Higher order quantum operations of unitaries and their implications

Mio Murao

Department of Physics, Graduate School of Science, The University of Tokyo

Collaboration with:

Akihito Soeda, Marco Tulio Quintino, Jisho Miyazaki, Atsuhi Shimbo, Ryosuke Sakai, Qingxiuxiong Dong (U Tokyo), Shojun Nakayama (NII) and Yuichiro Matsuzaki (NTT)

Quantum computer

How to compute?

- Computers using quantum systems and their quantum properties

  quantum processing

What to compute?

- Computational problems for classical inputs/outputs

  a map (function) $f : \{0,1\}^n \to \{0,1\}^m$

  Shor's factoring algorithm, Grover search algorithm, ...

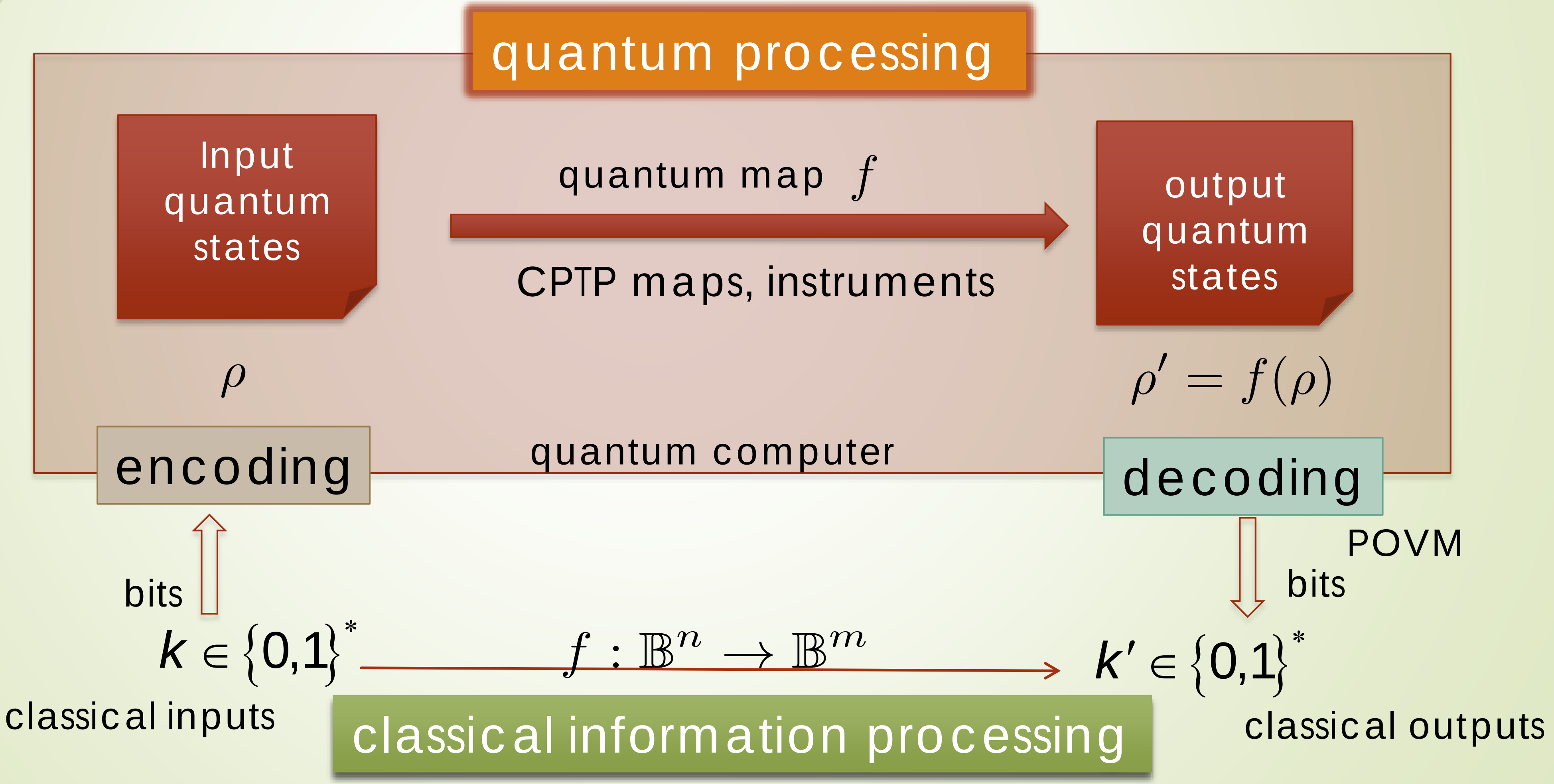
  Quantum "information processing"
  = Quantum processing of "classical information"

  Transforming classical input bits to classical output bits via quantum systems

- What else we can compute? Any more applications of quantum computers?

Quantum “information processing”

Quantum computer performs quantum processing of “classical information”



Quantum “information processing”

Quantum computer performs quantum processing of “classical information”

But
(as you know)

quantum computer can “compute” more!

bits

POVM

bits

$k \in \{0,1\}^*$ $\quad f : \mathbb{B}^n \to \mathbb{B}^m \quad$ $k' \in \{0,1\}^*$

classical inputs

classical information processing

classical outputs

“Quantum information” processing

Interpreting input **quantum states** as “**quantum information”**….

“quantum information” processing

input “quantum information” → quantum map f → output

CPTP maps, instruments, POVM

quantum states
ρ

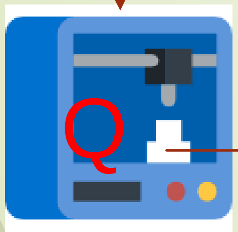
quantum states
$\rho' = f(\rho)$
(and/or some number
$r = f(\rho)$)

quantum computer

quantum states ρ

command

$f = \dots$



$\rho' = f(\rho)$

quantum states

Quantum computer can perform quantum processing of “quantum information”

=

quantum systems (states and dynamics)

Systems can be given without knowing what they are!

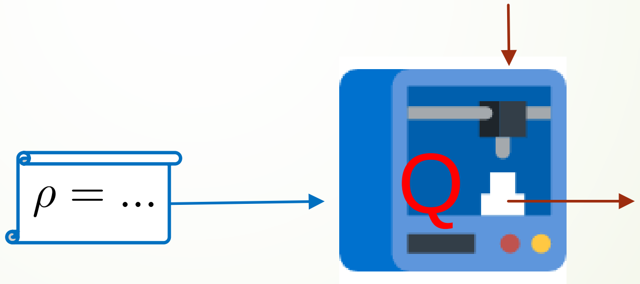
Quantum computer
for processing quantum systems (type 0)

In addition to be useful for efficiently processing ”classical information”, quantum computer can be used for

- **Type 0: Preparing (known) quantum states** $\rho_0 \to \rho$

ρ_0 : A fixed initial quantum state
(a known state)

A classical data
(command) describing
the state to be prepared



ρ : a quantum state

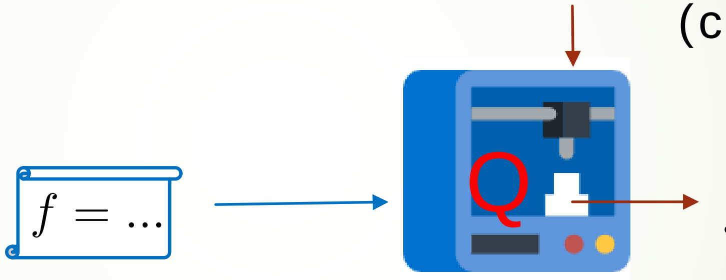
”quantum state printer”

Quantum computer
for processing quantum systems (type 1)

Type 1: Processing quantum states $\rho \to f(\rho)$

ρ : an arbitrary quantum state
(can be an unknown state)

A classical data
(command) describing the
quantum map* to apply



$f(\rho)$: output quantum state
(can be unknown)

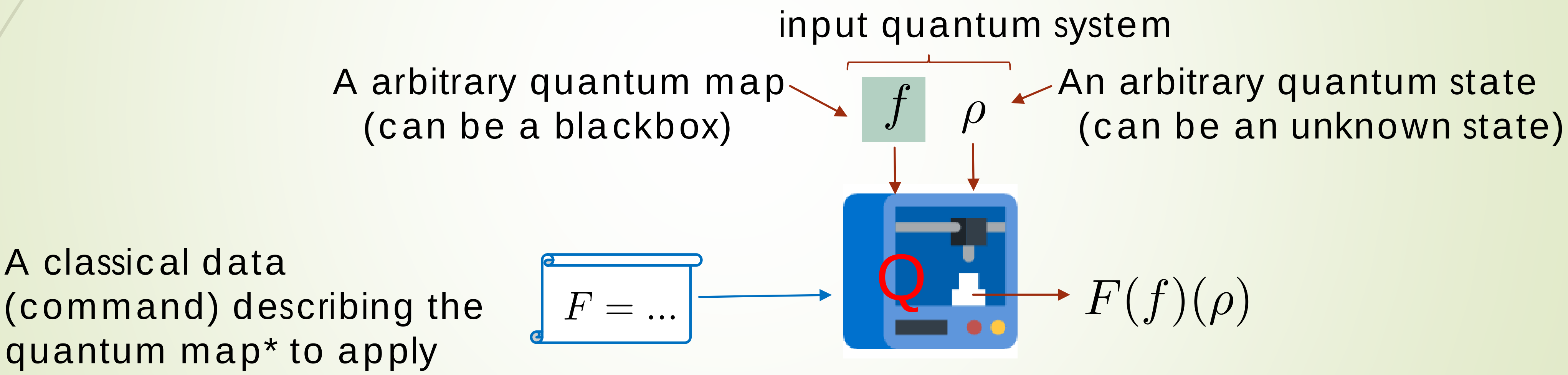
"quantum state processor"

Quantum maps := a set of operations on states allowed in quantum mechanics
Completely Positive Trace Decreasing (CPTD) maps,
(can be deterministic, or probabilistic)

Quantum computer
for processing quantum systems (type 2)

Type 2: Processing quantum maps $\rho \to F(f)(\rho)$

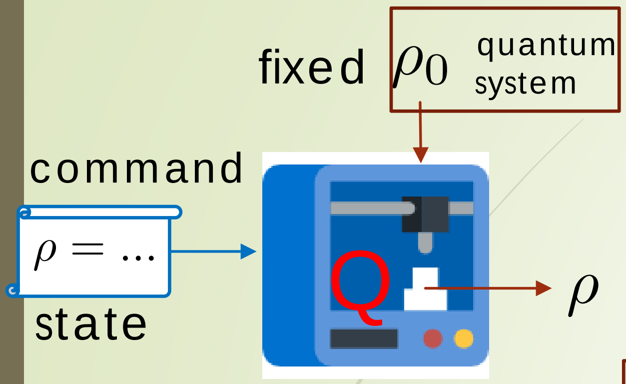
where $f \to F(f)$ is a **supermap**, a map of a map



”processed process quantum state processor”

We call this type of “quantum formation processing” as **“higher order quantum operations”**

