Dining Philosophers, Leader Election and Ring Size problems, in the quantum setting

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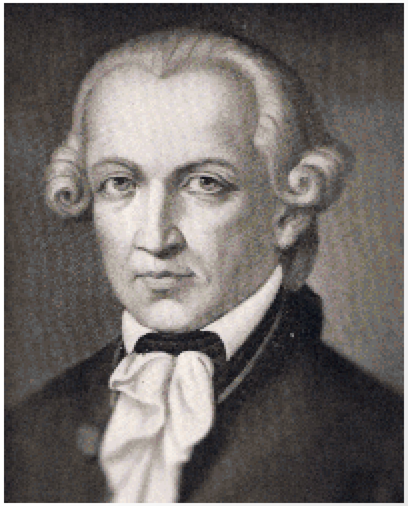
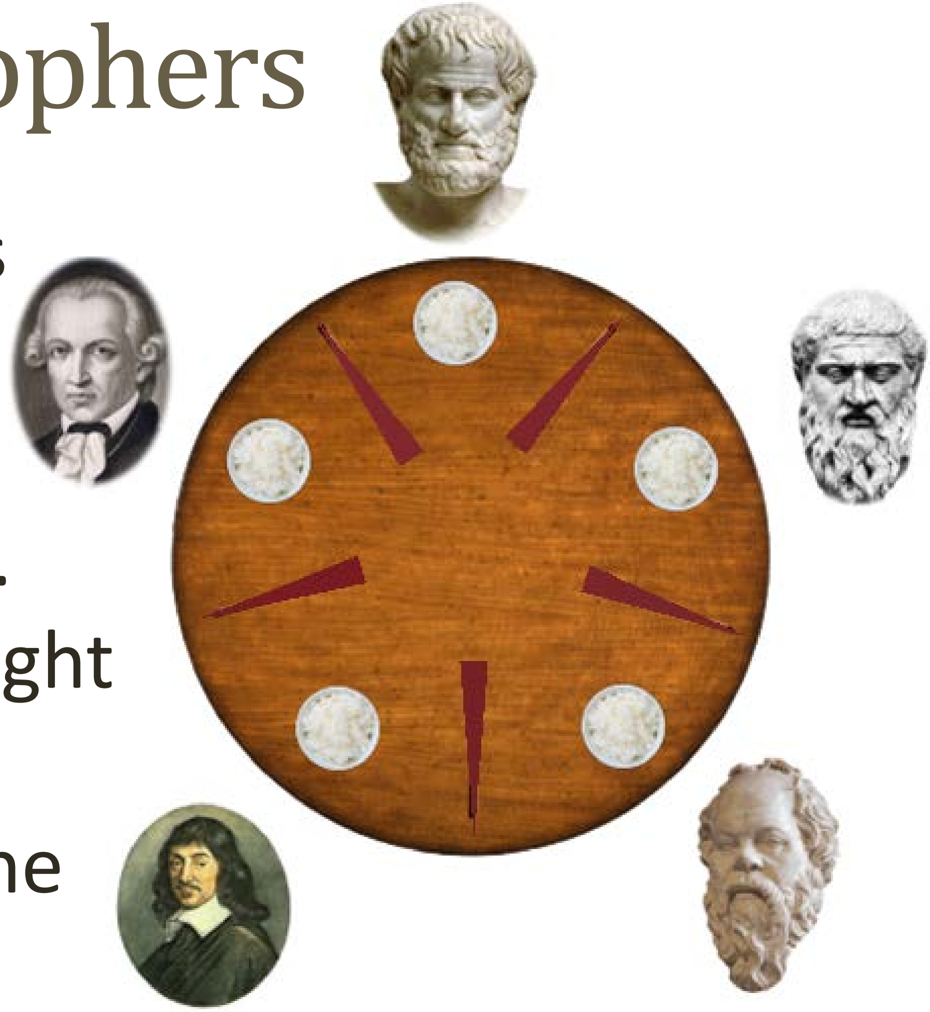


Dining philosophers



Group of philosophers sitting around a table. Between each pair, there is one chopstick. At some point they might get hungry. They are anonymous and run the same algorithm.

