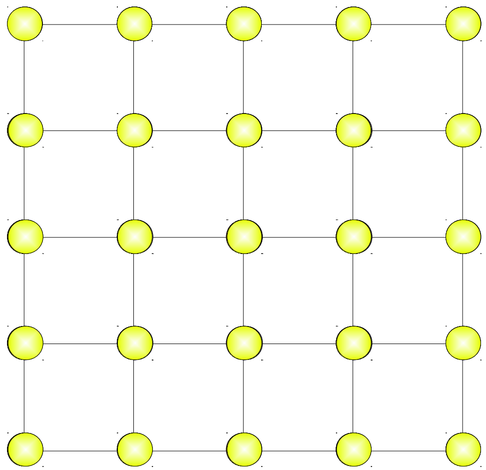
$$\gamma_j \ll \kappa \ll \Omega_j \text{ and low } T_j$$

[Houhou, Aissaoui, AF, PRA '15]

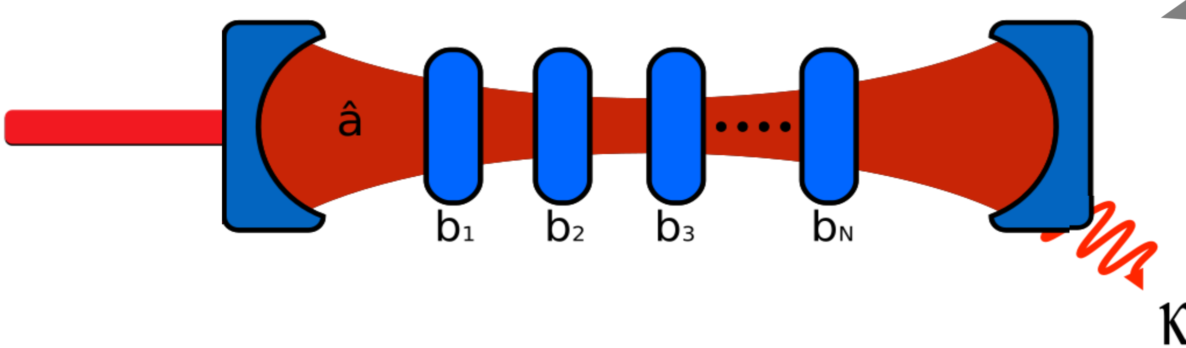
Outline

- Measurement-based quantum computation with CVs
- Generation of universal resources for CV quantum computation
- Arbitrary Gaussian computation

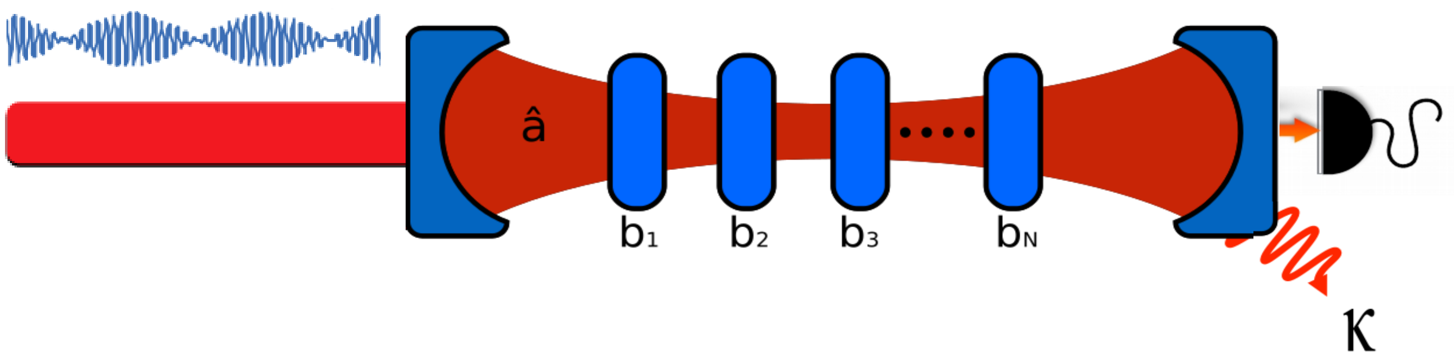
General Gaussian Computation



Once the cluster is generated (and verified), single- and two-mode Gaussian gates are implemented via projective **quadrature measurements**



