Resource destroying maps, with new applications

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Liu-Hu-Lloyd, PRL 118, 060502 (2017)
Liu-Takagi-Lloyd, arXiv:1708.09076

Outline

Quantum resource theories

Resource destroying maps: a theory of resource theories

Applications: coherence and discord

Monotonicity of diagonal discord, a simple measure of q. correlation

Quantum resource theories

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- Operational aspects: Rules for possible transformations; Rate of certain transformations as operational measures...

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General theory relating monotones and free operations?

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- Reveals fundamental connections among elements.
- Generates easily computable monotones (without optimizations) under typical free operations.
- Applies to *all* theories including nonconvex ones e.g. discord. (Frameworks for convex theories Brandão-Gour '15, Regula '17 etc.)

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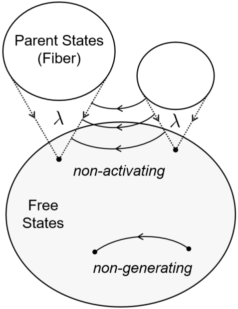

Defines a fiber bundle structure of all states:
Base—free states; fiber (of a free state)—parent states

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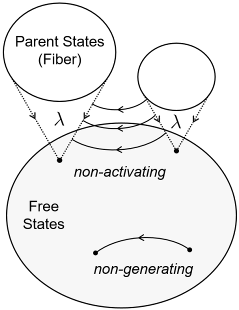

F is nonconvex $\Rightarrow$ no linear RD maps $\Rightarrow$ no RD channels/CPTP maps. Exact conditions Gour '16.

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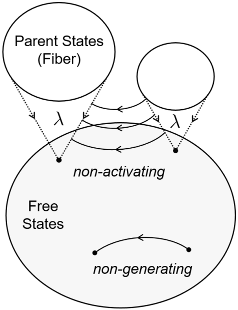

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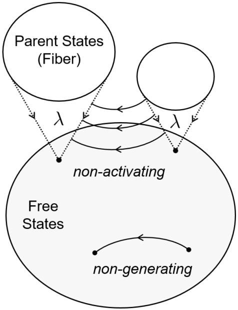

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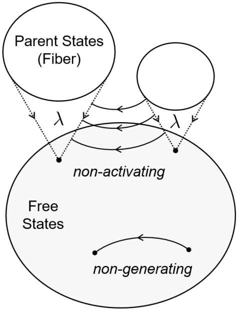

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- *Selective* (subscript s): $\exists$ Kraus decomposition s.t. all arms satisfy $\uparrow$ (property holds even if the measurement outcome is retained).

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General properties of free classes

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Free conditions hold for:

- Compositions;
- Convex combinations when the RD map is linear.

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Monotonicity theorems

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Proving commutativity can be nontrivial.

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Let D be a distance measure that further satisfies $D(\rho, \sigma) = \sum_\mu p_\mu D(\rho_\mu, \sigma_\mu)$ (true for eg relative entropy). Then selective monotonicity holds for $\mathfrak{D}$ under selective commuting operations $\mathbb{X}_s$.

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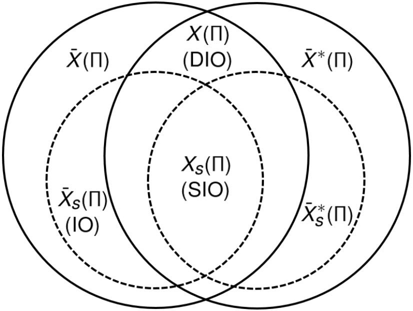

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DIO $\sim$; Chitambar-Gour '16
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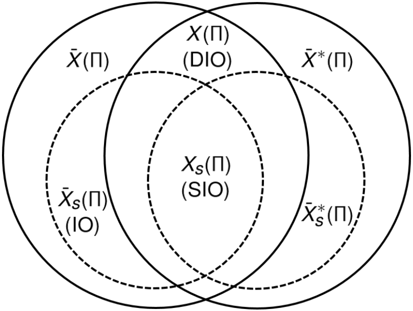

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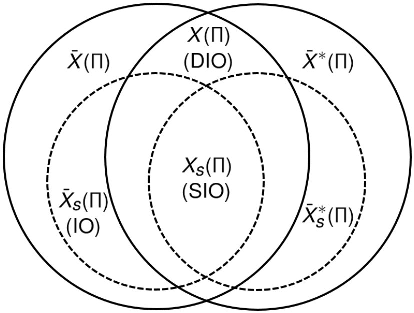

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Reviews: 1609.02439; 1703.01852.

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- Notorious problems:
  - Evaluation is very hard (NP-complete), including similar measures, eg deficit. Reason: the optimization over local POVMs (or even proj.) is intractable;
  - No strong physical correspondences;
  - No good resource theory treatments yet. Difficulty: nonconvexity—no known RT frameworks apply.

