Quantum non-malleability and authentication

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Motivation: a classical story...

Crypto for bank transfers

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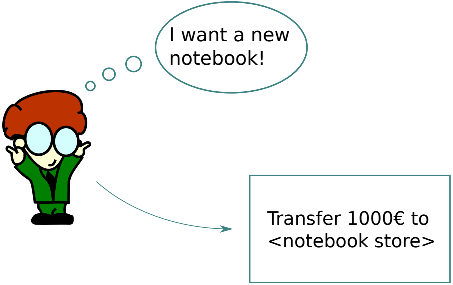

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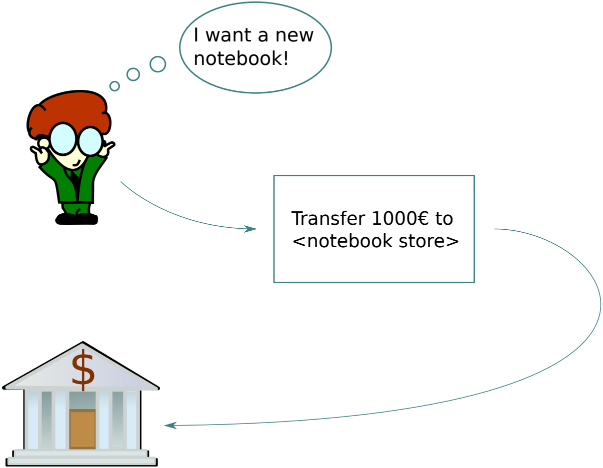

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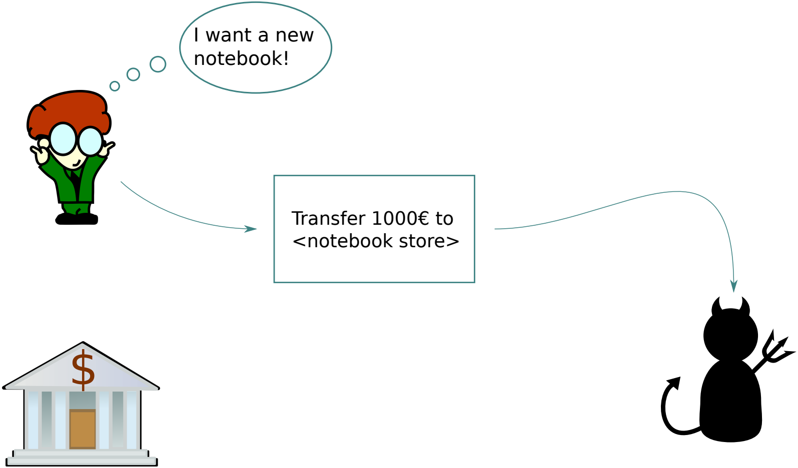

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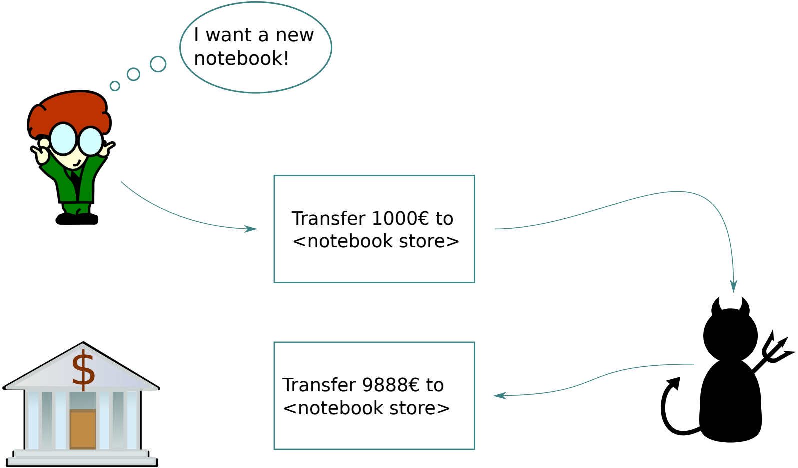

