COMPRESSION FOR QUANTUM POPULATION CODING

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INTRODUCTION

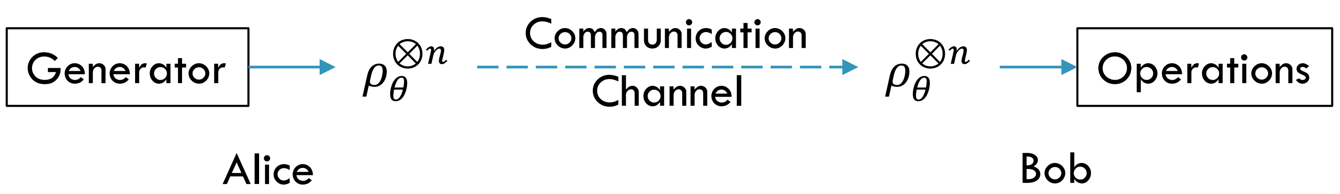
Population: A group of identical states

ENCODING OF IDENTICAL PREPARED STATES

Alice has a device that generates n identical states

Bob wants to use the states

- Eg. to perform tomography



Faithful transmission, minimal memory usage

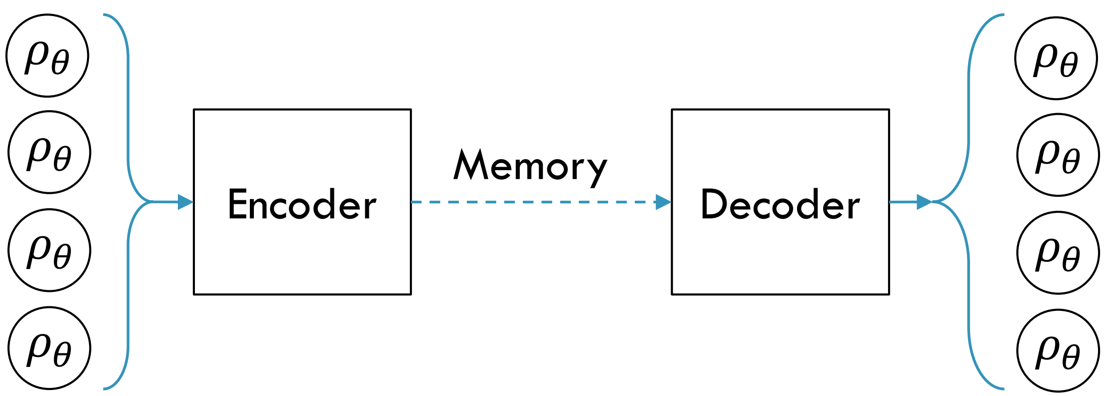
COMPRESSION FOR QUANTUM POPULATION CODING

Input: quantum population coding $\rho_\theta^{\otimes n}$ with unknown θ

Encoder/decoder: quantum channels

Faithfulness: output a state almost the same as the input $\rho_\theta^{\otimes n}$

Optimality: minimizing memory size



RELATED WORKS

Memory cost for compressing n identical

1. Qubits — $\log n$ qubits + $1/2 \log n$ bits
   [YY, GC, MH; PRL 16]
2. Clock states — $1/2 \log n$ qubits
   [YY, GC, MH; arxiv:1703.05876]
3. General qudits — $O(\log n)$
   [YY, GC, Ebler; PRL 16]
4. Classical population coding — $1/2 \log n$ bits
   [MH, Vincent Tan; 17]

MAIN RESULT

Memory cost for compressing $\rho_\theta^{\otimes n}$

PARAMETRIZATION
CLASSICAL AND QUANTUM PARAMETERS

A state family $\left\{\rho_\theta^{\otimes n} : \theta = (\mu, \xi) \in \Theta\right\}$ characterized by two kinds of parameters:

$$\rho_\theta = U_\xi \rho_\mu U_\xi^\dagger$$

Classical parameters $\mu \in \mathbb{R}^{d-1}$: determining the spectrum

Quantum parameters $\xi \in \mathbb{R}^{d^2-d}$: determining the eigenbasis

f_c, f_q: number of free classical/quantum parameters

PARAMETRIZATION EXAMPLES

Full qudit state family: $f_c = d - 1$ and $f_q = d^2 - d$

Phase-covariant state family: $f_c = 0$ and $f_q = d - 1$

$$\rho_\theta = U_\theta \rho_0 U_\theta^\dagger \quad U_\theta = \sum_k e^{i\theta_k} |k\rangle\langle k|$$

MAIN RESULT
MEMORY SIZE

A state family $\{\rho_\theta^{\otimes n}: \theta \in \Theta\}$ can be faithfully compressed into:

$(1/2 + \delta) \log n$ bits	per free classical parameter
$1/2 \log n$ bits + $\delta \log n$ qubits	per free quantum parameter

where $\delta > 0$ can be made arbitrarily small (but not zero)

Mixed qubits [YY, GC, MH; PRL 16]	$3/2 \log n$ (qu)bits
Classical population coding [MH, Vincent Tan, 17]	$1/2 \log n$ bits

MAIN RESULT
OPTIMALITY

Optimality of memory size:

Any compression protocol using $(1/2 - \epsilon) \log n$ (qu)bits cannot be faithful

Necessity of quantum memory:

Any compression protocol using only classical memory cannot be faithful unless the family is **classical** (no free quantum parameter)

BUILDING THE PROTOCOL

Achieving the minimal memory cost

BUILDING BLOCKS
QUANTUM LAN

Quantum local asymptotic normality (Q-LAN) [Kahn, Guta; CMP 09]

In a neighborhood of θ_0, $\rho_\theta^{\otimes n}$ is asymptotically ($n \to \infty$) equivalent to a classical-quantum Gaussian state:

$$\rho_\theta^{\otimes n} \quad \underset{\longleftarrow}{\xrightarrow{\text{Quantum channels}}} \quad \gamma_\mu^{class} \otimes \gamma_\xi^{quant}$$

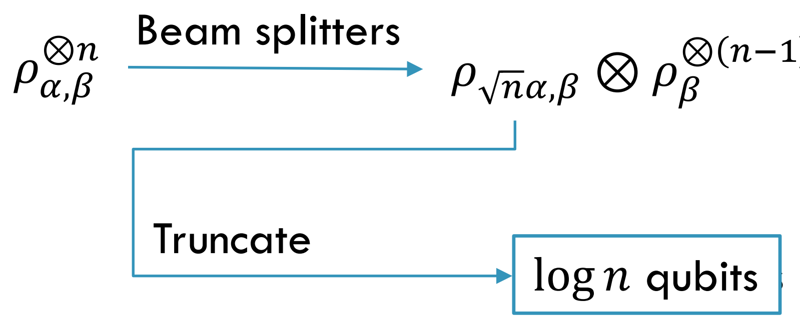
Classical mode γ_μ^{class} : a Gaussian distribution with f_c variates

Quantum mode γ_ξ^{quant}: a multimode (number of modes depending on f_q) displaced thermal state

BUILDING BLOCKS
DISPLACED THERMAL STATE ENCODER

Displaced thermal states $\rho_{\alpha,\beta} = D_\alpha \rho_\beta D_\alpha^\dagger$
where $\alpha \in \mathbb{C}, \beta \in \mathbb{R}$, D_α is the displacement operator and ρ_β is a **fixed** thermal state (state of a system in equilibrium)

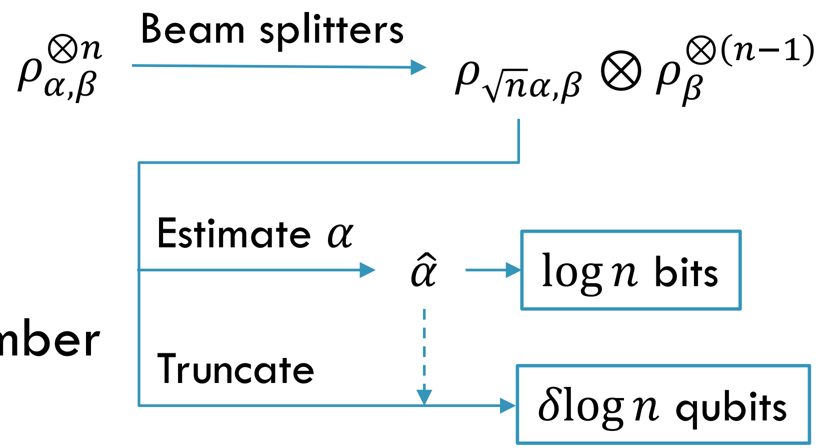
1. Concentrate the displacement
2. Truncate in photon number basis