Continuous-monitoring strategy

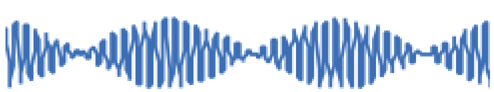


1) Modulate a driving with the mechanical frequency of the oscillator to be measured (the phase determines the addressed quadrature)

2) Continuously monitoring the output light

$$H = g(t)\,X \sum_j (b_j + b_j^\dagger) + \sum_j \Omega_j b_j^\dagger b_j$$

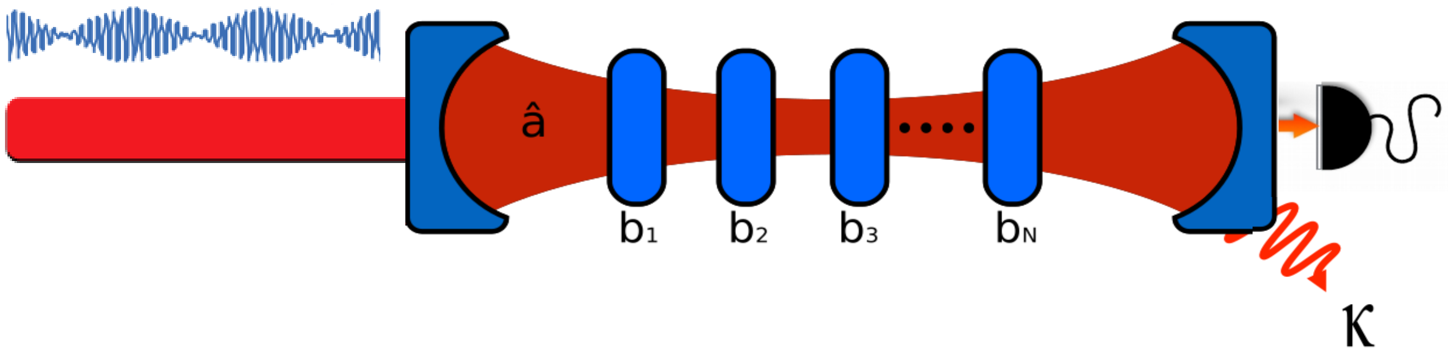
$$g(t) \equiv 2g' \cos(\Omega_j t + \theta)$$



$$H_{\rm int} = g' X \left[Q_{\theta,j} + Q_{0,j} \cos(2\Omega_j t + \theta) + Q_{\pi/2,j} \sin(2\Omega_j t + \theta) \right]$$

[Clerk, Marquardt, Jacobs, NJP (2008)]