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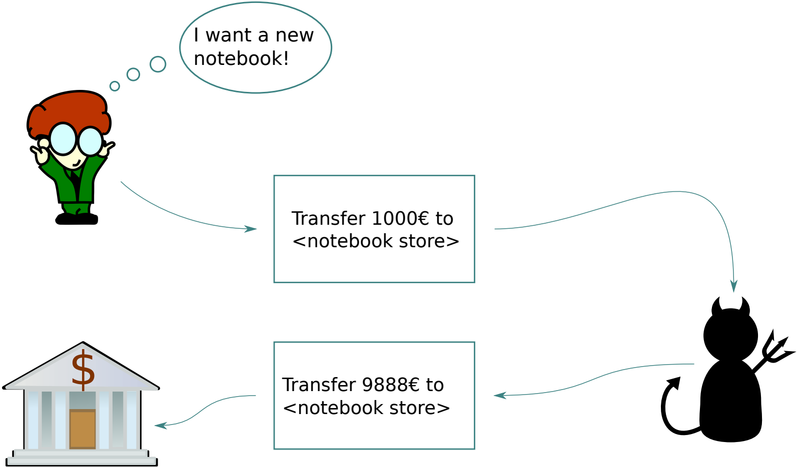

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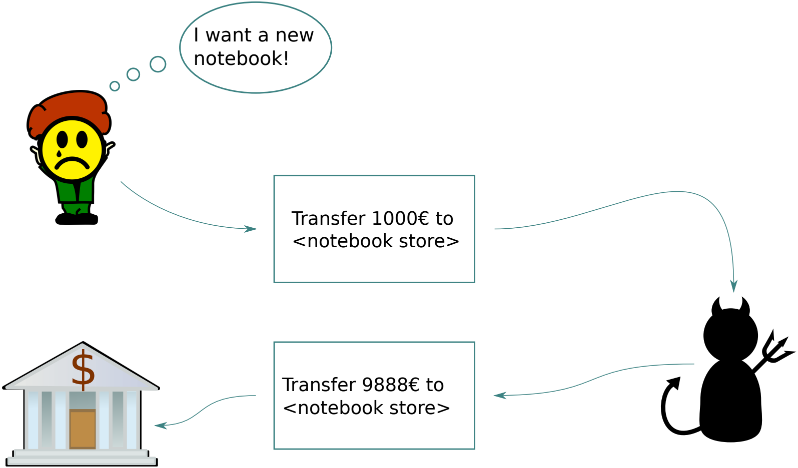

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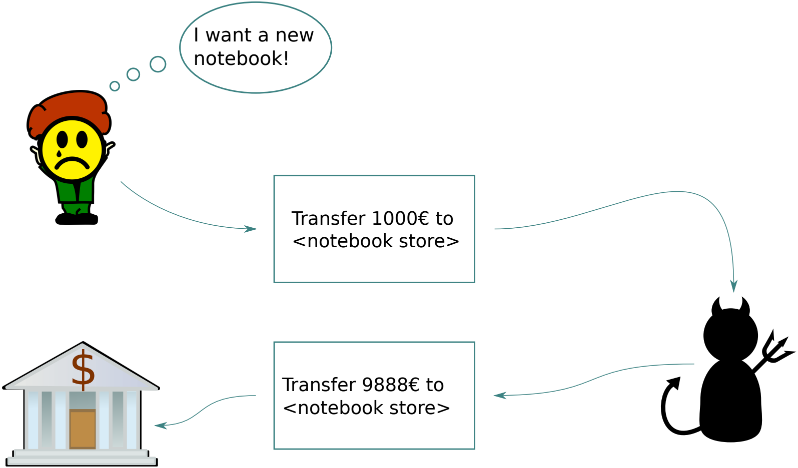

- What cryptographic security notions would fix this problem?

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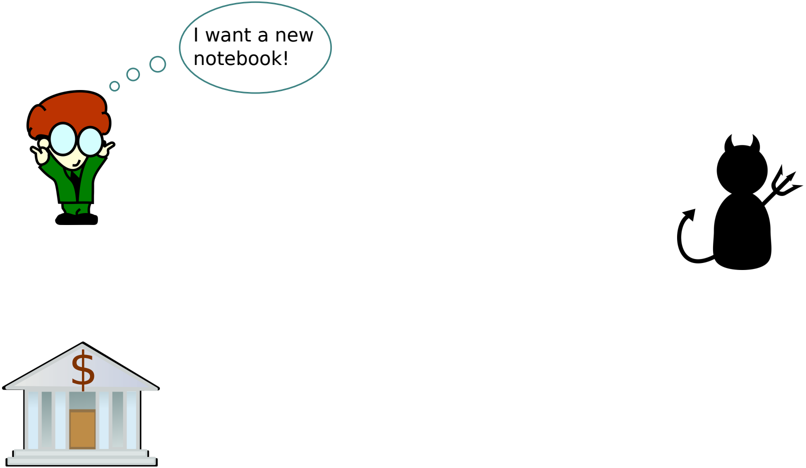