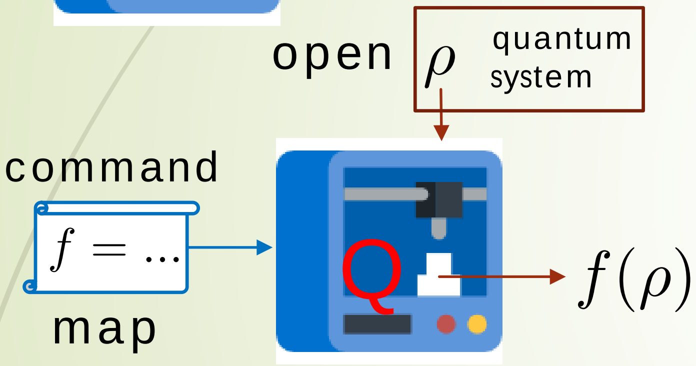
Higher/lower order quantum operations



Type 0 ("0th order operation) $\rho_0 \to \rho$

"quantum state printer"

"**lower order quantum operations**"



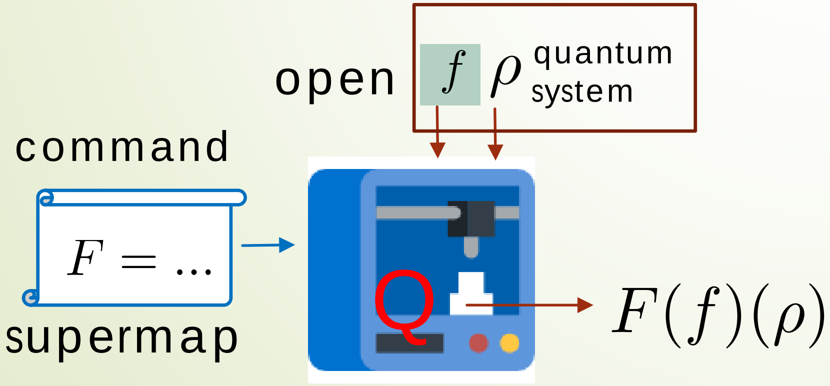
Type 1 ("1st order operation") $\rho \to f(\rho)$

"quantum state processor"

"**quantum operations**"

ρ — f — $f(\rho)$

quantum circuit representation



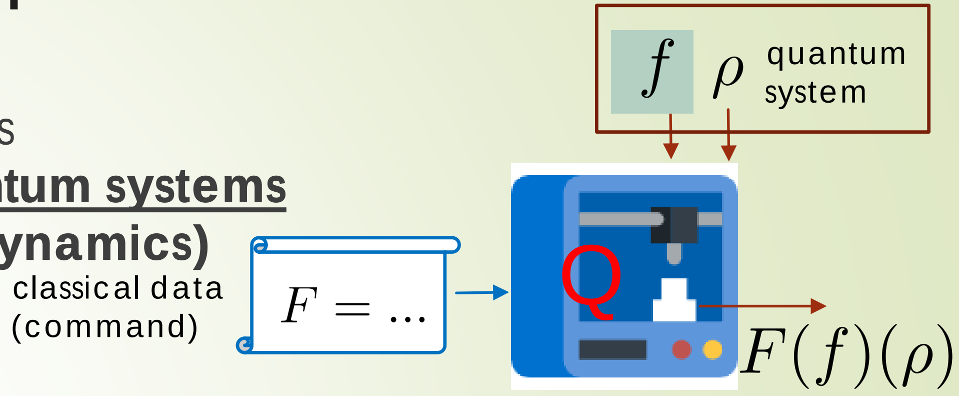
Type 2 ("2nd order operation") $\rho \to F(f)(\rho)$

"processed process quantum state processor"

"Higher order quantum operations"

Which F is possible in quantum mechanics?

Why higher order quantum operations?



- Understanding the power of quantum computers as a **quantum system processor processing quantum systems** including unknown quantum **states and maps (dynamics)**

- It may extend **usefulness of quantum computers**
- It may extend our **understanding of quantum mechanics**

Why we need to process quantum systems?

- Their properties are **yet not well understood** except the pioneering works in terms of quantum combs

  G. Chiribella, G. M. D'Ariano, and P. Perinotti, Phys. Rev. Lett. (2008)
  G. Chiribella, G. M. D'Ariano, and P. Perinotti, Phys. Rev. A (2009)

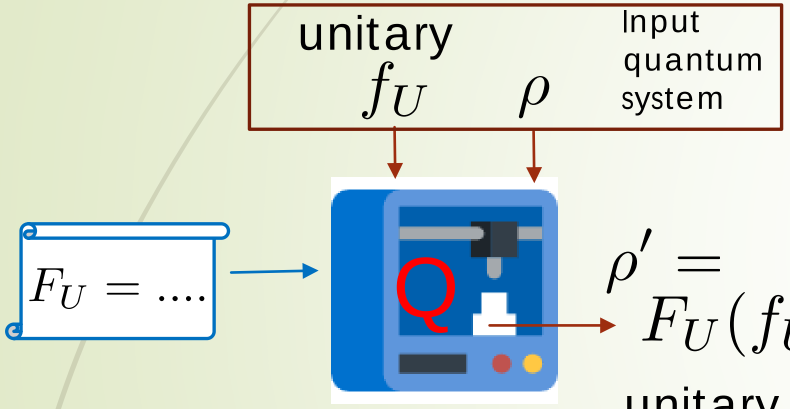
- Function of functions → **Functional** quantum computing?

  T. M. Rambo, J. B. Altepeter, P. Kumer and G. M. D'Ariano, Phys. Rev. A (2016)

- **Controllization of a unitary:** one of the higher order quantum operations and key elements many quantum algorithms ➡ **A useful subroutine for Q algorithms**

Outline of this talk

In this talk, I focus on higher order quantum operations for **unitaries** since it looks more useful in QIP (and easier to analyze…) (+ finite dimensional systems)



$\rho' = F_U(f_U)(\rho) = g_U(\rho)$

unitary

$$U, V_U := V(U) \in \text{unitary}$$

$$f_U(\rho) := U \rho U^\dagger$$

$$g_U(\rho) := F_U(f_U)(\rho) = V_U \rho V_U^\dagger = \rho'$$

1. Definitions and properties of higher order quantum operations
2. Universal conjugation algorithm of unitaries and its implications
3. Universal controllization algorithm of Hamiltonian dynamics and its implications
4. “Quantum learning” as higher order quantum operations

1. Definitions and properties of higher order quantum operations

Purely mathematical

Supermap mapping unitaries to unitaries

- A supermap is a map from an input map to an output map $f \to F(f)$
- We consider a supermap F mapping an input unitary U to output **unitary** V_U, that is, for $U, V_U := V(U) \in \mathrm{unitary}$ and $f(\rho) = U\rho U^\dagger =: f_U(\rho)$,

$$F(f)(\rho) = F(f_U)(\rho) = V_U \rho V_U^\dagger =: F_U(\rho)$$

$U \to V_U$

- Examples supermaps looking useful for processing quantum systems:
  - Replication $V_U = U \otimes U$ G. Chiribella, G. M. D'Ariano, and P. Perinotti, PRL (2008)
  - Inverse $V_U := U^{-1} = U^\dagger$ "undo"
  - Complex conjugation in terms of a fixed basis $V_U := U^*$ "time-reversal"
  - Transposition in terms of a fixed basis $V_U := U^T$
  - Controllization up to phase $V_U = |0\rangle\langle 0| \otimes \mathbb{I} + e^{i\theta_U} |1\rangle\langle 1| \otimes U$
  - "Quantum switch operation" $V_{U_1,U_2} := |0\rangle\langle 0| \otimes U_1 U_2 + |1\rangle\langle 1| \otimes U_2 U_1$
  - Neutralization $V_U := \mathbb{I}$ G. Chiribella, G. M. D'Ariano, P. Perinotti, and B. Valiron, PRA (2013)

error correction

How to applying F_U in quantum computer (Case 1)

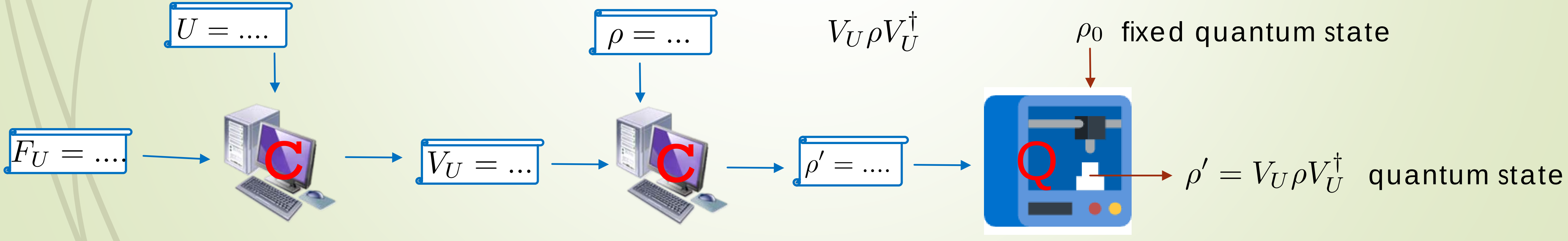
F_U (which supermap to apply) is given as classical data $F_U = ...$

Task: Using a system performing a unitary U, apply F_U on an arbitrary state ρ to create a quantum state $\rho' = F_U(\rho) = V_U \rho V_U^\dagger$

The strategy depends on how U and ρ are given in quantum computer

<u>Case 1</u> If both U and ρ are given as classical data $U = ...$ $\rho = ...$,

- classically calculate $\rho' = ...$ for $\rho' = V_U \rho V_U^\dagger$ and prepare ρ' by type 0 QC

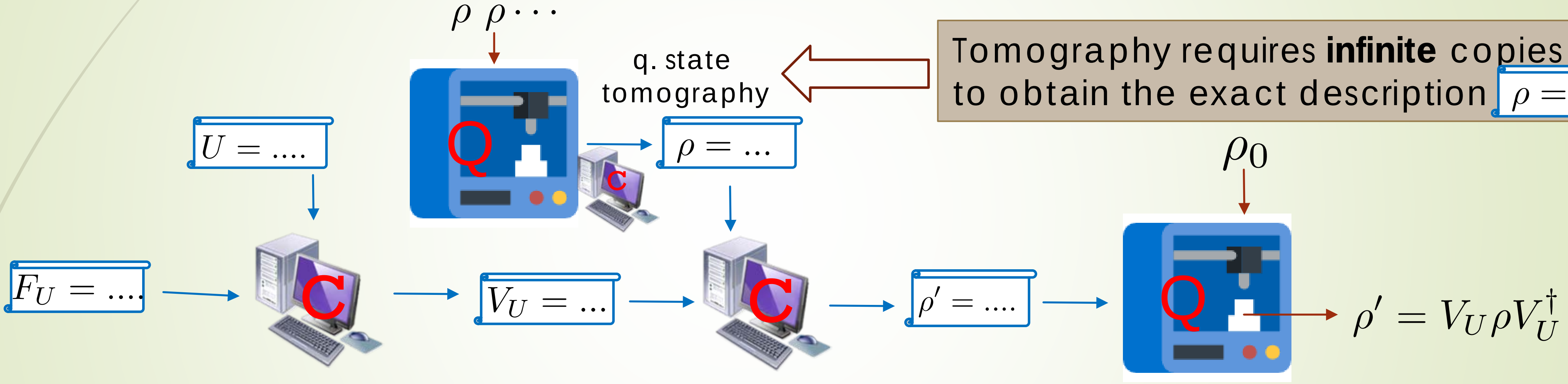


Classical information processing ➡ A use of quantum computer as "quantum state printer"

How to applying F_U in quantum computer (Case 2)

<u>Case 2</u> If ρ is given as an (unknown) quantum state, but U is given by $U = ...$