Continuous-monitoring strategy



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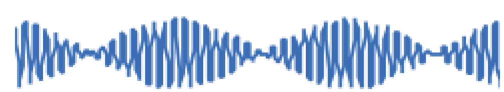
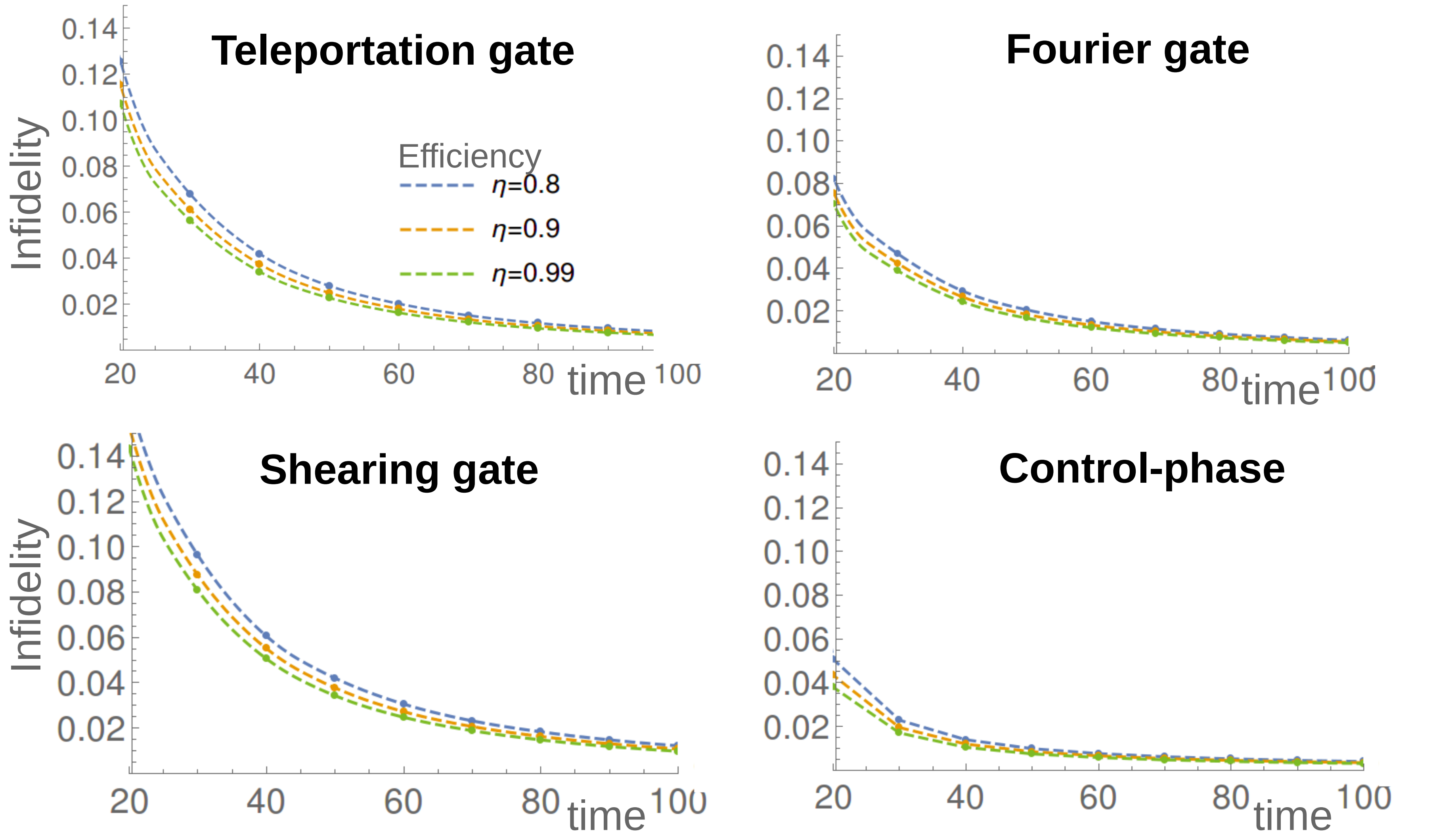


$$\boxed{H_{\rm int} = g'\, X\, [Q_{\theta,j}} + Q_{0,j}\cos(2\Omega_j t + \theta) + Q_{\pi/2,j}\sin(2\Omega_j t + \theta)]$$

QND measurement of an arbitrary quadrature of an arbitrary oscillator

[Clerk, Marquardt, Jacobs, NJP (2008)]