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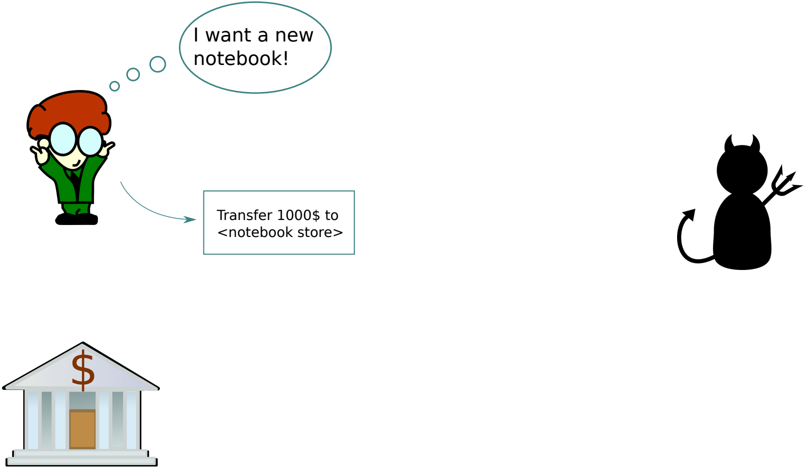

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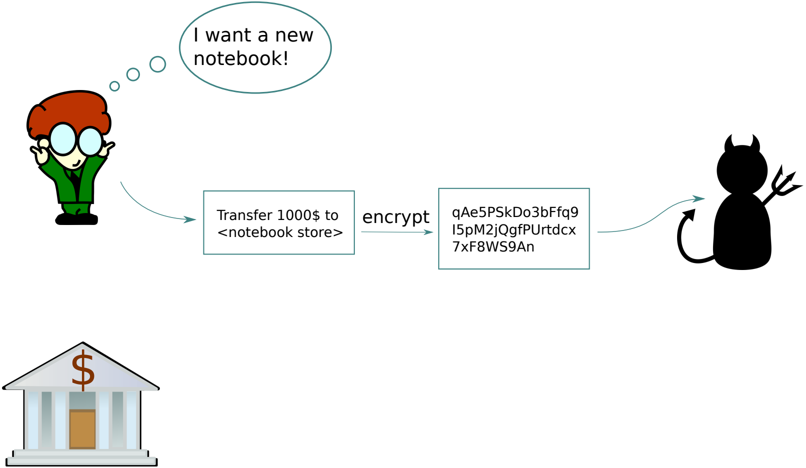

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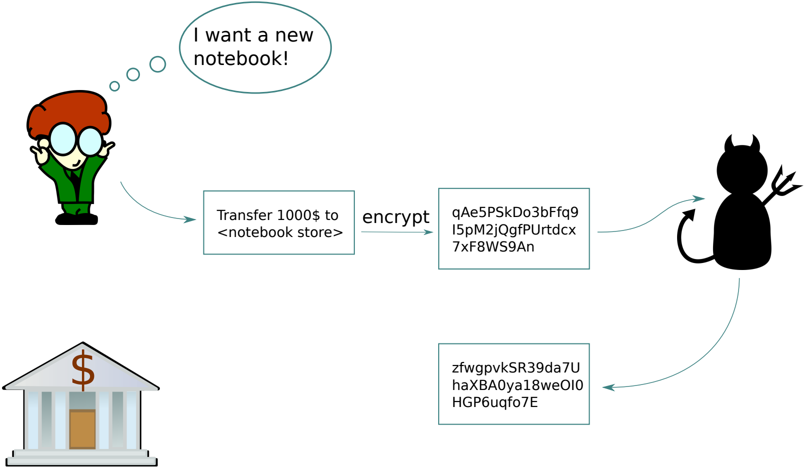