Discord

- Discord destroying map (no discord destroying channel due to nonconvexity):
  Local dephasing in the eigenbasis:

$$\pi_A(\rho_{AB}) := \sum_i (|i\rangle_A\langle i| \otimes I_B)\rho_{AB}(|i\rangle_A\langle i| \otimes I_B).$$

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- Physical correspondences: eg thermo
  - Heat flow. No-go theorem: no energy transport without discord. Instantaneous heat flow rate, thermal initial states at different temperatures: $\Delta E \approx \frac{1}{\beta_A - \beta_B} \bar{D}_A$ ($\propto$ infinitesimal DD).
  - Work extraction. Difference in extractable work, local vs global demons/classical vs quantum channels Brodutch-Terno '10.

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  - Qubit: semiclassical ∪ unital/mixed-unitary HFZL; Streltsov-Kampermann-Bruβ '11
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- Qubit: $\exists$ mixed-unitary $\notin X_A(\pi_A)$ (note: ISO $\subsetneq$ MU); Explicit condition; Open: all MU$\setminus$ISO $\notin X_A(\pi_A)$?

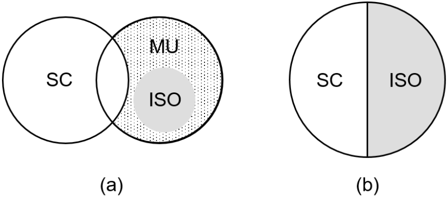

Figure: (a) qubit; (b) qudit $d > 2$.

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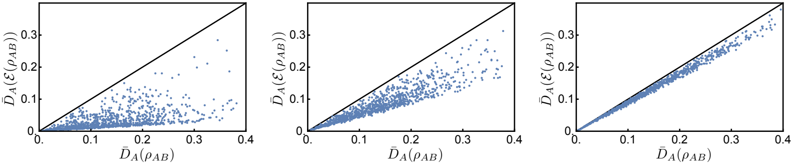

Figure: (a) $\frac{1}{3}\rho + \frac{2}{3}H\rho H$
(b) $\frac{1}{3}\rho + \frac{2}{3}R_n(\pi/2)\rho R_n(\pi/2)^\dagger$ where $R_{\mathbf{n}}(\pi/2)$ is the $\pi/2$ rotation with respect to the axis $\mathbf{n} \propto (1,1,1)$
(c) $\frac{1}{6}\rho + \frac{1}{3}R_X(\pi/10)\rho R_X(\pi/10)^\dagger + \frac{1}{2}R_Z(\pi/5)\rho R_Z(\pi/5)^\dagger$ where R_X and R_Z are rotations with respect to X axis and Z axis respectively.

Diagonal discord

Monotonicity

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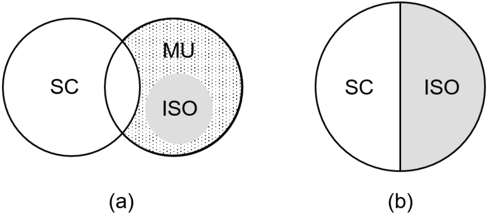

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Conclusion: for qudits, DD is monotone under all local commutativity-preserving (nongenerating) channels; for qubits, very likely.

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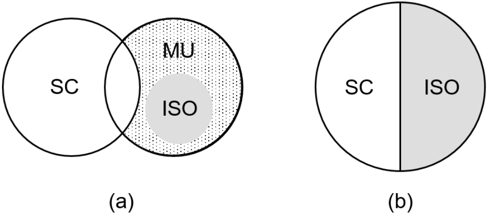

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DD (surprisingly) exhibits almost “maximal” monotonicity!

Diagonal discord

Continuity

Another desirable property: continuity.

- Examples are known that DD can be discontinuous (but all in the vicinity of degeneracies).
- We show that, when ρ_A is nondegenerate, DD is continuous. Fannes-type bound:

Theorem

Δ: smallest gap in the spectrum. $\|\rho'_{AB} - \rho_{AB}\|_1 \leq \epsilon$. For sufficiently small ϵ:

$$\begin{aligned} & \left|\bar{D}_A(\rho'_{AB}) - \bar{D}_A(\rho_{AB})\right| \\ \leq \; & 4\left(\frac{3\sqrt{d_A^3}d_B^2}{\Delta}+1\right)\epsilon\log d_A d_B + 2H\left[\left(\frac{6\sqrt{d_A^3}d_B^2}{\Delta}+1\right)\epsilon\right] \\ & +2H(\epsilon). \end{aligned}$$

Diagonal discord

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Diagonal discord

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- The above monotonicity and continuity results generalize to multisided measures.

Summary

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  - Generalizations to resource theories beyond states (eg channels, measurements)?

RD map: 1606.03723
Diagonal discord: 1708.09076

Thanks for your attention!