The algorithm should make sure that every hungry philosopher would eventually eat.

Random solution

- Lehman Rabin [LR96] showed a random solution.
- Idea: When hungry, each philosopher randomly picks a side, and waits until he can lift the corresponding chopstick. Then he tries to lift the other.
  Upon failing, he puts down the first chopstick and repeats: choose a random side, etc…

No exact DP in classical world

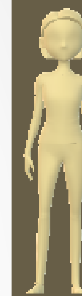
- In the classical settings, there is no exact (deterministic) solution to the DP problem [LR96].
- Essentially, since the philosophers are identical, and start from the same state, they will always stay in the same state.
- One can not break the symmetry.

First result

- The DP was not studied in the quantum setting before.
- We prove an exact (deterministic) solution.
- Theorem 1: There exists a deterministic protocol for the DP problem that guarantees that every hungry philosopher will eat in a finite amount of time.

We do this using:

Leader Election


- N anonymous, symmetric parties want to elect a unique leader. If they had unique I.D, they could elect the highest I.D. as the leader.
- Notorious problem in distributed algorithms.



- When N is known, randomized solution exists. [IR90]
- Like DP, there is no classical exact solution.

Quantum exact solution

- Tani et al [Tani12] showed that in the Quantum setting, it is possible to break the symmetry, and solve the exact LE problem.
- In order to break the symmetry, they introduce the *magic unitary* U_n*:*
- *Given the state* $|0^n\rangle + |1^n\rangle$ *over* n *parties, if each party applies the local unitary* U_n*, the system transpose to a non-symmetric state (with 0 support over* $span(|0^n\rangle, |1^n\rangle)$ *)*

Our results

- Theorem 2: There exists a deterministic quantum protocol to the DP problem, when n or an upper bound on n is known, which uses O(1) quantum memory and O(log n) classical memory per philosopher and O(n^2) time complexity.
- Theorem 3: Using this algorithm we achieve a significantly more efficient LE protocol on a ring, in time, memory and communication complexity, than the best known algorithm [Tani12].

Remainder of talk

- More rigorous definition of DP.
- Symmetry breaking.
- Simple DP solution.
- Efficient DP solution.
- Efficient LE solution using DP.
- Summary.