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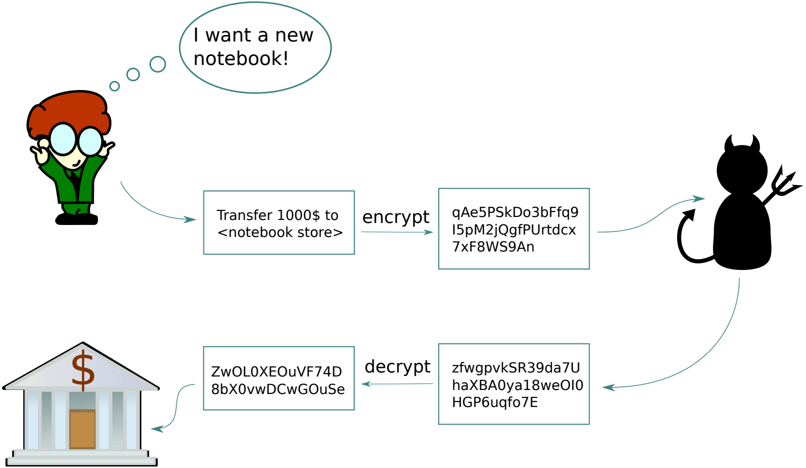

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♠ Additional result: The new definition of quantum authentication with key recycling by Garg, Yuen, Zhandry, '16, can be fulfilled using unitary 2-designs.

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- Information theoretic definition using entropy:
  (X, C), $(\tilde{X}, \tilde{C})$ two plaintext ciphertext pairs, $C \neq \tilde{C}$

def: scheme is NM if $I(\tilde{X} : \tilde{C}|XC) = 0$ (Hanaoka et al. '02)

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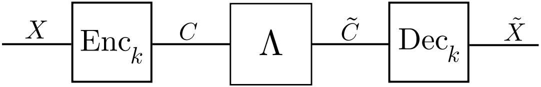

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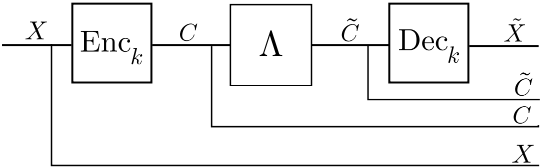

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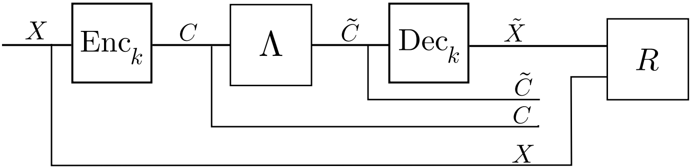

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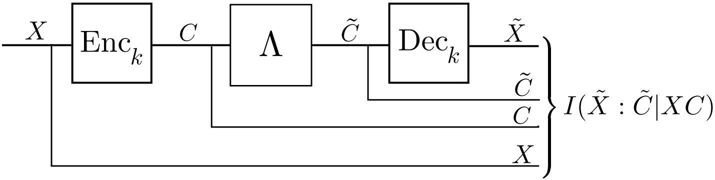

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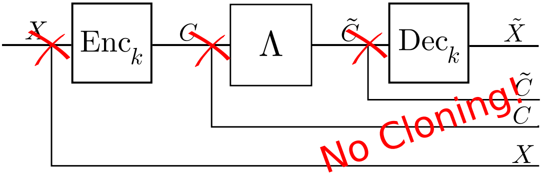

Quantum symmetric key encryption

def: Quantum encryption scheme: $(\mathrm{Enc}_k, \mathrm{Dec}_k)$

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- encryption map $(\mathrm{Enc}_k)_{A \to C}$, decryption map $(\mathrm{Dec}_k)_{C \to \bar{A}}$

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- correctness: $\mathrm{Dec}_k \circ \mathrm{Enc}_k = \mathrm{id}_A$
- average encryption map: $\mathrm{Enc}_K = \mathbb{E}_k \mathrm{Enc}_k$

Setup for q-non-malleability

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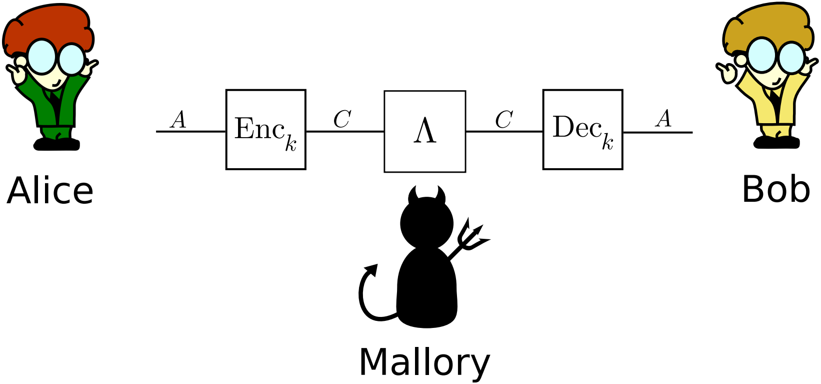