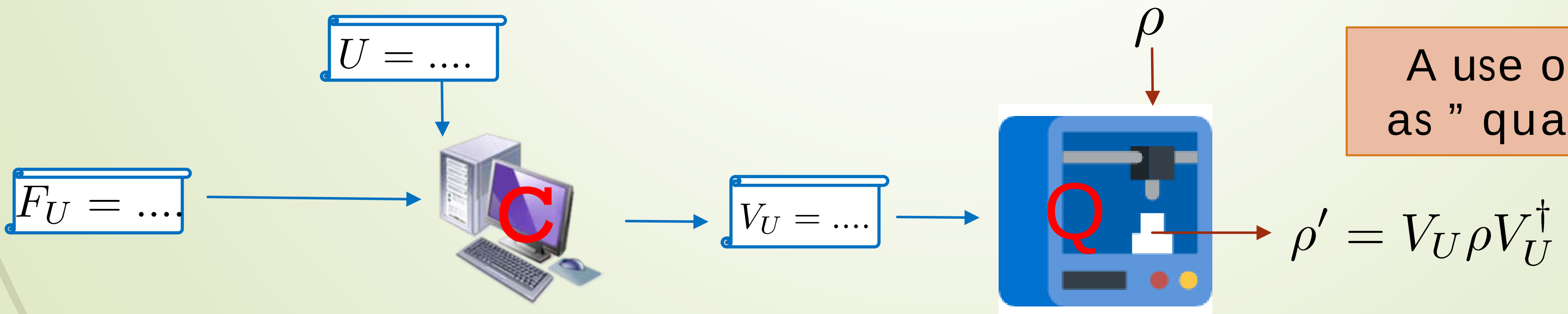
- Using multiple copies of ρ to obtain $\rho = ...$ by state tomography and use <u>type 0 QC</u>



Tomography requires **infinite** copies of ρ to obtain the exact description $\rho = ...$

Alternatively,

- Classically calculate $V_U = ...$ and apply it on ρ to obtain $\rho' = V_U \rho V_U^\dagger$ using <u>type 1 QC</u>



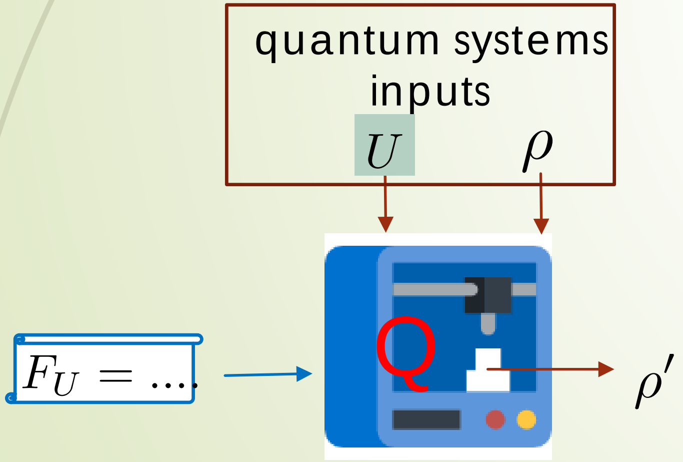
A use of quantum computer as " quantum state processor"

How to applying F_U in quantum computer (Case 3)

<u>Case 3</u> If ρ is given as an (**unknown**) quantum state, and U is given by a **blackbox** physical system (a blackbox quantum gate) U

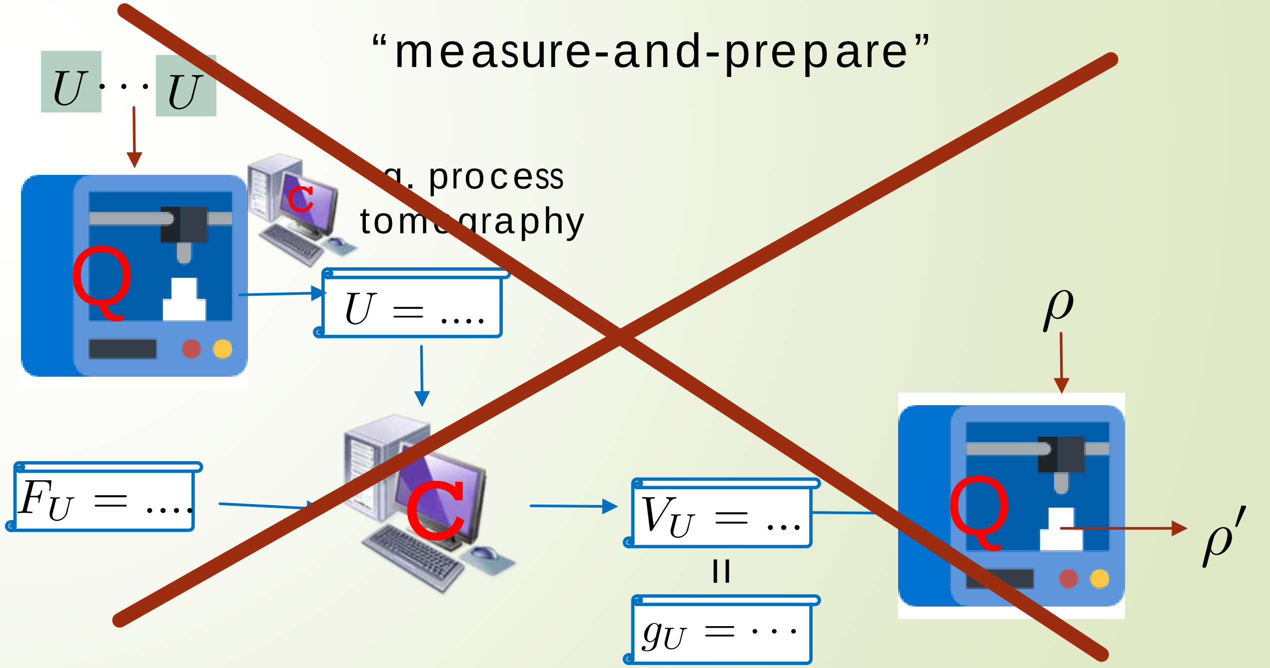
we use **higher order quantum operations**: directly transforming ρ and the blackbox U into $\rho' = F_U(\rho) = V_U \rho V_U^\dagger$ without help of classical computer by <u>type 2 QC</u>

A use of quantum computer as "processed process quantum state processor"



without process tomography or classical computer

No output of classical information!

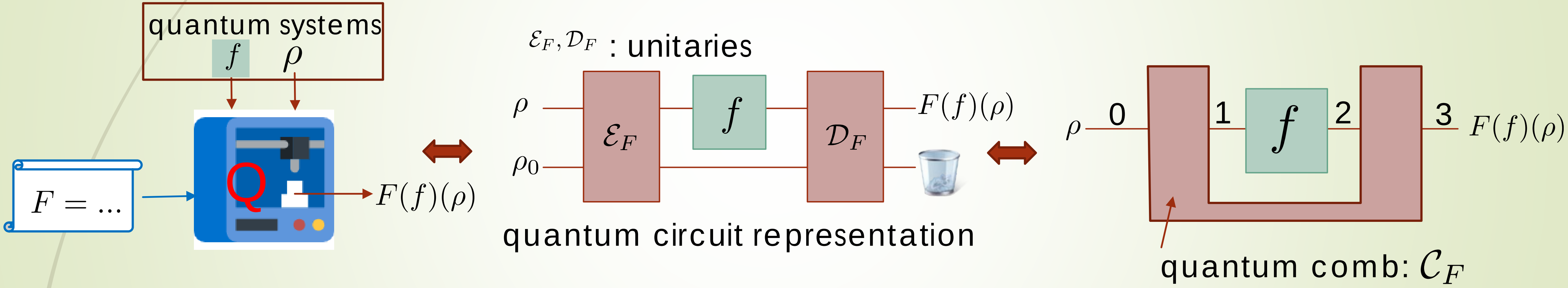


Quantum process tomography and <u>type 1 QC</u>

Implementing supermaps in quantum realm

How to implement supermaps in type 3 QIP? Which supermaps are implementable?

- If a given quantum map f is applicable **only once** for implementing $F(f)(\rho)$,



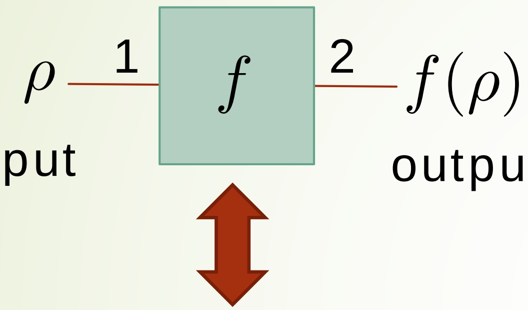
- **The quantum comb formalism*** provides the condition to implement $F(f)(\rho)$ in quantum mechanics for a general quantum map f by the conditions for the **Choi state** of the quantum comb $\mathcal{C}_F$ representing supermap F

*G. Chiribella, G. M. D'Ariano, and P. Perinotti, PRL (2008)
*G. Chiribella, G. M. D'Ariano, and P. Perinotti, PRA (2009)

Quick review of

The Choi state of a quantum map f

General quantum maps f on systems be represented by the Choi states (matrices)



input output

A linear map $f : \mathcal{B}(\mathcal{H}_1) \to \mathcal{B}(\mathcal{H}_2)$

completely positive and trace preserving (CPTP)
for implementable by quantum computer

The Choi state (or Choi matrix, CJ matrix) of f: $M_f \in \mathcal{B}(\mathcal{H}_1 \otimes \mathcal{H}_2)$

$$M_f = \sum_{i,j} |i\rangle\langle j| \otimes f(|i\rangle\langle j|) = (\mathcal{I} \otimes f)|I\rangle\rangle\langle\langle I|$$

where

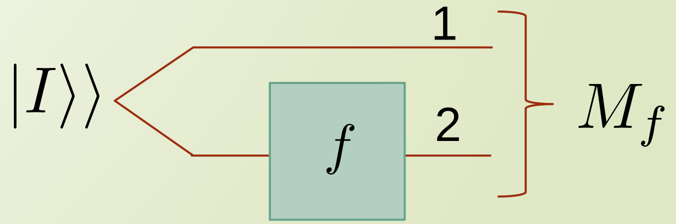
$$|I\rangle\rangle := \sum_i |i\rangle \otimes |i\rangle$$

Unnormalized state

$$M_f \geq 0 \Leftrightarrow f \text{ is completely positive}$$

$$\mathrm{tr}_2 M_f = \mathbb{I}_1 \Leftrightarrow f \text{ is trace preserving}$$

⇨ $f(\rho) = \mathrm{tr}_1[M_f(\rho^T \otimes \mathbb{I})]$



*G. Chiribella, G. M. D'Ariano, and P. Perinotti, Phys. Rev. Lett. (2008)
*G. Chiribella, G. M. D'Ariano, and P. Perinotti, Phys. Rev. A (2009)

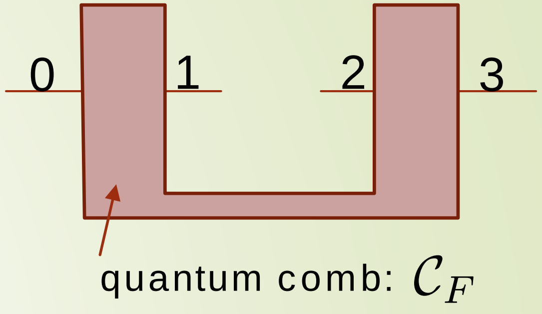
The Choi state of quantum comb $\mathcal{C}_F$