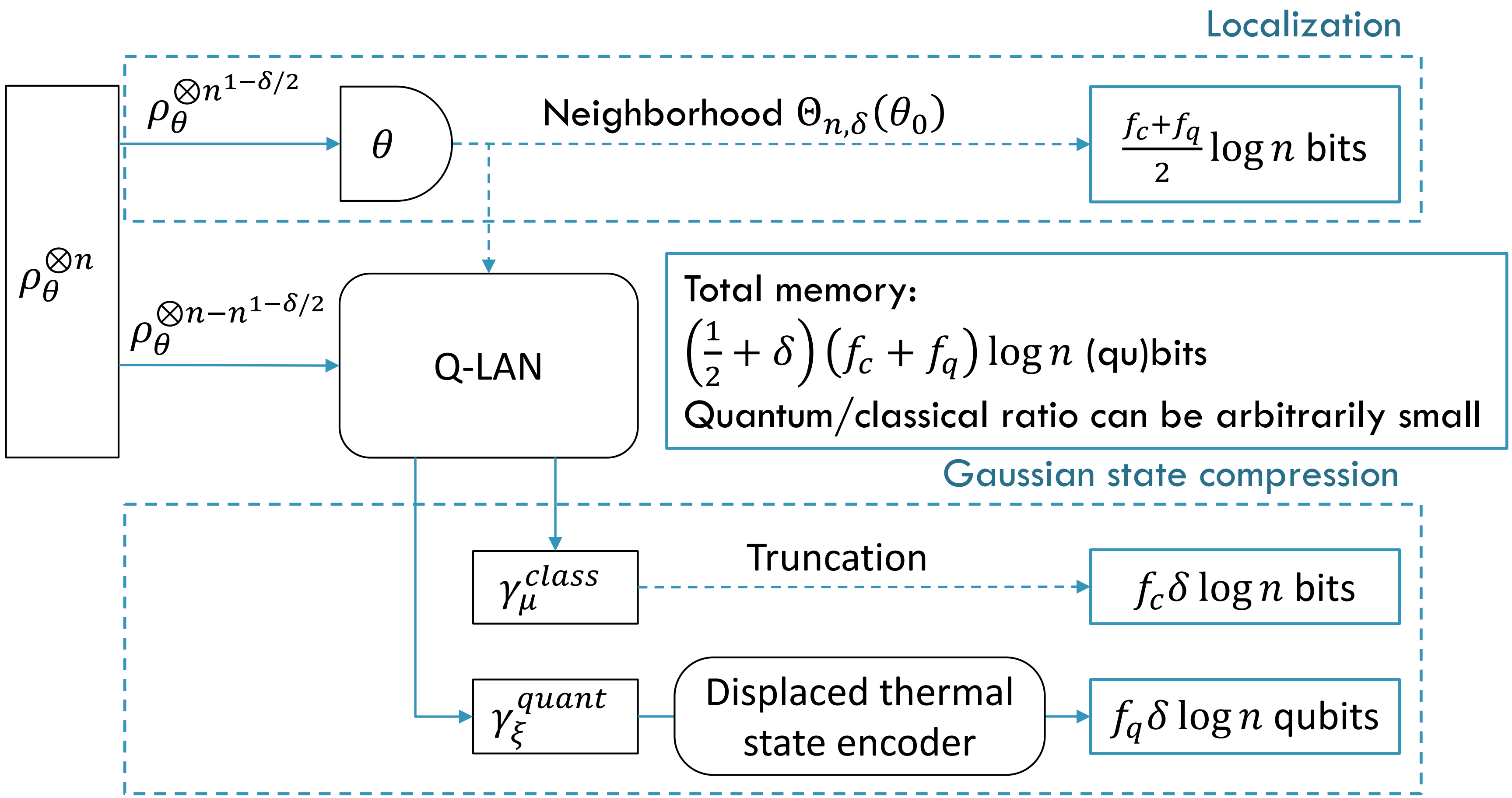
BUILDING BLOCKS
LESS QUANTUM MEMORY

Compressing $\rho_{\alpha,\beta} = D_\alpha \rho_\beta D_\alpha^\dagger$ with less quantum memory

1. Concentrate the displacement
2. Estimate α by heterodyne measurement
3. Truncate in displaced photon number basis
   - According to estimated α



THE COMPRESSION PROTOCOL



OPTIMALITY & ERROR BOUND

The information-theoretical limit

OPTIMALITY OF MEMORY SIZE

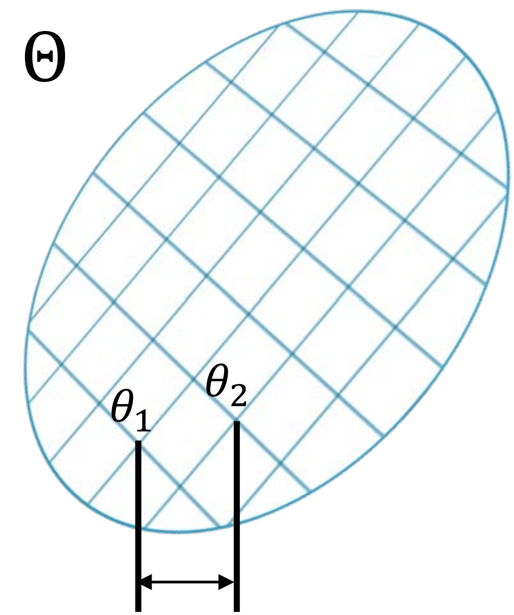
Construct a mesh M on Θ containing $\mathrm{n}^{f/2-\epsilon}$ ($f = f_c + f_q$) mutually distinguishable states