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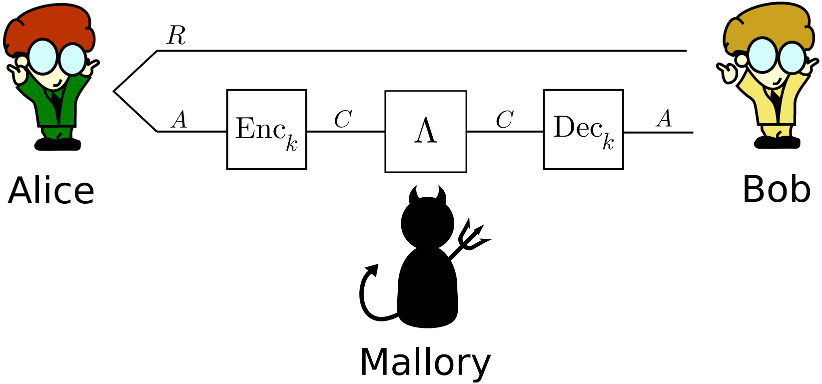

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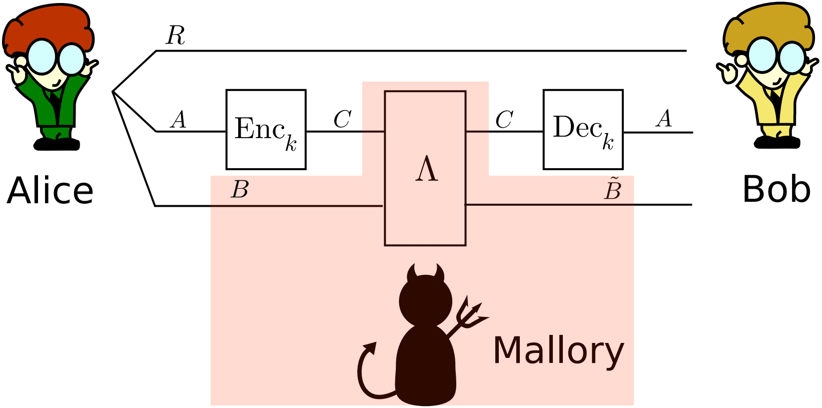

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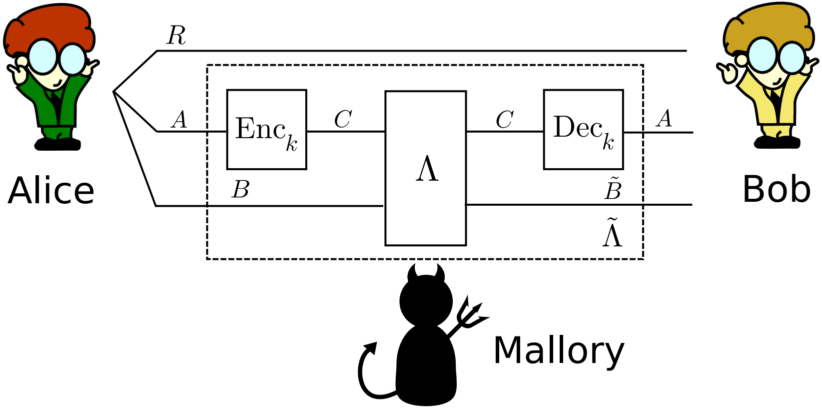
def: effective map on plaintexts and side info
$\tilde{\Lambda} = \mathbb{E}_k[\mathrm{Dec}_k \circ \Lambda \circ \mathrm{Enc}_k]$



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- "probability of Λ acting as the identity on C"

$\Rightarrow$ $p_{=}(\Lambda) = p$ for the example if $\mathrm{tr}(U_C) = 0$.

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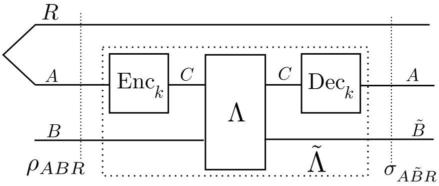
Definition (Quantum non-malleability (qNM))

A scheme $\Pi = (\mathrm{Enc}_k, \mathrm{Dec}_k)$ *is non-malleable, if for all states* ρ_{ABR} *and all attacks* $\Lambda_{CB\to C\tilde{B}}$,

$$I(AR:\tilde{B})_\sigma \leq I(AR:B)_\rho$$

with $\sigma_{A\tilde{B}R} = \tilde{\Lambda}_{AB\to A\tilde{B}}(\rho_{ABR})$.