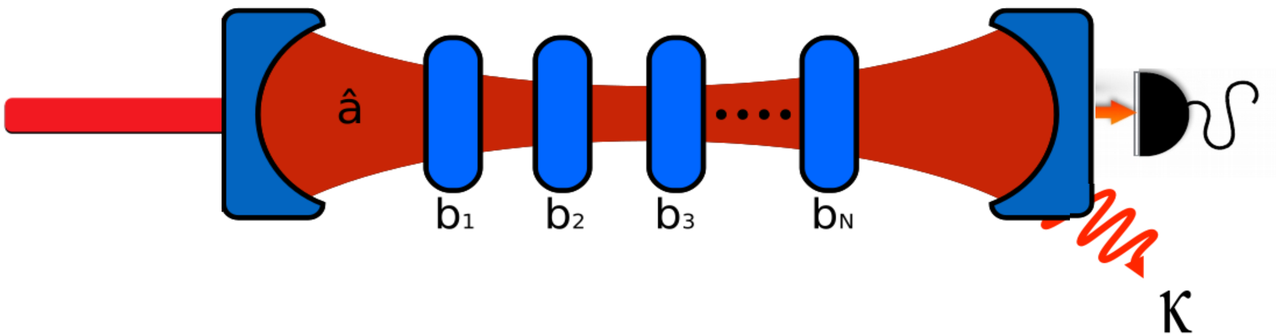
Universal single- and two-mode gates



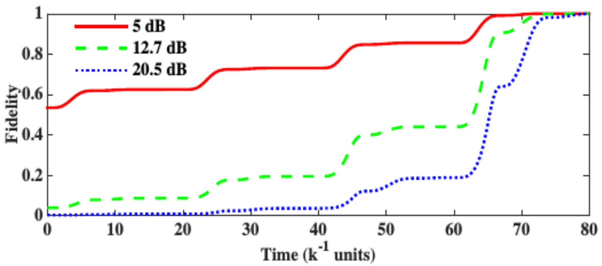
The difference (infidelity) between implementing the gates via ideal projective measurements and via continuous monitoring vanishes for long monitoring times (and low losses)

[Moore, Houhou, AF, arXiv:1609XXX]