- Quantum comb $\mathcal{C}_F : \mathcal{B}(\mathcal{H}_1, \mathcal{H}_2) \to \mathcal{B}(\mathcal{H}_0, \mathcal{H}_3)$

map → Choi state, map → Choi state

$$\mathcal{B}(\mathcal{H}_1 \otimes \mathcal{H}_2) \to \mathcal{B}(\mathcal{H}_0 \otimes \mathcal{H}_3)$$

A supermap implementable by quantum computer in predefined causal order



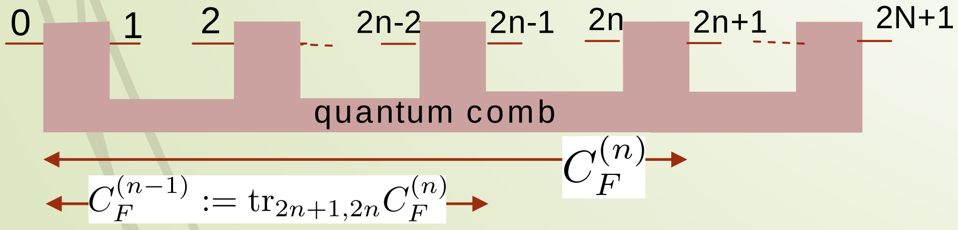
- The Choi state C_F of the quantum comb $\mathcal{C}_F$:

$$C_F \in \mathcal{B}(\mathcal{H}_1 \otimes \mathcal{H}_2) \otimes (\mathcal{H}_0 \otimes \mathcal{H}_3))$$

such that (1) $C_F \geq 0 \Leftrightarrow \mathcal{C}_F$ is completely positive

(2) $\mathrm{tr}_{123} C_F = \mathbb{I}_0 \Leftrightarrow \mathcal{C}_F$ is trace preserving

(3) $\mathrm{tr}_3 C_F = \mathrm{tr}_{23} C_F \otimes \mathbb{I}_2 \leftarrow$ The requirement of causal order



Quantum comb formalism* provides conditions of causal order for more general quantum combs

$$\mathrm{tr}_{2n+1} C_F^{(n)} = C_F^{(n-1)} \otimes \mathbb{I}_{2n}, n = 0, 1, \cdots, N$$

Supermaps not implementable by quantum comb

- The Choi states of many useful supermaps for unitaries **do not satisfy the conditions** required for the quantum combs when a single copy of ρ is provided and a single use of U is allowed. $\rightarrow$ many no-go theorems
- Examples of F_U not implementable by quantum computer in this setting:

the Choi state of F_U

  - Controlled-unitary: Even linearity does not hold*
  - Complex conjugation of a unitary: The CP condition (1) does not hold if $d \geq 3$
  - Transposition of a unitary : The causal order condition (3) does not hold ((1),(2) is OK)
  - Quantum switch: The causal order condition (3) does not hold

G. Chiribella, G. M. D'Ariano, P. Perinotti, and B. Valiron, PRA (2013)

Seems to have different types of reasons behind the No-go theorems ....

*No-go theorem of universal controllization in this setting: J. Thompson, et al., arXiv:1310.2927 (2013)
M. Araujo, et al., New J. Phys. (2014)

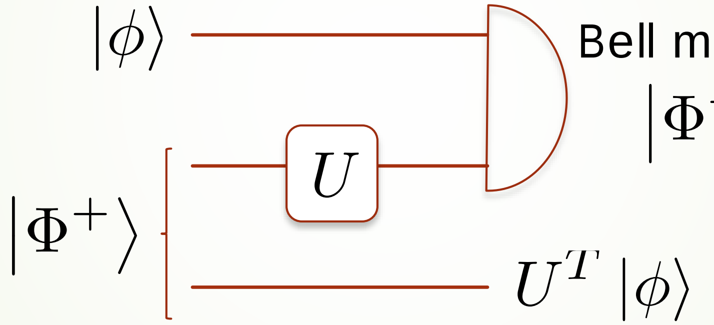
But never give up…

By relaxing the setting, we can still try to implement F_U from a single use of U and ρ + QC

For example,

- Allowing <u>probabilistic</u> implementations

  Transposition: Gate teleportation achieves transposition with probability $\frac{1}{d^2}$



Bell measurement

$|\Phi^+\rangle\langle\Phi^+|$ ← probability $\frac{1}{d^2}$

How about other supermaps?

- Consider <u>approximate</u> implementation and find optimal under some figure of merit

  Inverse, Controlled-unitary, complex conjugation, G. Chiribella and D. Ebler, New J Phys. (2016)

- <u>Restricting the set </u>of U, namely, partial information about U is provided

  Controlled-unitary: A. Bisio, M. Dall'Arno, and P. Perinotti, PRA (2016)

This type of relaxations still requires linearity of the map in terms of U and ρ

Our approach 1

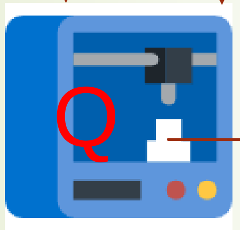
Adding a **nonlinear** power in terms of U

- We relax the setting that multiple but finite uses of U is allowed but keep a single ρ

quantum systems inputs

$U \cdots U$ ρ

$F_U = \ldots$



$\rho' = g_U(\rho) = V_U \rho V_U^\dagger$

output quantum state

or if it is not possible, try probabilistic but exact implementations

For transposition F_U, port based teleportation* is expected to be useful

*S. Ishizaka and T. Hiroshima, PRL (2008), PRA (2009)

Why considering this relaxation?

- Infinitely many use of U provides $U = \ldots.$ via process tomography, and then any F_U is implementable by type 1 QC
  = measure-and-prepare strategy
- Can higher order quantum operations be more efficient by type 2 QC than by type 1 QC?

What is a minimum use of U to implement F_U ?

Our approach 2

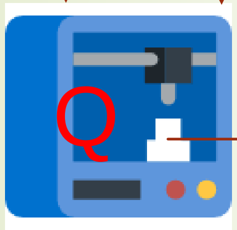
- We relax the setting that n uses of U is allowed but keep a single ρ

and try to implement $\rho' = g_{U^n} = V_{U^n} \rho V_{U^n}$

quantum systems inputs

$U \cdots U \quad \rho$

$F_{U^n} = ...$



$\rho' = g_{U^n} = V_{U^n} \rho V_{U^n}$

output quantum state

We "**divide**" U^n into n parts of U and apply F_{U^n}

(or allow fractional queries)

Getting *n*-th root of unitary is not possible in general
But possible for Hamiltonian dynamics

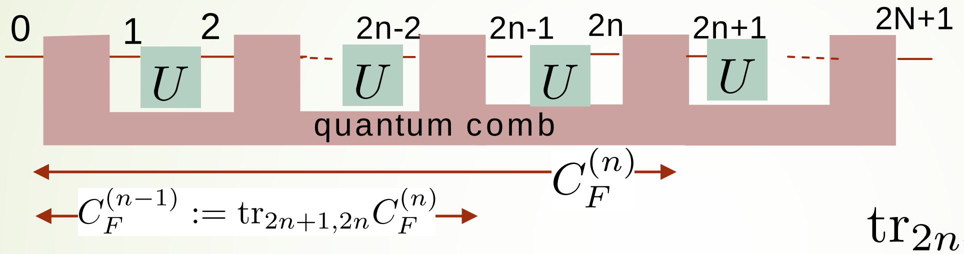
Why considering this relaxation?

- Infinitely many use of U provides $U = ...$ via process tomography, and then any F_U is implementable by type 1 QC
- Can we perform higher order quantum operations more efficiently by type 2 QC than by type 1 QC?

How does the divisibility of U affect implementability of F_U ?

In principle…

- Implementability of F_U can be found by analyzing the existence of a quantum comb with *N* uses of U



Conditions
(1) CP
(2) TP
(3) Causal order

$$\mathrm{tr}_{2n+1} C_F^{(n)} = C_F^{(n-1)} \otimes \mathbb{I}_{2n}, n = 0, 1, \cdots, N$$

- This means to analyze a state in $\mathcal{H}^{\otimes 2(N+1)}$ $\rightarrow$ very difficult to solve in general F_U

- We show implementable F_U with finite uses of U by construction and the analyze the properties of such higher order quantum operations

-- complex conjugation of a unitary: approach 1
-- controllization of a unitary: approach 2

2. Universal conjugation algorithm of unitaries and its implications

Complex conjugate of a unitary

- By applying U implement U^*, i.e. obtaining $\rho' = U^* \rho U^T$ for all ρ (single copy)

or Appling *U* but implementing its complex conjugate *U**

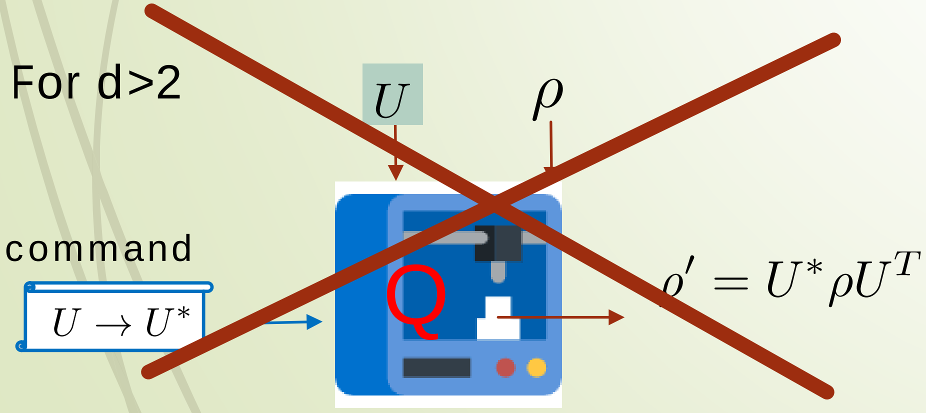
Anti-linear operations

- For a qubit unitary (d=2) it is possible $\sigma_y U \sigma_y^\dagger = U^*$ for $U \in SU(2)$

- For d>3, with a single use of U, the Choi state of the supermap is not completely positive, so it is not deterministically implementable

G. Chiribella and D. Ebler, New J. Phys (2016)

For d>2



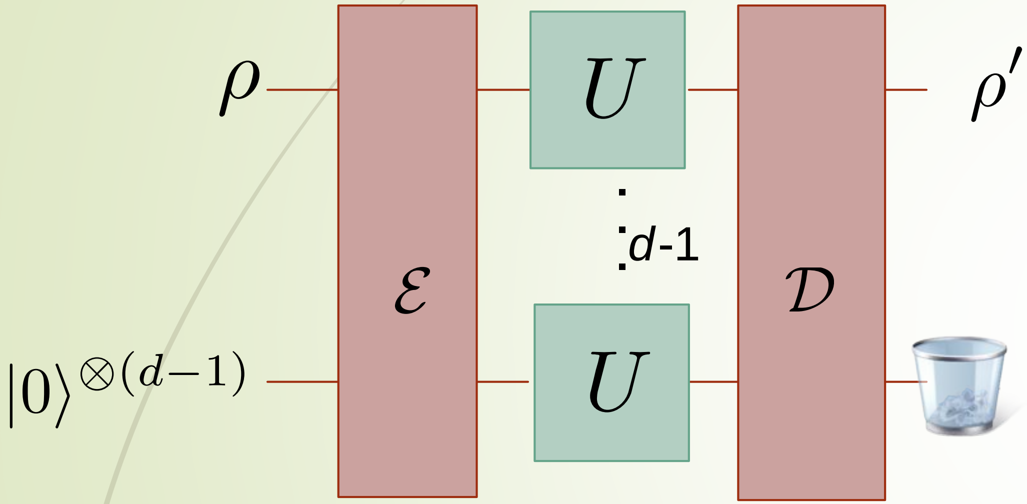
We add a bit of a “nonlinear” resource for solving this problem

How about we allows multiple but finite use of U ?