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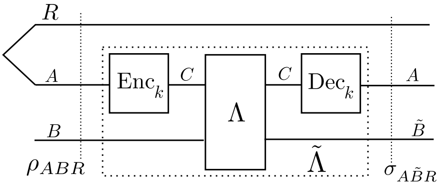

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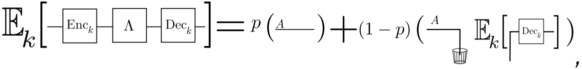
Definition (ABW-NM, Ambainis, Bouda, Winter '09)

Let $\Pi = (\mathrm{Enc}_k, \mathrm{Dec}_k)$ *be a quantum encryption scheme.* Π *is ABW-NM if*



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Comparison to previous definition

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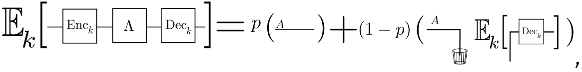

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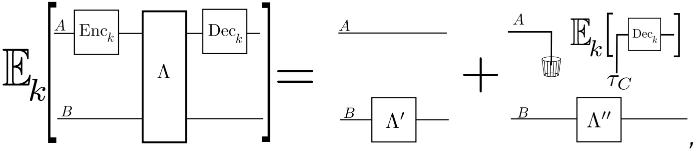
Theorem (Alagic, CM)

Let $\Pi = (\mathrm{Enc}_k, \mathrm{Dec}_k)$ *be a quantum encryption scheme.* Π *is qNM if and only if*



where Λ' *and* Λ'' *are explicitly given in terms of* Λ.

Improvements

The new definition

... allows adversaries with side information

... prevents plaintext injection attack

... provides *ciphertext* non-malleability

while ABW-NM does not.

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- ! qNM $\Rightarrow$ information theoretic IND
- qNM serves as primitive for quantum authentication schemes
  $\Rightarrow$ last part of the talk

Summary non-malleability

	ABW-NM	qNM
assumes secrecy	✓	✗
implies secrecy	✗	✓
secure against plaintext injection	✗	✓
primitive for authentication	✗	✓

Authentication

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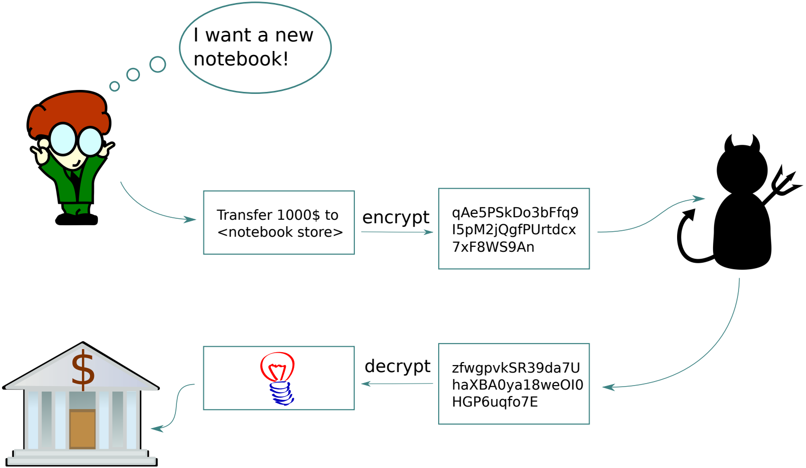

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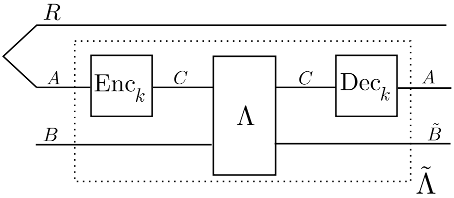
Definition (GYZ Authentication; Garg, Yuen and Zhandry)

$\Pi = (\mathrm{Enc}_k, \mathrm{Dec}_k)$ *is* ε*-GYZ-authenticating if, for any attack* $\Lambda_{CB \to CB'}$*, there exists* $\Lambda^{acc}_{B \to \tilde{B}}$ *such that for all* ρ_{AB}

$$\mathbb{E}_k \left[\left\| \Pi_{acc} \left[\mathrm{Dec}_k \circ \Lambda \circ \mathrm{Enc}_k(\rho_{AB}) \right] \Pi_{acc} - \left(\mathrm{id}_A \otimes \Lambda^{acc} \right)(\rho_{AB}) \right\|_1 \right] \leq \varepsilon$$

with $\Pi_{acc} = \mathbb{1} - \perp$.