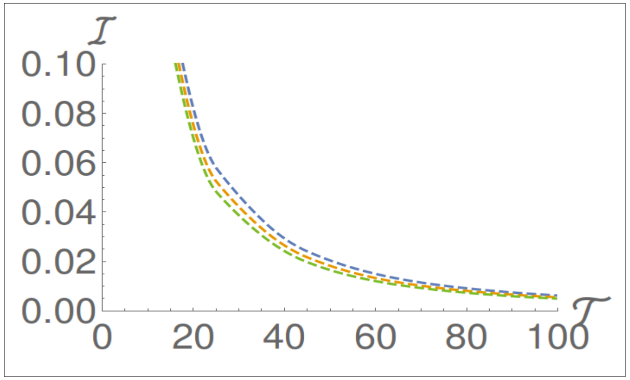
To Conclude



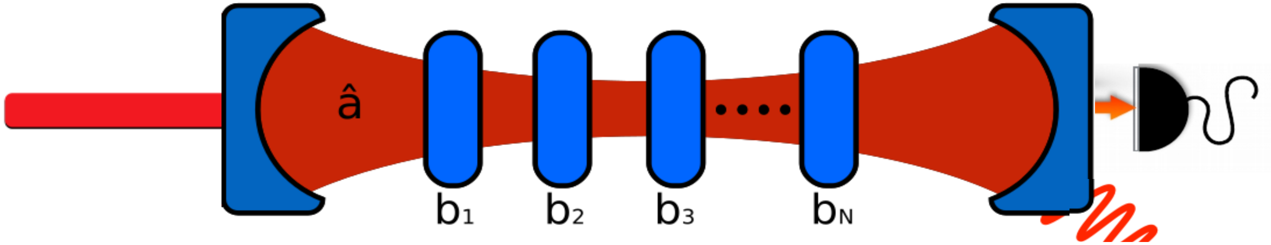
Cluster-state generation



Gaussian computation



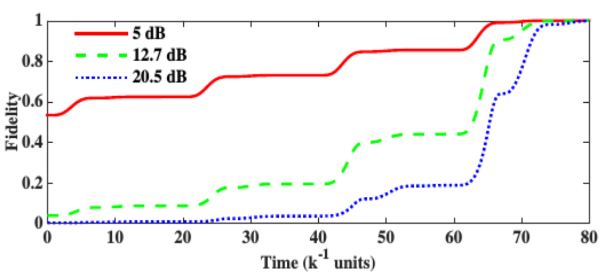
To Conclude



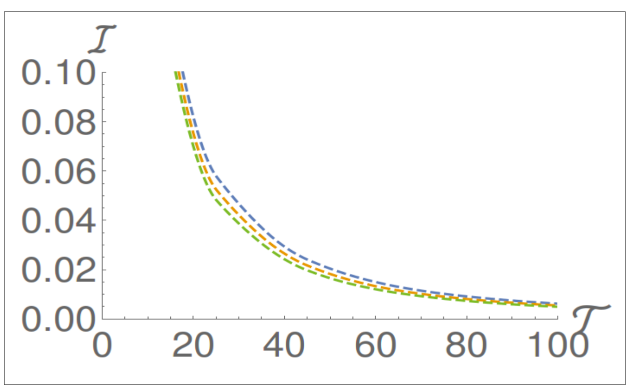
Opto-mechanics

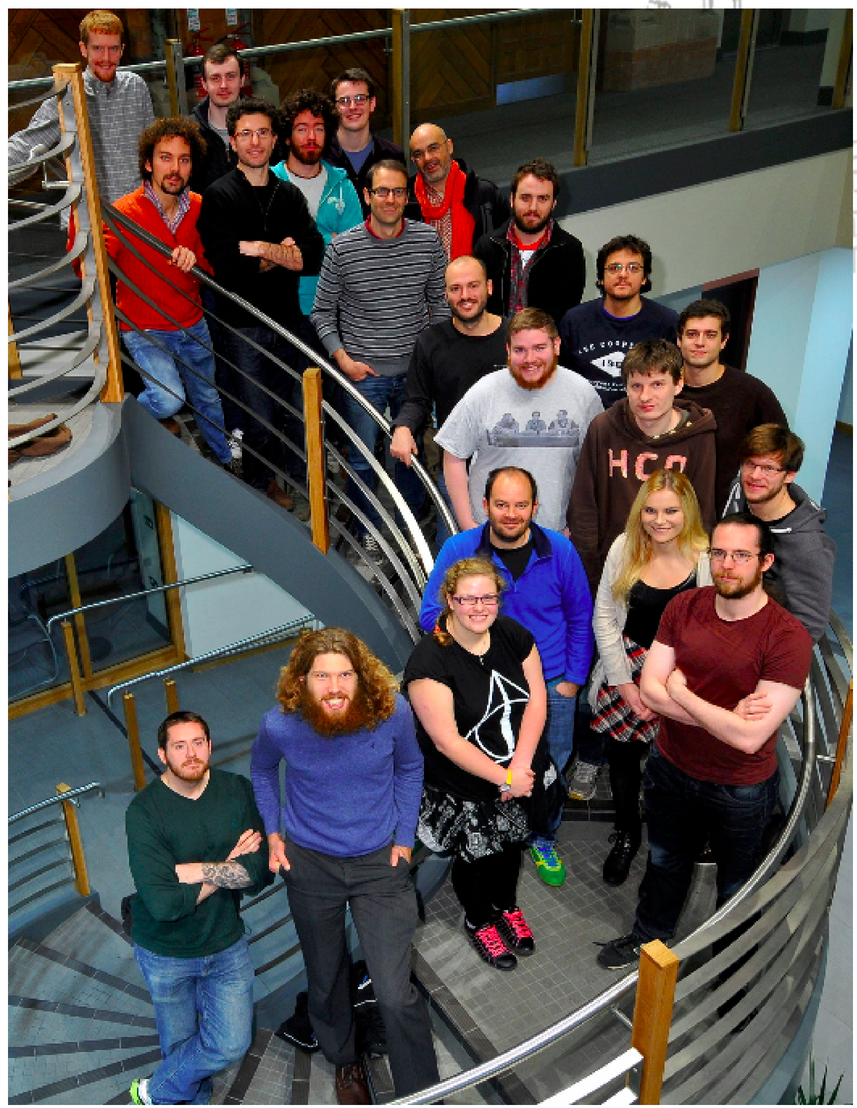
CV Quantum Computation

Cluster-state generation



Gaussian computation





QTEQ QUANTUM TECHNOLOGY at QUEEN'S

EPSRC

John Templeton Foundation

O. Houhou

(U Constantine,QUB)

