Characterizing Quantum Supremacy in Near-Term Devices

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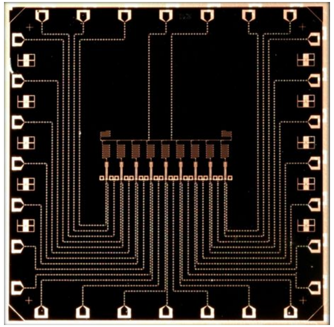
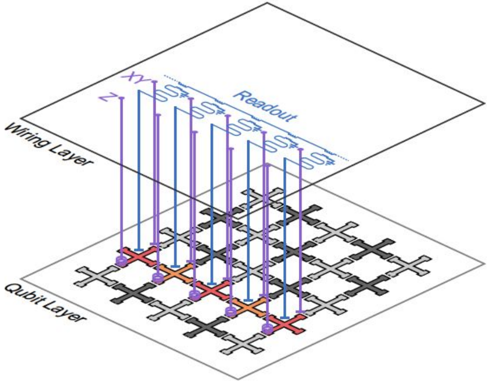
Google mission: build a quantum computer that does something useful, ASAP





Aiming for medium scale planar platform (e.g. 7 x 7 qubits) in near future

Long road to error correction; can intermediate devices perform useful computation?

We will solve *useless* and intractable problems before *useful* and intractable problems

1608.00263: minimal resource experimental demonstration of "Quantum Supremacy"

What is “Quantum Supremacy”?

Preskill (1203.5813): “when well controlled quantum systems can perform tasks surpassing what can be done in the classical world”

Boixo et al. (1608.00263): “when existing quantum devices can solve formal computational problems that cannot be solved on existing classical devices by implementing known algorithms in a reasonable amount of time”

Relative concept: “supremacy” implies competition with classical state-of-the-art

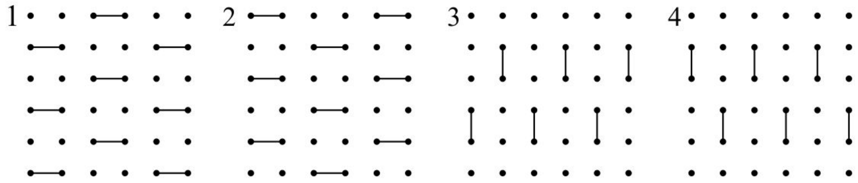
Practical: a description of status quo, not complexity theory result, not asymptotic

Task: sample from the output of a random quantum circuit

Select RQC U, apply to $|+\rangle^n$ state, sample bit strings $|x\rangle$ in computational basis

Produce samples with high likelihood, $\Pi_m \, p_U(x_m)$ where $p_U(x) = |\langle x|U|+\rangle^{\mathbf{n}}|^2$

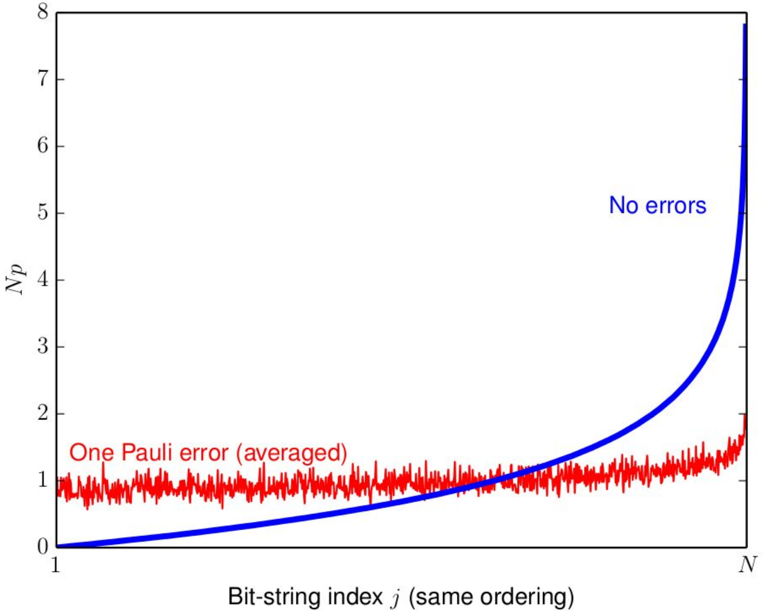
Our RQCs consist of CZ cycles on planar lattice with random $X^{1/2}$, $Y^{1/2}$ and T gates