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- advantages: shorter keys, nice constructions (Clifford group)

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consider pure states and attack isometries (Stinespring)

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want to bound

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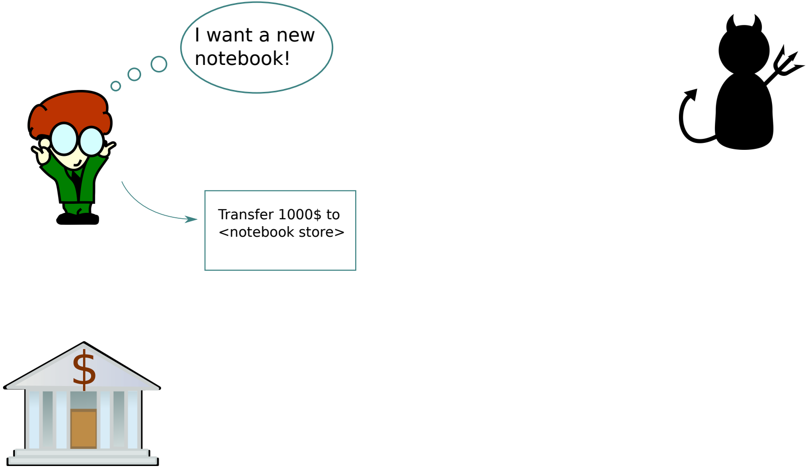
Use "swap trick" $\mathrm{tr} A_X B_X = \mathrm{tr} S_{XX'} A_X \otimes B_{X'}$ and Schur's lemma for $U \mapsto U\otimes U$

Authentication from NM: Intuition



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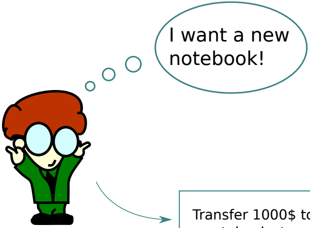

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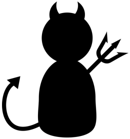
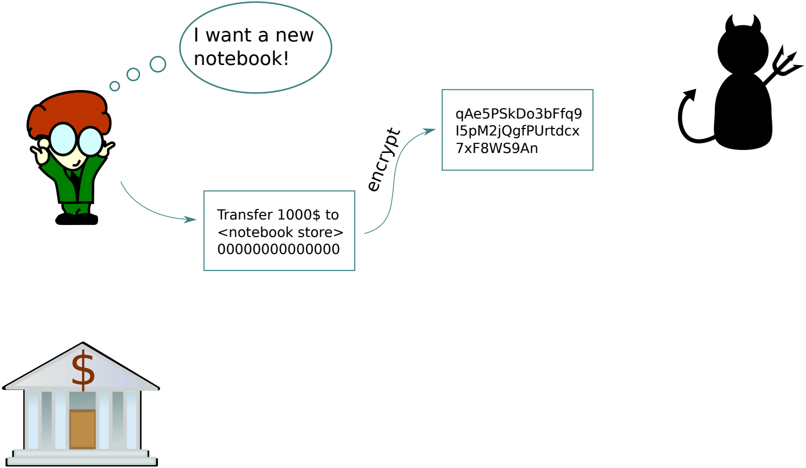
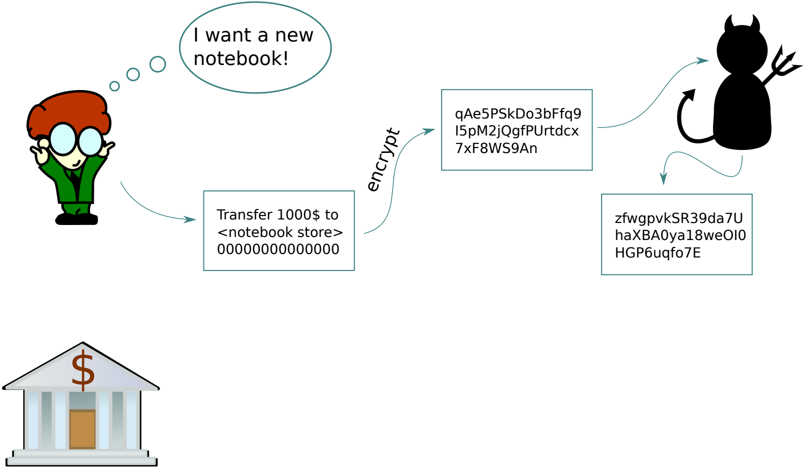