J. Miyazaki, A. Soeda and M. Murao, arXiv1706.03481

A universal algorithm for $U \to U^*$

We found an universal implementation algorithm with *d* time use of *U!*



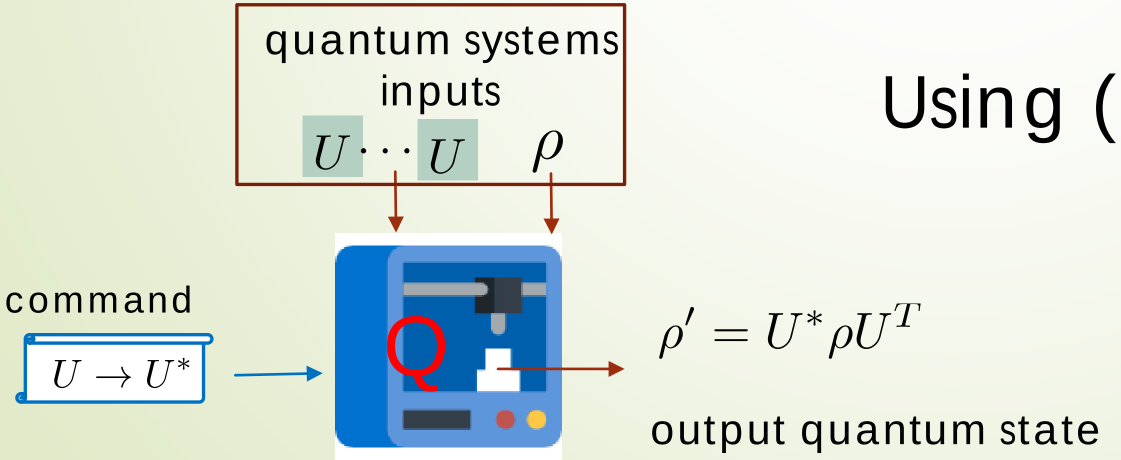
$$\rho' = U^* \rho U^T$$

$$\mathcal{E}(\rho) = A\rho A^\dagger, \ \mathcal{D}(\rho) = A^\dagger \rho A$$

$$A = \frac{1}{\sqrt{(d-1)!}} \sum_{\sigma \in S_d} \mathrm{sgn}(\sigma) |\sigma_2\rangle \otimes \cdots \otimes |\sigma_d\rangle \langle \sigma_1|$$

A: isometry to the (*d*-*1*)-dim antisymmetric subspace

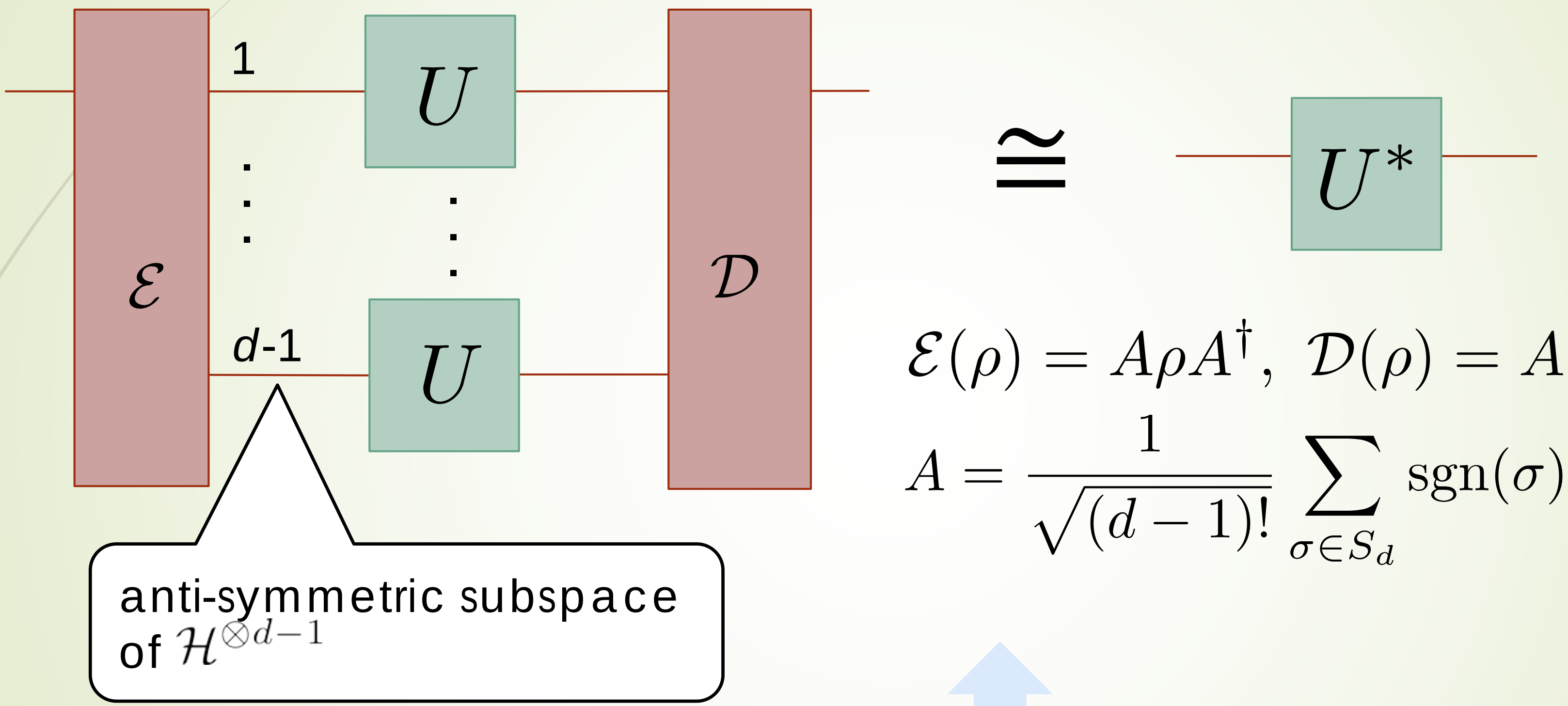
Using (d-1)× U Deterministic&exact



Recovers the d=2 case as well

But we do not know this is optimal for d>3

Interpretation in terms of group theory

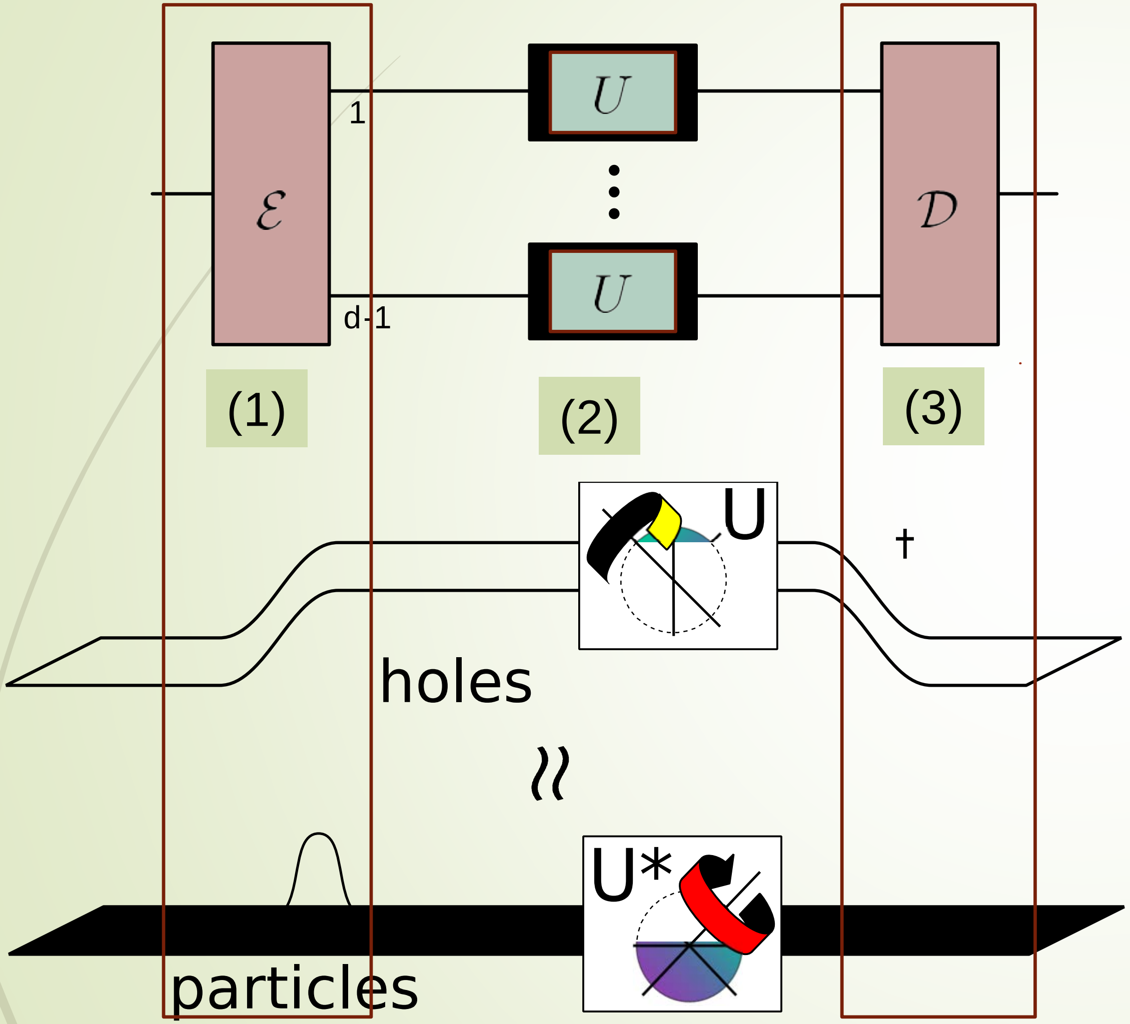


$$\mathcal{E}(\rho) = A\rho A^\dagger,\ \mathcal{D}(\rho) = A^\dagger \rho A$$

$$A = \frac{1}{\sqrt{(d-1)!}} \sum_{\sigma \in S_d} \mathrm{sgn}(\sigma) |\sigma_2\rangle \otimes \cdots \otimes |\sigma_d\rangle\langle\sigma_1|$$

"Tensor product representation $\mathrm{SU}(d)^{\otimes d-1}$ includes conjugate representation $\mathrm{SU}(d)^*$ as an irreducible representation"

Particle-hole interpretation of fermionic systems



$$A = \frac{1}{\sqrt{(d-1)!}} \sum_{\sigma \in S_d} \mathrm{sgn}(\sigma) |\sigma_2\rangle \otimes \cdots \otimes |\sigma_d\rangle\langle\sigma_1|$$

A d-mode fermionic ystem

particle: $a_{|i\rangle}, a^\dagger_{|i\rangle} (i = 1, \cdots d)$

(1) particle-hole conversion

$$\begin{cases} a_{|i\rangle} \rightarrow b^\dagger_{|i\rangle} \\ a^\dagger_{|i\rangle} \rightarrow b_{|i\rangle} \end{cases}$$

: hole

(2) mode transformation

hole $b^\dagger_{|i\rangle} \rightarrow b^\dagger_{U|i\rangle} = \sum_{j=1}^{d} [U]_{ij} b^\dagger_{|i\rangle}$

particle $a^\dagger_{|i\rangle} \rightarrow a^\dagger_{U^*|i\rangle} = \sum_{j=1}^{d} [U]^*_{ij} a^\dagger_{|i\rangle}$

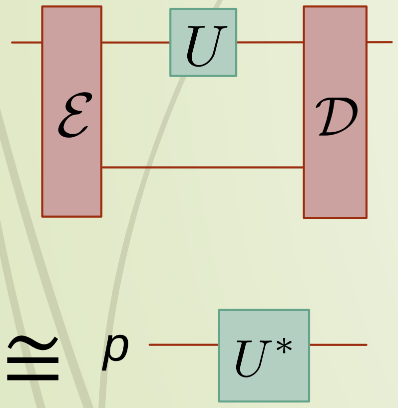
(3) hole-particle conversion

$$\begin{cases} b_{|i\rangle} \rightarrow a^\dagger_{|i\rangle} \\ b^\dagger_{|i\rangle} \rightarrow a_{|i\rangle} \end{cases}$$

No-go theorem for probabilistic implementation

For d>3 with a **single** use of U , exact implementation is impossible **even probabilistically** (p>0).