RQCs converge to finite moments of Haar measure

Even single qubit Pauli errors decorrelate samples

Classical cost exponential in min(n, d $n^{1/2}$, τ)



A practical method to estimate the fidelity of RQCs

Cross entropy $H_{U,A} \equiv -\sum_x p_A(x) \log p_U(x)$ measures similarity of $p_U(x)$ and $p_A(x)$

$p_U(x) = |\langle x|U|+\rangle|^2$ follows Porter-Thomas distribution: $\mathrm{Prob}[p_U(x) > p] = N\, e^{-Np}$
When $p_A(x) = p_U(x)$ then $H_{U,A} = -\int p \log p\, (N e^{-Np})\, N\, dp = \log N - 1 + \gamma \equiv H_1$
For average U, when $p_A(x)$ uncorrelated with $p_U(x)$ then $\langle H_{U,A}\rangle_U = H_1 + 1 \equiv H_0$

Cross entropy difference is $\alpha_A \equiv H_0 - \langle H_{U,A}\rangle_U$ and approximates fidelity for RQCs
Consider noisy quantum circuit with fidelity f so that $p_A(x) = f\, p_U(x) + (1 - f)\, p_U^{\varepsilon}(x)$
$\alpha_A = H_0 + f \sum_x p_U(x) \log p_U(x) + (1 - f)\langle \sum_x p_U^{\varepsilon}(x) \log p_U(x)\rangle_U \approx H_0 - f H_1 - (1 - f) H_0 = f$

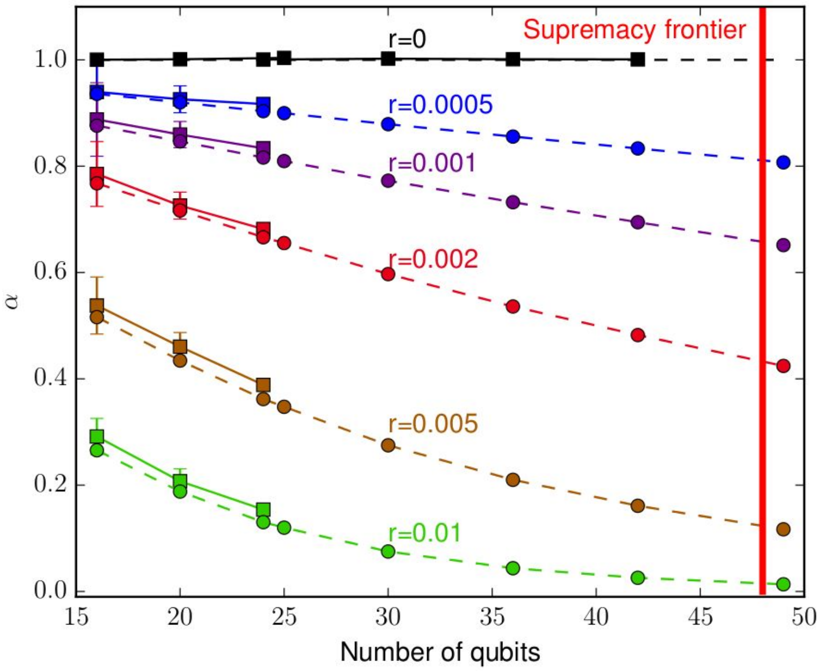
If $p_U(x)$ computable, CED efficiently measured as $H_0 + m^{-1} \sum_k \log p_U(x_k^A) + O(m^{-1/2})$

The experimental proposal for quantum supremacy

1. Randomly select RQCs, experimentally implement $U|+\rangle^n$; sample m strings $\{x_1, x_2, \dots, x_m\}$ in computational basis

2. Use classical supercomputer to compute probabilities $\{p_U(x_1), p_U(x_2), \dots, p_U(x_m)\}$ then, $f = \alpha = \log N + \gamma + m^{-1} \sum_k \log p_U(x_k) + O(m^{-1/2})$

3. Extrapolate α to supremacy regime; expect $f \propto \exp(- r * \text{number of gates})$



Probabilistic verification procedure possible

Summary of results

Planar RQCs with d > 27 and n = 7 x 7 likely cost $\Omega(2^n)$ to classically simulate

Would require at least 4 Petabytes of RAM with fast interconnect (doesn't exist)

Cross entropy difference is useful metric for benchmarking quantum circuits

Approximates fidelity for RQCs, efficient to measure if classical simulation possible