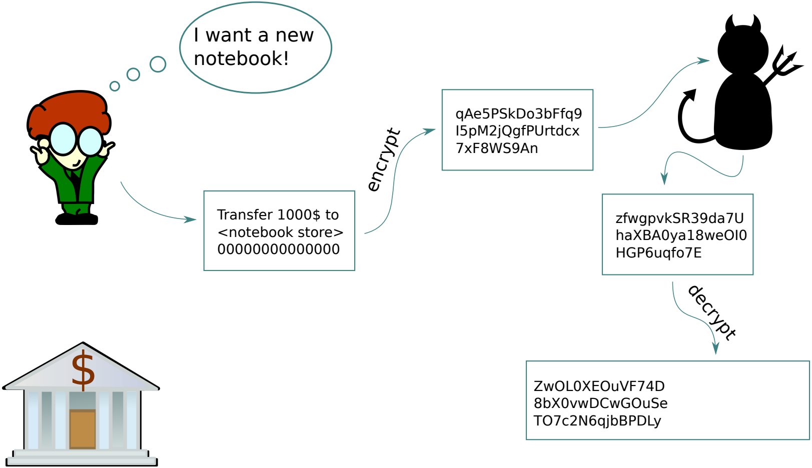
Authentication from NM: Intuition



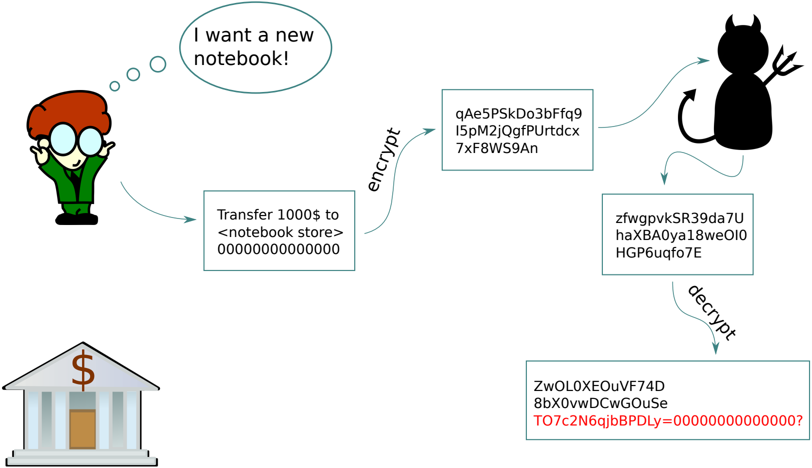
Authentication from NM: Intuition



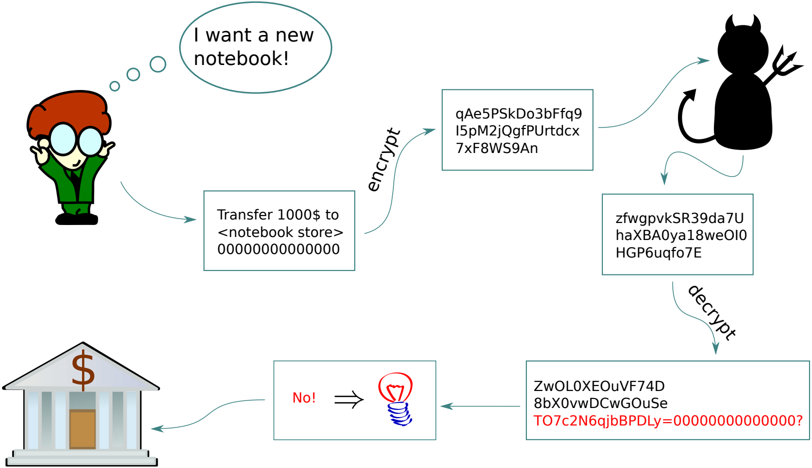
Authentication from NM: Intuition



Authentication from NM: Intuition



Authentication from NM: Intuition



Authentication from qNM

Theorem (Alagic, CM)

Adding a constant tag to a quantum message and encrypting it with an qNM scheme achieves DNS-authentication

Summary authentication

- ✓ DNS authentication from qNM schemes via tagging
- ✓ GYZ authentication from 2-designs instead of 8-designs

Open questions

Computational security?

Current work with Christian Majenz and Tommaso Gagliardoni

Can we improve the Λ-dependence of NM?

NM with high probability?