Outline of proof)



1. Assume there exists a pair of completely positive trace decreasing maps $\mathcal{E}, \mathcal{D}$ which implements complex conjugation of a unitary with non-zero probablity.
2. Regard the pair as a supermap and represent the supermap by a Choi state
3. A certain symmetry should be hold for the supermap
4. The symmetry restricts the form of the Choi state
5. Show that the restriction and the complex conjugation property is mutual exclusive

Formal statement and proof: J. Miyazaki, A. Soeda and M. Murao, arXiv1706.03481

No-go theorem for probabilistic exact pure state conjugation with finite copies

- Unitary conjugation is possible with (d-1) uses of U

different!

Pure state conjugation $|\phi\rangle \to |\phi^*\rangle$ in the computationall basis for unknown $|\phi\rangle$

- State conjugation is impossible **even probabilistically** with finite copies of $|\phi\rangle$ (p>0)

- If not, by repeating the probabilistic implementation, we can achieve the fidelity of the approximate state conjugation which is higher than the optimal implementation fidelity derived by

*V. Vuzek, M. Hillery and R.F. Werner, PRA (1999)

$$\max_{\Gamma} \min_{\rho:\mathrm{pure}} \mathrm{Tr}[\rho^{\perp}\Gamma(\rho^{\otimes n})] = 1 - \frac{1}{n+2}$$

Bi-product: Conjugation-based quantities

- Concurrence and several other functions of a state is defined using complex conjugation of a given state.

construction of "observable"

- Osterloh&Siewert,PRA2012
- Kus&Bengtsson PRA2009
- Kotowski et. al., PRA2010
- Mintert,APB2007
- Oszmaniec PhD2014

transpose (≅conjugation) based functions of states

- Θ-cocurrence Uhlmann,PRA2000 $|\langle\psi|\sigma_y\otimes\sigma_y|\psi^*\rangle|$
- PT-anti-unitary asymmetry measure Bu *et.al.*,arXiv2016
- Filtering Osterloh&Siewert,PRA2005
- I-concurrence Rungta, et.al.,PRA2001
- G-concurrence and its variants Gour,PRA2005

Unitary conjugation algorithm helps to uniformly understand

$$C_{\mathcal{F}}^{n;m}(|\psi\rangle) := \mathrm{Tr}[(|\psi\rangle\langle\psi|)^{\otimes m}\mathcal{F}((|\psi^*\rangle\langle\psi^*|)^{\otimes n})] \quad \mathcal{F}:\mathcal{B}(\mathcal{H}^{\otimes n})\to\mathcal{B}(\mathcal{H}^{\otimes n})$$

+ CP

$$C_{\mathcal{F}}^{n;m}(|\psi\rangle) = \langle\psi|^{\otimes m+n}\tilde{\mathcal{F}}|\psi\rangle^{\otimes m+n}$$

($\tilde{\mathcal{F}}$: The Choi state of $\mathcal{F}$)

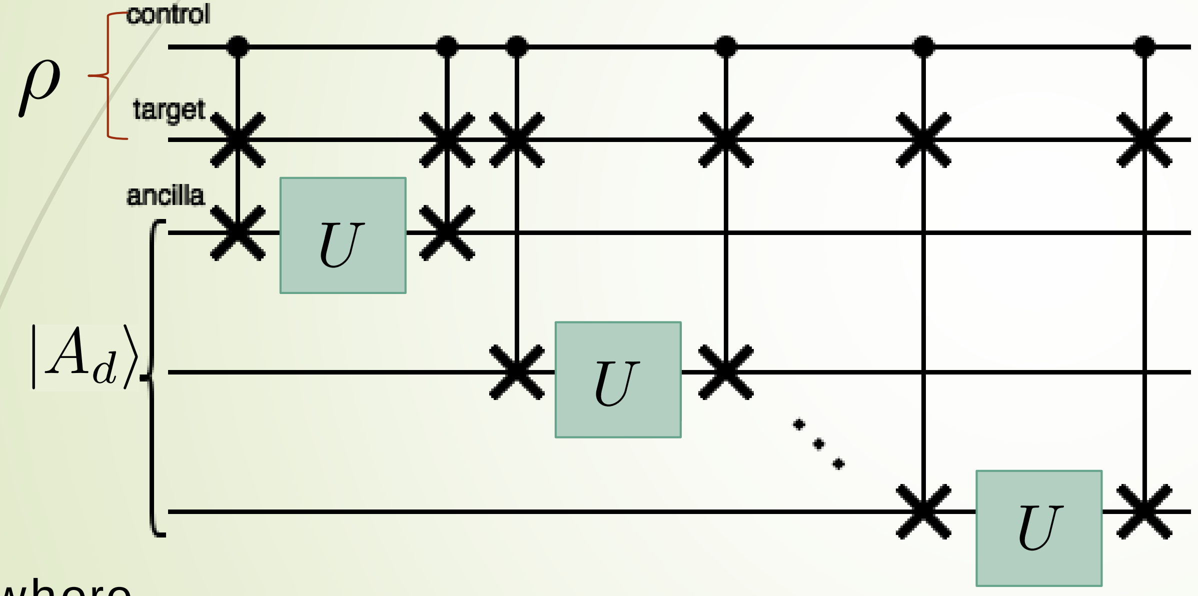
- By considering "nonlinear" observables (in terms of multiple copies of $|\psi\rangle$)
  ⇒We can evaluate the value of the function without knowing $|\psi\rangle$

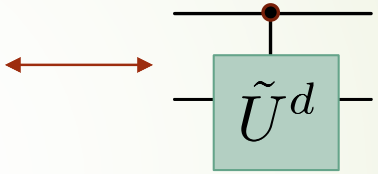
Universal controllization algorithm of Hamiltonian dynamics and its implications

Controllization of a unitary operation by using $\boxed{U}^{\times d}$

Q. Dong, S. Nakayama, A. Soeda and M. Murao, in preparation

- **Exact and deterministic** controllization algorithm with ***d*** calls of a *d*-dimensional unitary operations
- Implementing $|0\rangle\langle 0| \otimes \mathbb{I} + e^{i\theta_U}|1\rangle\langle 1| \otimes U^d$ instead of $|0\rangle\langle 0| \otimes \mathbb{I} + e^{i\theta_U}|1\rangle\langle 1| \otimes U$

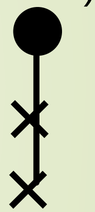




where

$$\tilde{U}^d = e^{i\theta_U} U^d$$

Fredkin gate
(Ctrl-SWAP)



The power of "root" allows to implement controllization

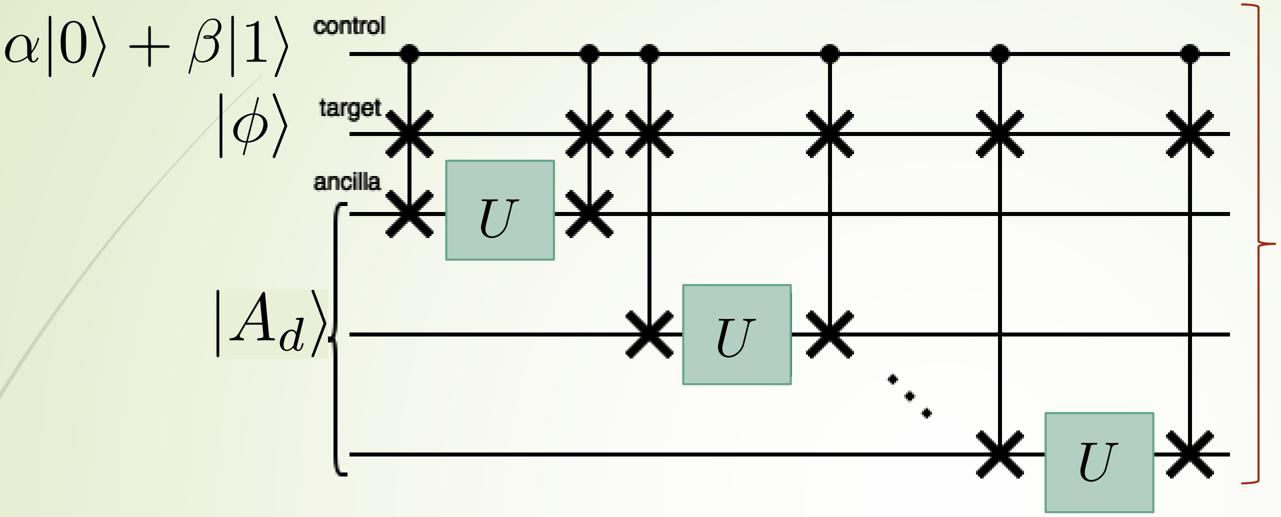
where

$$|A_d\rangle = \frac{1}{\sqrt{d!}} \sum_{\sigma \in S_d} \mathrm{sgn}(\sigma) |\sigma(1)\rangle \otimes |\sigma(2)\rangle \otimes \cdots \otimes |\sigma(d)\rangle$$

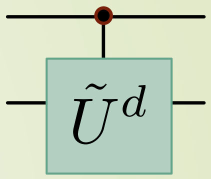
The totally antisymmetric state of a *d*-dimensional system

An algorithm to perform a root supermap by using C-U and C-U was proposed by L. Sheridan, D. Maslov, and M. Mosca, J. Phys. A Math. Theor. 42, 185302 (2009)

How the controllization algorithm works



$|output(\phi)\rangle$



$\tilde{U}^d = U^d/\mathrm{Det}(U)$

$$
\begin{aligned}
|output(\phi)\rangle &= \alpha|0\rangle \otimes |\phi\rangle \otimes U^{\otimes d}|A_d\rangle + \beta|1\rangle \otimes U^d|\phi\rangle \otimes |A_d\rangle \\
&= \{\alpha|0\rangle \otimes |\phi\rangle + \beta|1\rangle \otimes U^d/\mathrm{Det}(U)|\phi\rangle\} \otimes \mathrm{Det}(U)|A_d\rangle
\end{aligned}
$$