D. Moore

(QUB)

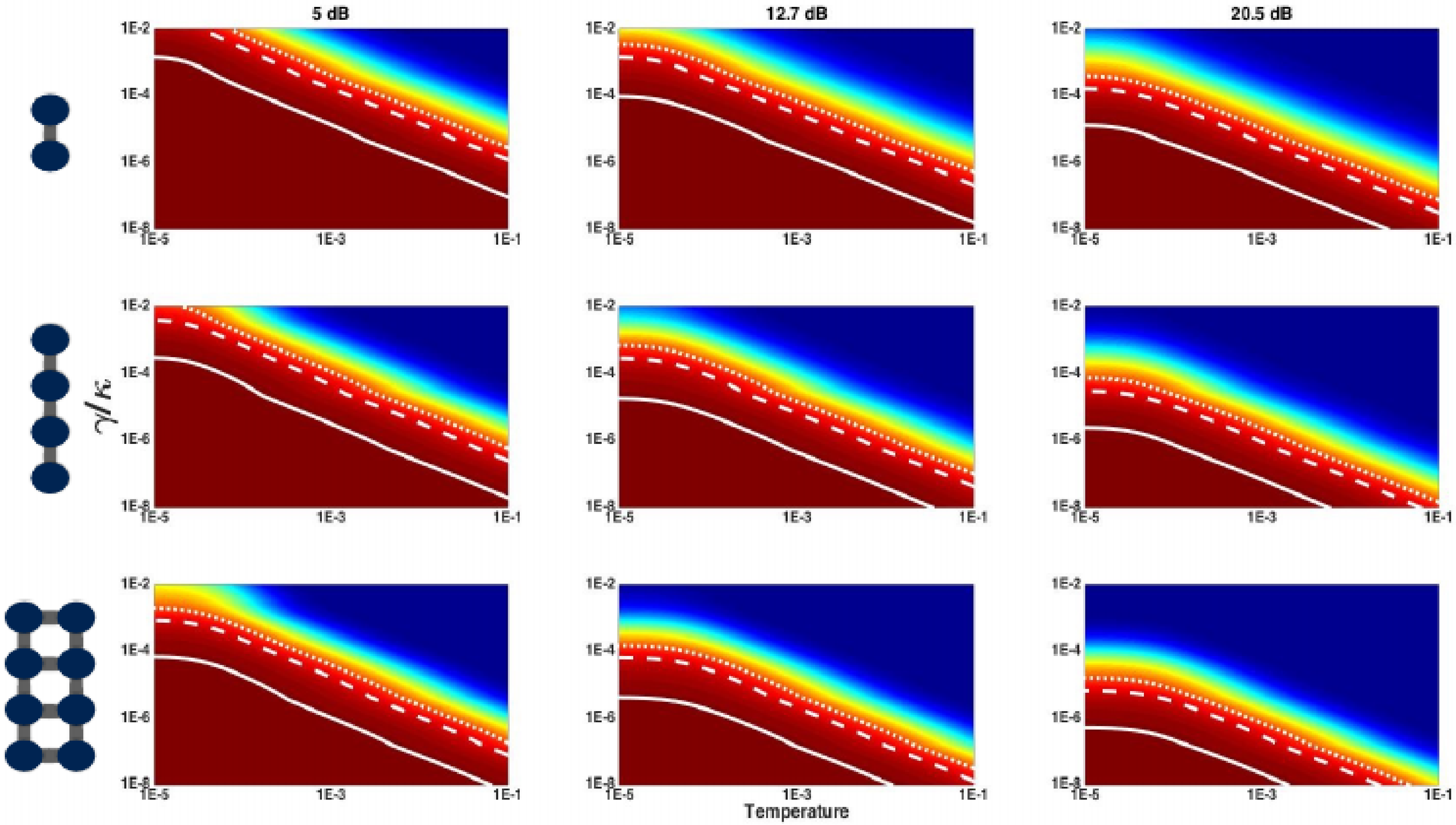
Effects of mechanical noise

Consider mechanical noise at temperature T_j and damping rate γ_j :