Distributed dining philosophers

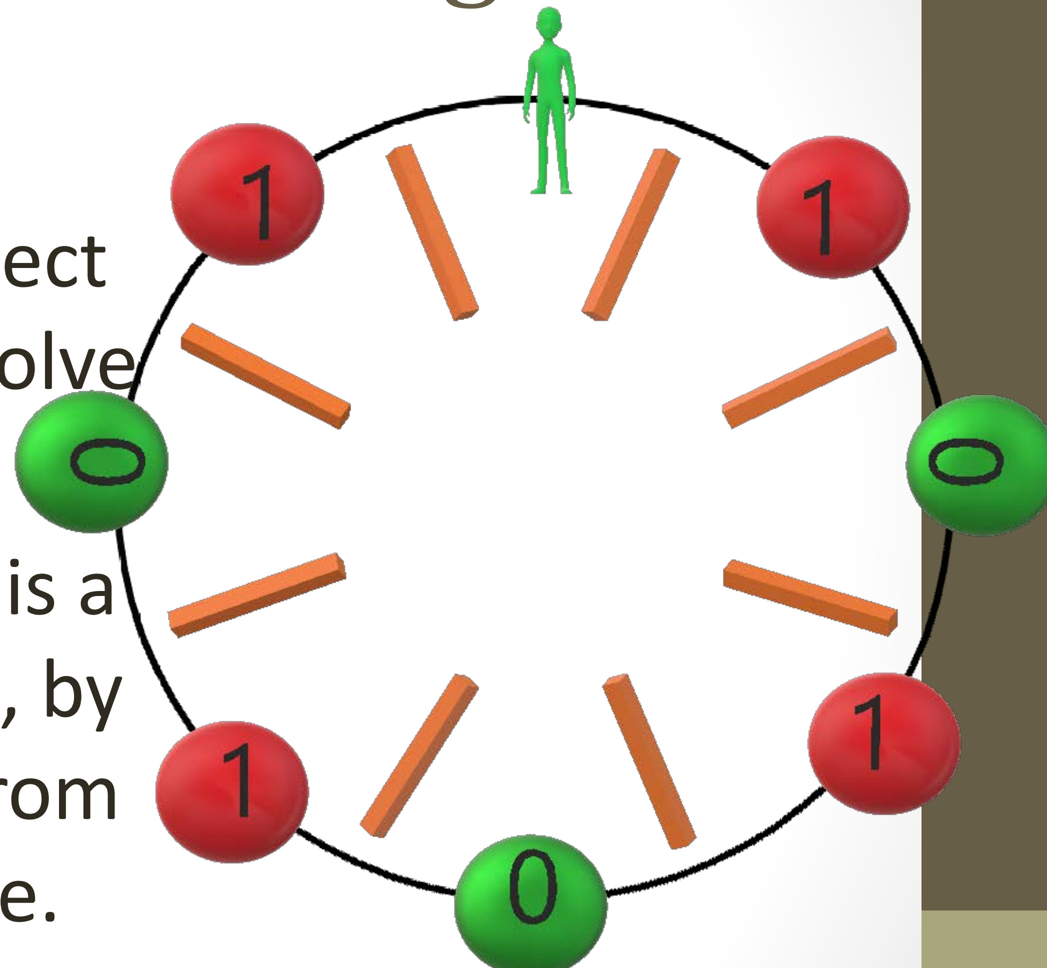
- The setting is asynchronous, anonymous.
- Assume one evil scheduler (CPU), and each philosopher a thread, and the scheduler gives the token.
- We want to ensure that one of the hungry philosophers will eventually eat.
- An algorithm that ensures that, can ensure (with slight modification) that no-one will starve.

DP by breaking symmetry

- Lemma: given a division into 2 groups, one can solve the DP.
  The 0 group will try to lift left chopstick first, and the 1 group will try to lift the right chopstick first.
- Proof idea: assume deadlock, look at a philosopher that lifted a chopstick, his neighbor is in the same group etc…

Simple DP solution using LE

- Given that the philosophers can elect a leader, they can solve the DP problem.
- For example, there is a symmetry breaking, by parity of distance from the leader clockwise.



- Exact [Tani12] solution for LE implies exact DP. But inefficient.