Key properties

- For any $U \in U(d)$ $U^{\otimes d}$ has a one-dimensional invariant subspace $U^{\otimes d} = \mathrm{Det}(U) \oplus \cdots$
- The invariant subspace is spanned by $|A_d\rangle = \frac{1}{\sqrt{d!}} \sum_{\sigma \in S_d} \mathrm{sgn}(\sigma)|\sigma(1)\rangle \otimes |\sigma(2)\rangle \otimes \cdots \otimes |\sigma(d)\rangle$

But large well controlled ancilla system required

Controllization of Hamiltonian dynamics

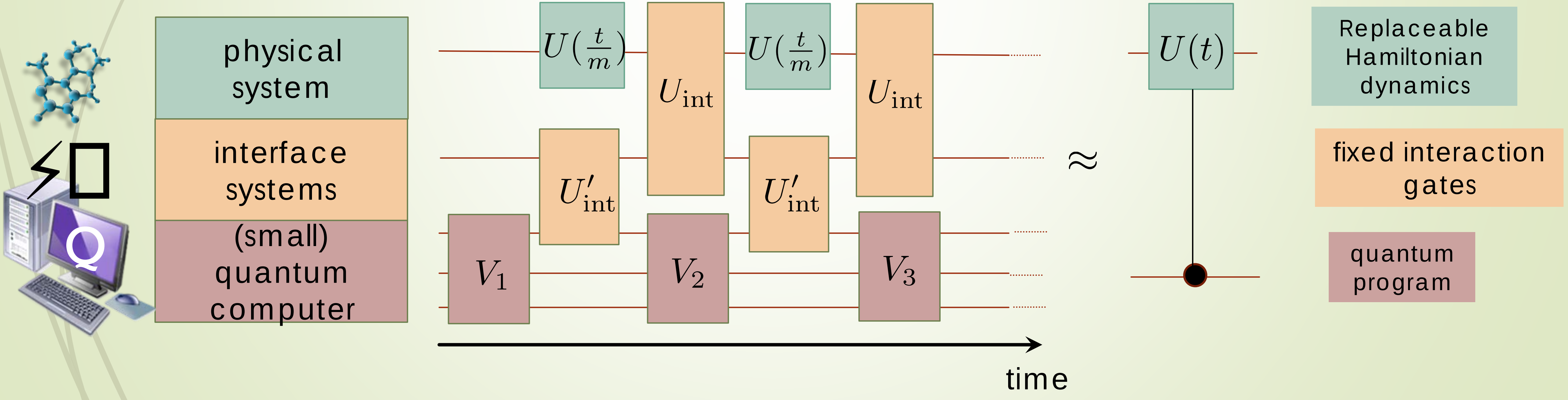
If the blackbox unitary is given as a physical system $U(t) = e^{-iHt}$

where we can control the time parameter t , Hamiltonian H is unknown,

we can perform universal approximate controllization up to phase

$$|0\rangle\langle 0| \otimes \mathbb{I} + e^{i\theta_{U(t)}} |1\rangle\langle 1| \otimes U(t)$$

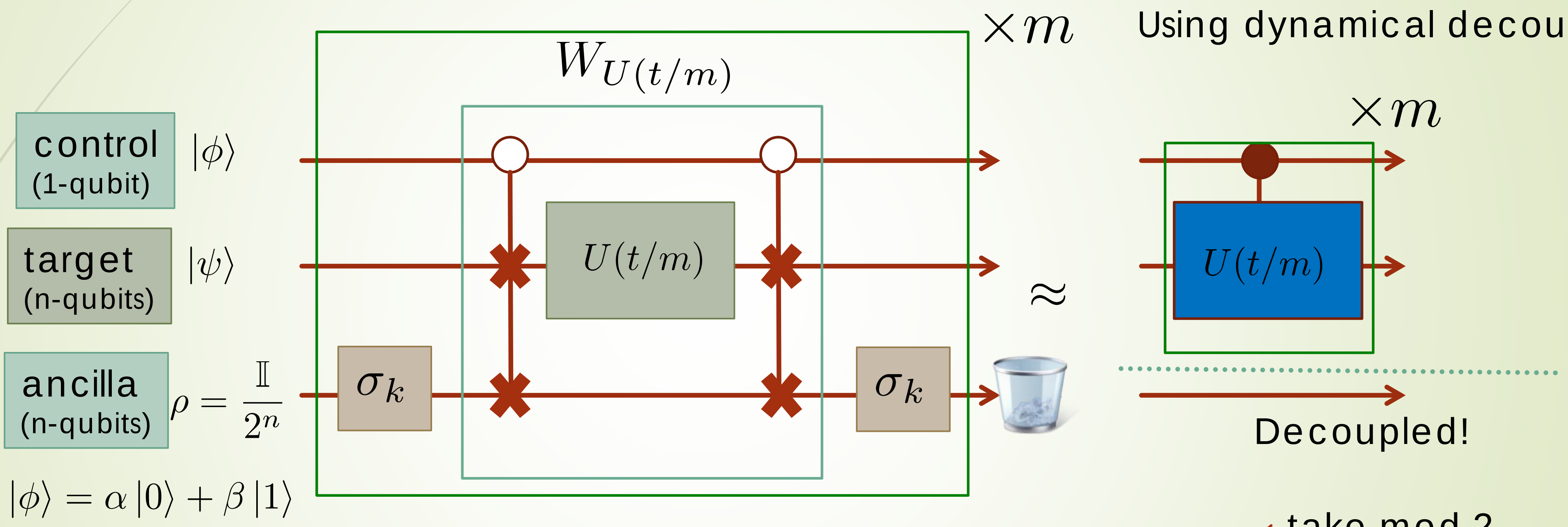
with less ancilla spaces using a randomized quantum algorithm
and less control for initial ancilla states



S. Nakayama, A. Soeda and M. Murao, PRL 114, 190501(2015)

S. Nakayama, A. Soeda and M. Murao, PRL 114, 190501(2015)

Universal controllization algorithm for unknown Hamiltonian dynamics



take mod 2

$$W_{U(t/m)} \rho W^{\dagger}_{U(t/m)} = \sum_{j,k=0,1} |j\rangle\langle k| \otimes U(jt/m)\, \rho_{j,k} U^{\dagger}(kt/m) \otimes \frac{1}{d} U((j-k)t/m)$$

Diagonal terms: correct for controlled U(t/m) since U(0)=**I**

Off-diagonal terms: causing error in O(1/m^2)

S. Nakayama, A. Soeda and M. Murao, PRL 114, 190501(2015)

An application of higher order quantum operations

Performing **projective measurement in energy eigenbasis** (PME) of unknown *H*

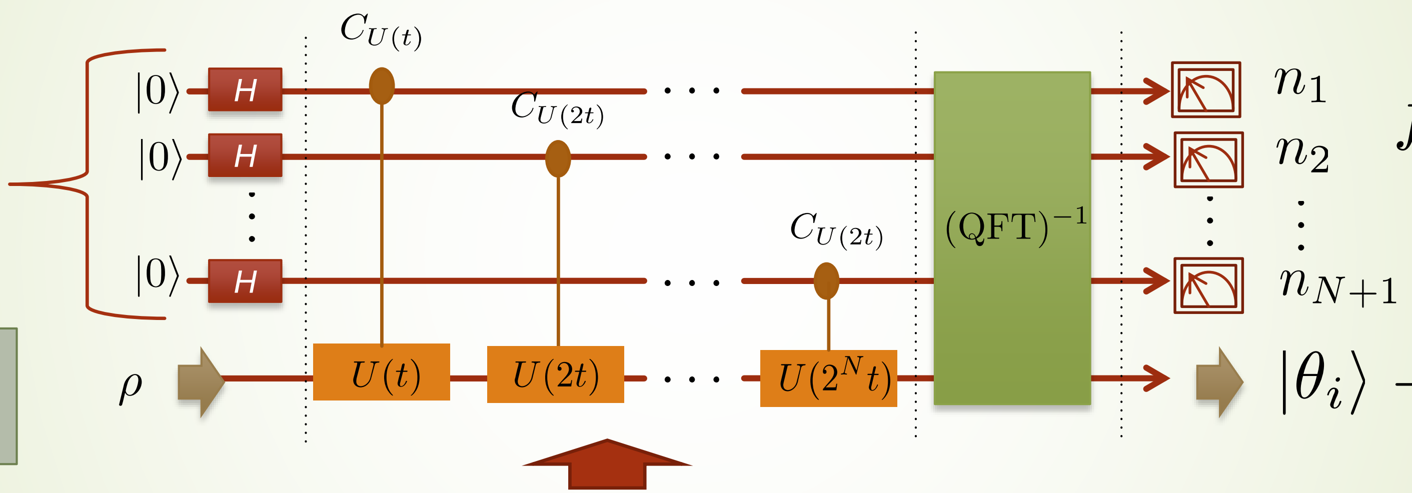
by combining the phase estimation algorithm (if |*H*| is known)

$$U(t) = e^{-iHt}$$
$$H|E_i\rangle = E_i|E_i\rangle$$

N+1 qubit
Q Computer

Unknown
Hamiltonian system

$$n_k \in \{0,1\}$$



$$f_i = 0.n_1 \cdots n_{N+1} \rightarrow E_i$$

$$|\theta_i\rangle \longrightarrow |E_i\rangle$$

use universal controllization up to phase

We can perform PME without obtaining $H = \ldots$ nor $|E_i\rangle = \ldots$

A proposal for adopting PME for the superconducting qubit system

Y. Matsuzaki, S, Nakayama, A. Soeda, S. Saito and M. Murao, PRA (2017)

4. “Quantum learning” as higher order quantum operations

"Quantum learning" of quantum maps

- A system implementing a black box quantum map f is given.

f We do not know what is f but we can perform f

- (Normal) learning of f aims to obtain the complete description
this can be done by process tomography by using f many times

  or a part of the complete description of f

  or evaluating properties of f, i.e. unitarity etc.

outputs are classical information

- "Quantum learning" aims to obtain ability to create a state $f(\rho)$ without using f, i.e. **mimicking the action of the map** for any input state ρ,

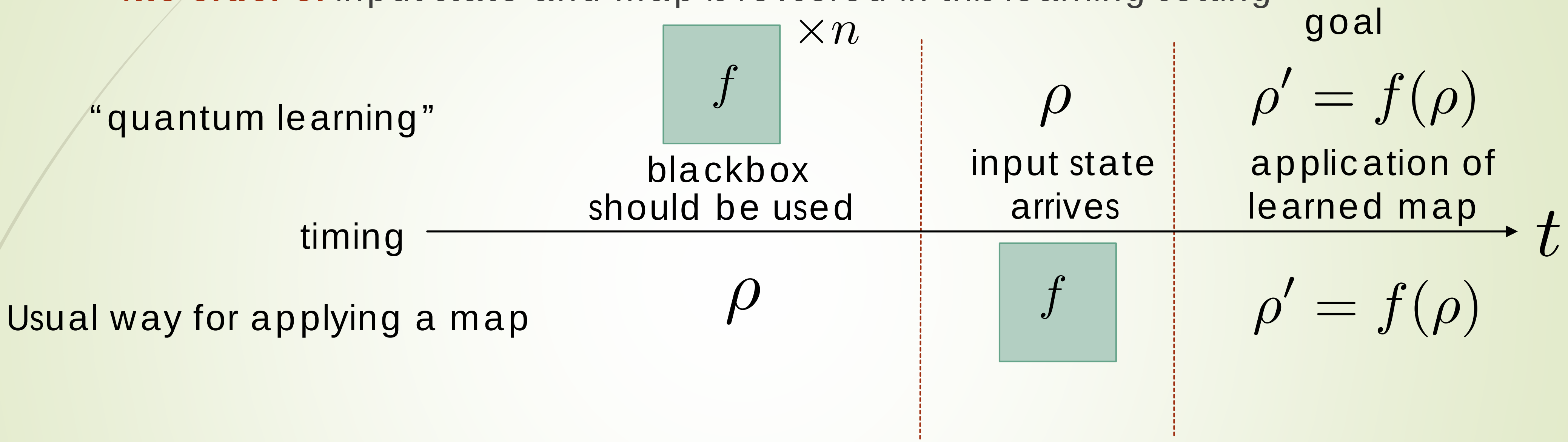
  $f = \ldots$ is not necessary to learn, but learning only the action of f