$$\frac{d\rho}{dt} = -i[H, \rho] + \kappa(a\rho a^\dagger - \frac{1}{2}a^\dagger a\rho - \frac{1}{2}\rho a^\dagger a) + \mathcal{L}_1 + \mathcal{L}_2$$

with γ_j , $\kappa \ll \Omega_j$:

$$\begin{aligned} \mathcal{L}_1 &= \sum_{j=1}^{N} \gamma_j(n_j + 1)\left(b_j\rho b_j^\dagger - \frac{1}{2}b_j^\dagger b_j\rho - \frac{1}{2}\rho b_j^\dagger b_j\right) \\ \mathcal{L}_2 &= \sum_{j=1}^{N} \gamma_j n_j \left(b_j^\dagger\rho b_j - \frac{1}{2}b_j b_j^\dagger\rho - \frac{1}{2}\rho b_j b_j^\dagger\right) \\ n_j &= \left(\exp\frac{\hbar\Omega_j}{K_B T_j} - 1\right)^{-1} \end{aligned}$$



- **The higher the target squeezing the less the tolerable noise**
- **The larger the target graph the less the tolerable noise**
- **Working regime:**

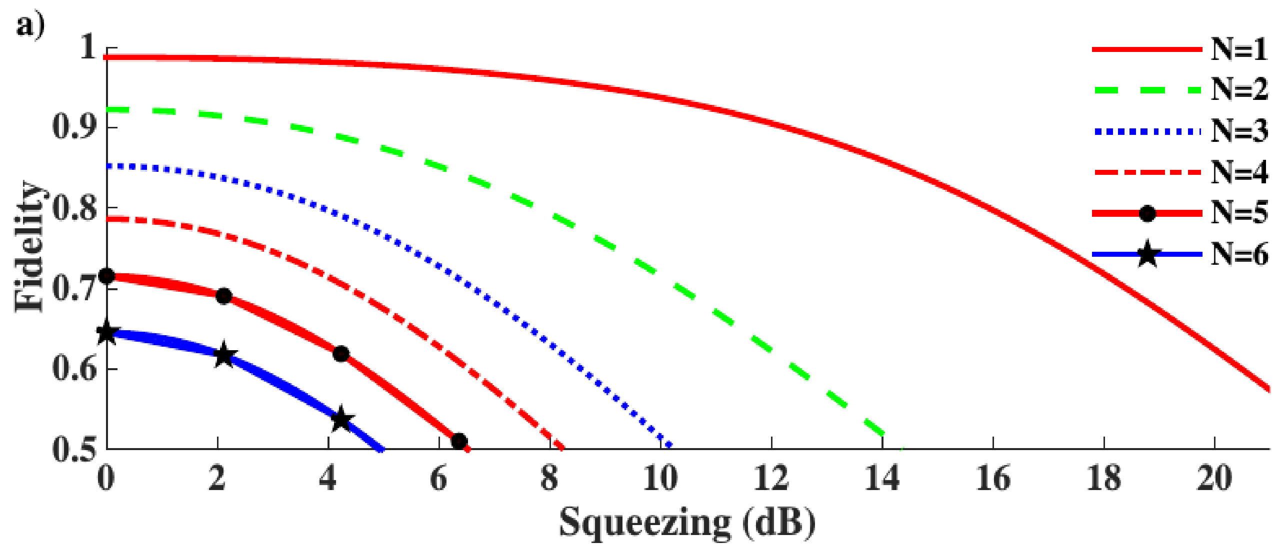
$$\gamma_j \ll \kappa \ll \Omega_j \text{ and low } T_j$$

Experimental feasibility

$$\Omega_j/2\pi = 11j \text{ MHz} \qquad \gamma/2\pi = 32 \text{ Hz} \qquad \kappa/2\pi = 0.2 \text{ MHz}$$

$T = 15$ mK

[Teufel et al., Nature (2011)]



$T = 1$ mK