Our more efficient DP algorithm

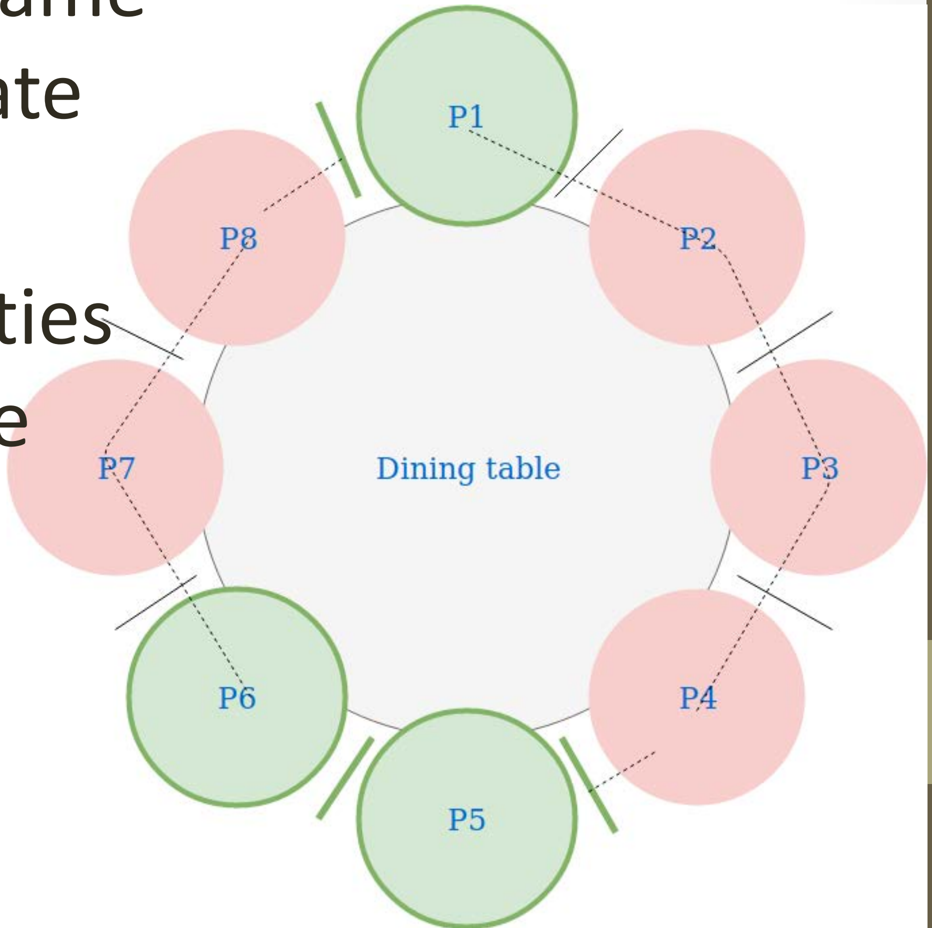
- Each philosopher creates EPR, gives one qubit to his right neighbor (hence also receives one).
- Each philosopher, checks if the values of the two qubits in the computational basis are identical or not. If not - symmetry was broken.
- If all qubits are identical, then the system is in the symmetric state $|0^n\rangle + |1^n\rangle$.
- Each philosopher applies [Tani12]'s magic unitary U_n and measure in the computational basis.
- The result defines the philosopher's group.

Our more efficient LE solution

- Interestingly, we can use the DP algorithm to simulate LE (note that in the classical case, DP does not imply LE which is a harder problem).
- Each phase, the parties that don't get to eat are eliminated.
- Each phase at least half the parties get eliminated, and at most half stay eligible.
- Total number of DP phases: O(logn).

One issue

- After the first round, we can't run the DP as is, because the same philosophers who ate can eat again.
- The eliminated parties will help the eligible parties to simulate chopstick lifts.



Comparisons table

n is known	Classical with error	Exact Quantum
DP	Constant space	**Constant quantum space**
LE	Logarithmic space	**Constant quantum space**

Open questions

- Can these exact protocols can be achieved with O(1) total (classical and quantum) memory?
- Related question with a physics flavor: Is there a constant depth, translation invariant quantum circuit over constant dimensional particles on a circle, that can break symmetry exactly?

No lockout

- In order to guarantee no lockout (there will be no starving philosopher), [LR96] introduced the *courteous condition*, which states:
  A philosopher who ate after his hungry neighbor, will not try to lift the chopstick.
- This solution guarantees no lockout.
- We used that idea in our algorithm to make them lockout free as well.

Random LE solution

- Random definition: we allow to not elect a leader, but choosing 2 leaders is forbidden.
- Random solution: If the parties know N in advance, each can randomly pick an I.D in the range $(1, N^2)$. And elect the highest, while ensuring no two such I.Ds.

What we didn't presented

- Complete distributed algorithms introduction.
- Fully detailed algorithms.
- Rigorous proofs.
- Exact complexities with exact definitions.
- Ring size problem.
- Case where only a bound on n is known.
- The exact relations between all these problems.

Questions?

Thank you

Deadlock and lockout

- There is the possibility of Deadlock.
  Which is the scenario where no philosopher ever gets to eat.
- Lockout condition is the scenario of a starving philosopher.

Ring size problem

- Definition.
- Classical impossibility.
- LE => Ring size.
- Quantum LE <= Ring size.

When a bound is known

- Still DP => LE.
- One need to be a bit more careful.