(Normal) learning of q. maps —type 1 QC→ "quantum learning" of q. maps

We can save the use of f

“Quantum learning” framework

- **The order of** input state and map is revsered in this learning setting



It is impossible to obtain the perfect learning from finite n, so we try to obtain optimal approximated state under some figure of merit

The terminology of quantum learning was introduced in

A. Bisio, G. Chiribella, G. M. D‘Ariano, S. Facchini, P. Perinotti, PRA (2010)

They analyzed in the case f is a unitary f_U

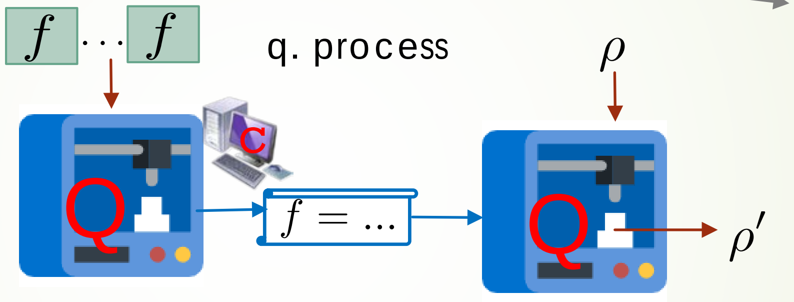
"Quantum learning*" framework

*A. Bisio, G. Chiribella, G. M. D'Ariano, S. Facchini, P. Perinotti, PRA (2010)

- We can learn $f = \ldots$ by clever quantum process tomography and then apply $f(\rho)$ by type 1 QC

Use the optimal measurement for finite copies

The "measure-and-prepare" strategy



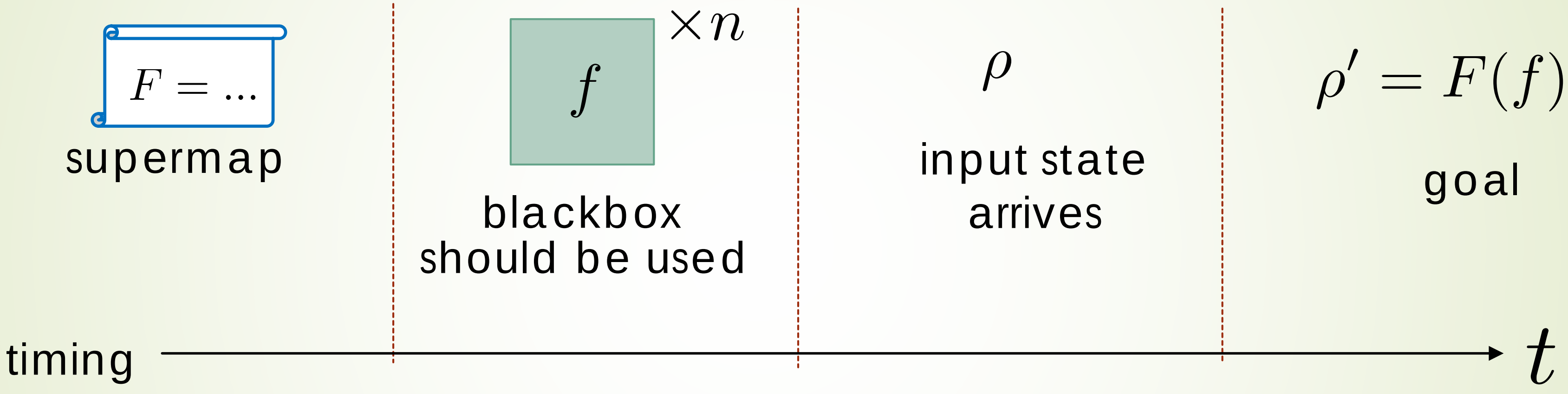
Can we do better with fully quantum type 2 QC?

- For random inputs ρ and random U, Bisio et al.* showed the average fidelity is obtained by the "measure-and-prepare" strategy
- This result is very interesting but this property is due to the uniform distribution of ρ

How about without assumption of the distribution of ρ ?

Sophisticated “Quantum learner”

- We can consider a bit sophisticated “quantum learner”, who can perform a function $F(f)$ of the learned map f, not just mimicking the map f



- The result of Bisio et al. PRA (2010) implies for the uniform distribution of ρ and U, and optimality is measured by the average fidelity, the optimal for this sophisticated quantum learning is achieved by the “measure-and-prepare” strategy.

<u>How about without assumption of the distribution of ρ ?</u>

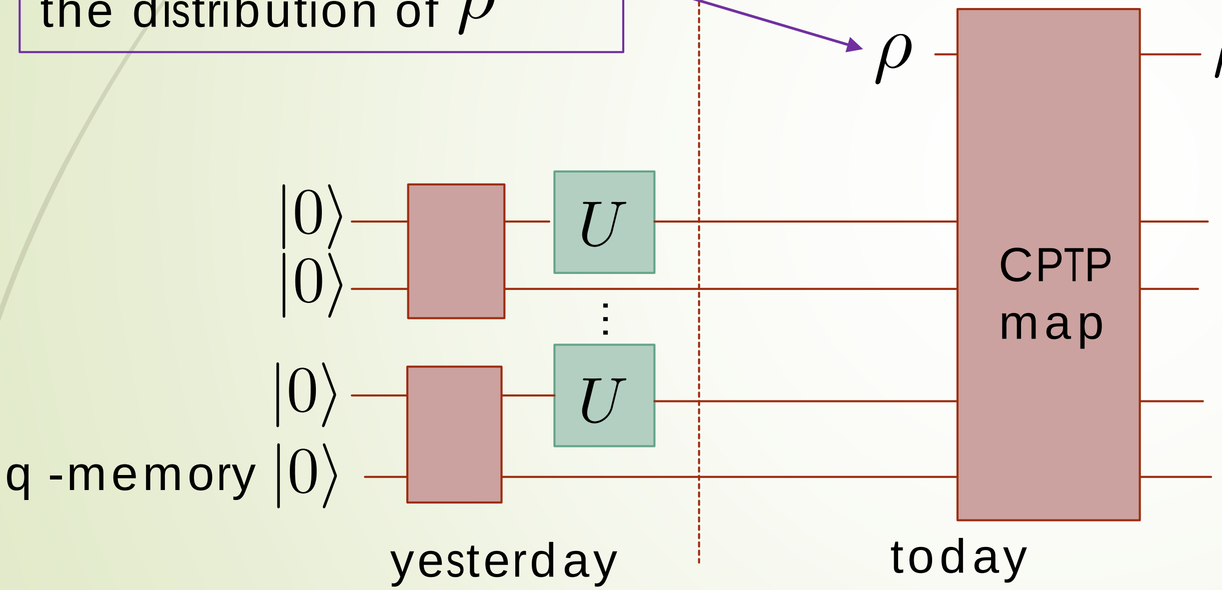
Transpose and dagger operations of a unitary

- Approximate implementation of $U^\dagger, U^*, U^T$ of an unknown unitary U in the quantum learning framework (details will be shown in the **poster session**)

Today: Poster #57

S. Nakayama, A. Soeda and M. Murao

Without assumption of the distribution of ρ



$$\rho' = F_U(f_U) = g_U(\rho) = V_U \rho V_U^\dagger$$

for

$$V_U = U^\dagger, U^*, U^T$$

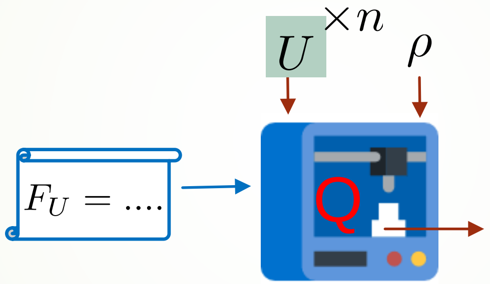
Efficiency is not as good as the "normal order" scheme for implementing U^* (as expected)

- The algorithm is a combination of
  - Adiabatic gate teleportation
  - Amplitude amplification (Grover search algorithm)
  - "State exponentiation*"

*Lloyd, Mohseni, and Rebentrost, Nat. Phys. (2014)

Summary

- To explore the power of quantum computers as a **quantum system processor processing quantum systems** including unknown quantum **states and maps** (dynamics), we analyze **higher order quantum operations of unitaries** implementing $F_U(f_U)(\rho)$ for supermaps F_U

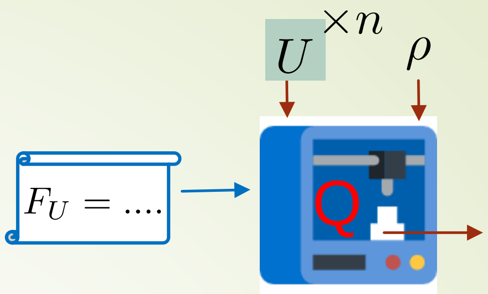


$$\rho' = F_U(f_U)(\rho) = V_U \rho V_U$$

- Many "useful" F_U cannot be applied from a single copy of ρ and a single use of U (**no-go theorems**). We show implementable F_U with finite uses of U by construction and the analyze the properties of such higher order quantum operations (complex conjugation and controllization of a unitary).

1. Complex conjugation: J. Miyazaki, A. Soeda and M. Murao, arXiv1706.03481
2. Cotrollization of unitary: Q. Dong, S. Nakayama, A. Soeda and M. Murao, in preparation
3. Cotrollization: of Hamiltonian dynamics: S. Nakayama, A. Soeda and M. Murao, PRL 114, 190501(2015)
4. "Quantum learning" type algorithm: S. Nakayama, A. Soeda and M. Murao, in preparation

What we found…



$\rho' = F_U(f_U)(\rho) = V_U \rho V_U$

Many “useful” F_U cannot be applied from a single copy of ρ and a single use of U (**no-go theorems**), so we consider relaxations of the settings.

1
- For complex conjugation, we show that a universal complex conjugation algorithm with ***d*-1 uses** of U for *d*>2 (*d*: dimension of the system)
- Exact complex conjugation cannot be implemented **even probabilistically** with a **single** use of U
- **Charge-hole** interpretation of the algorithm and conjugation-based quantities are presented

2&3
- For controllization, we analyze implementation of control-U^n by *n* use of U and present a universal exact algorithm and a universal approximate algorithm

4
- We present an algorithm for implementing supermaps in a (“sophisticated”) “**quantum learning**” **way**[4]

1. Complex conjugation: J. Miyazaki, A. Soeda and M. Murao, arXiv1706.03481
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3. Cotrollization: of Hamiltonian dynamics: S. Nakayama, A. Soeda and M. Murao, PRL 114, 190501(2015)
4. “Quantum learning” type algorithm: S. Nakayama, A. Soeda and M. Murao, in preparation

Thank you for your attention!