Sampling RQCs and achieving target CED defines minimal supremacy experiment

Based on current fidelities and digital error model, supremacy is within reach

Provided complexity theoretic reasons to believe this task is fundamentally hard

Extended results from IQP circuits (similar to BosonSampling) to broader class

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Complexity theoretic implications

$p_U(x) = |\langle x|U|+\rangle|^2 = \lambda\, |Z|^2$ computable using path-integral representation of U

For IQP circuits, Z is Ising partition function $Z = \sum_s e^{-i\, H\,(s)}$ with $H(s) = h\, s + s\, J\, s$

Z has sign problem; conjectured that Z is not approximable with NP-oracle

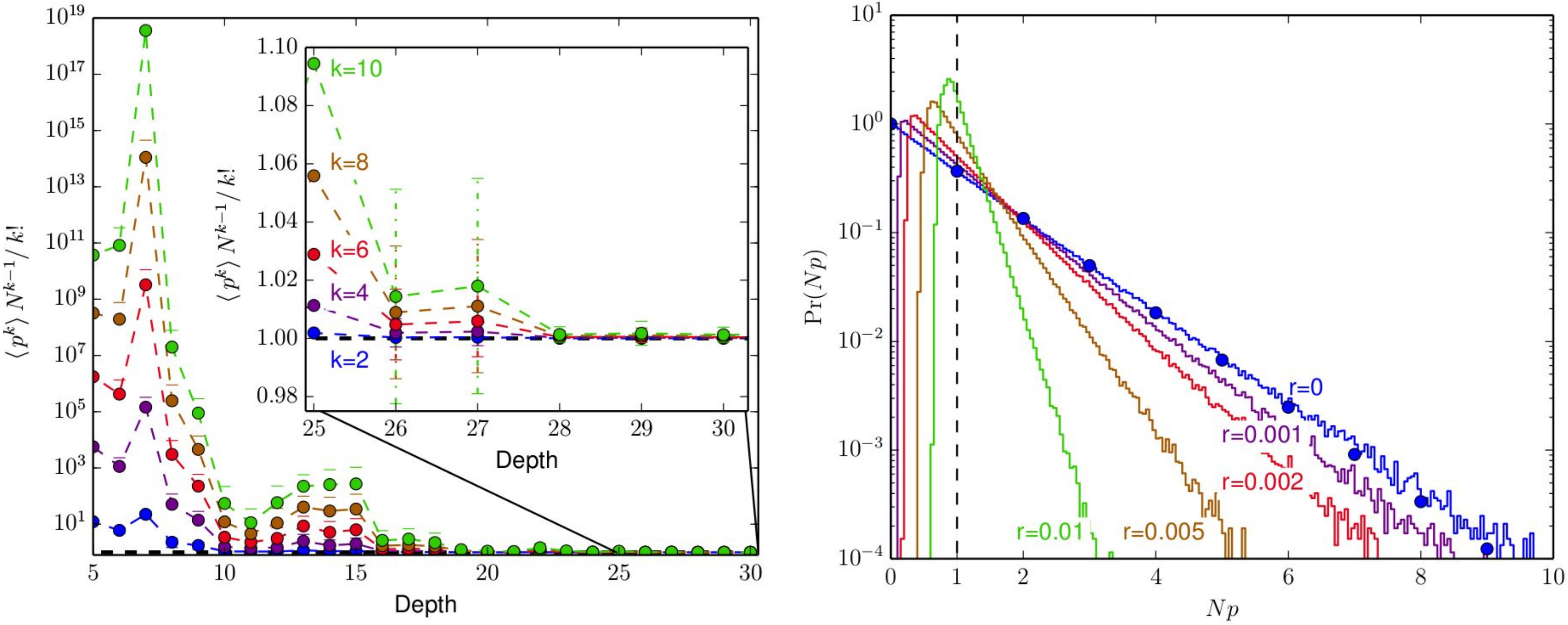
If $p_U(x)$ efficiently sampled, Z is approximable with NP-oracle (1504.07999)

Implies that efficient classically sampling would collapse polynomial hierarchy

Old result for BosonSampling (1011.3245) and IQP circuits (1005.1407)

We follow methodology of 1504.07999 and show same implications

Whereas IQP circuits map to 2D Ising model, our circuits map to 3D Ising model



$\langle p^k \rangle N^{k-1}/k!$
Depth
k=10
k=8
k=6
k=4
k=2
Depth
$\mathrm{Pr}(Np)$
Np
r=0
r=0.001
r=0.002
r=0.005
r=0.01