The following protocol

can faithfully communicate $\mathrm{n}^{f/2-\epsilon}$ different messages if $(\mathcal{E}, \mathcal{D})$ is faithful

The memory cost cannot be smaller than

$$\log \#\,\text{messages} = (f/2 - \epsilon)\log n$$



$O(n^{-1/2+\alpha}), \alpha > 0$

$\Rightarrow \rho_{\theta_1}^{\otimes n}$ distinguishable from $\rho_{\theta_2}^{\otimes n}$

ERROR BOUND

The compression error is upper bounded as

$$\epsilon_n = O(n^{-\delta/2}) + O\left(n^{-\kappa(\delta)}\right),$$

where the latter is the error of Q-LAN. $\kappa(\delta) > 0$ for $\delta \in (0,2/9)$

Faithfulness $\lim_{n\to\infty} \epsilon_n = 0$ is guaranteed as long as $\delta \in (0,2/9)$

The upper bound vanishes more slowly when less quantum memory (smaller δ) is used

QUANTUM MEMORY IS ESSENTIAL

The ratio between quantum/classical memory can be arbitrarily small

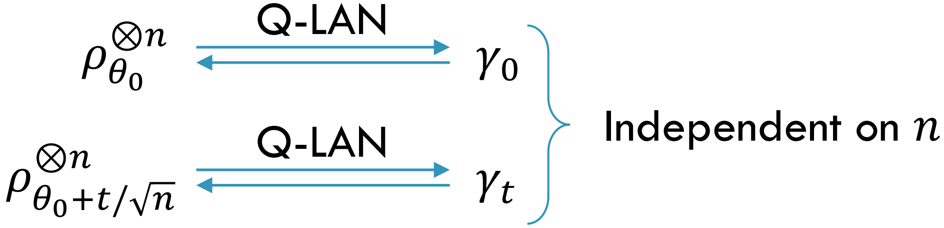
Can we use zero quantum memory? No.

$$\rho_\theta^{\otimes n} \neq \text{purely classical information}$$

Lemma: a state family can be perfectly compressed into classical memory if and only if it is **classical**, i.e. $[\rho_1, \rho_2] = 0$ for any ρ_1, ρ_2 from the family.

QUANTUM MEMORY IS ESSENTIAL

Pick states $\rho_{\theta_0}^{\otimes n}, \rho_{\theta_0+t/\sqrt{n}}^{\otimes n}$, t contains one free **quantum** parameter

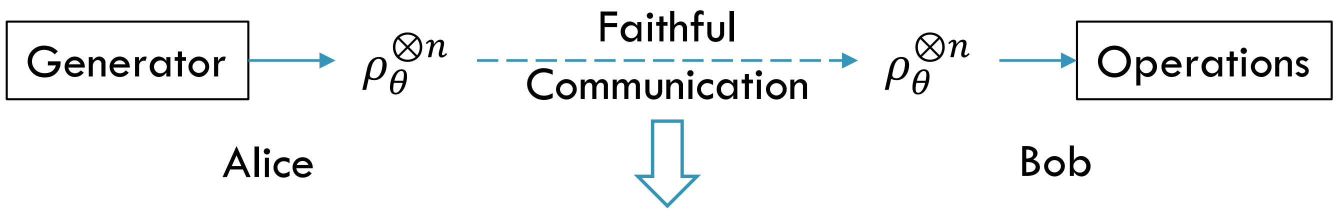


A protocol for $\rho_{\theta_0}^{\otimes n}, \rho_{\theta_0+t/\sqrt{n}}^{\otimes n} \Rightarrow$ a protocol for γ_0, γ_t

$n \to \infty \Rightarrow$ a protocol for γ_0, γ_t with arbitrarily small error

$\Rightarrow [\gamma_0, \gamma_t] = 0$ (which is false)

SUMMARY & FUTURE WORKS



$\sim 1/2 \log n$ for each free parameter

Arbitrarily small, but non-zero quantum memory

Extension to non-product states with symmetry

- Real population coding
- eg. states of bosonic systems

Non-zero quantum memory – how small can it be?

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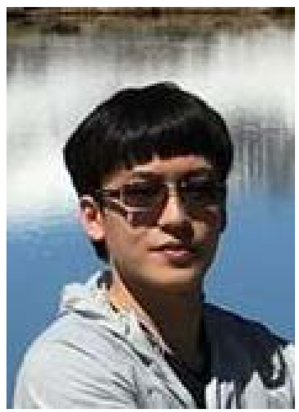

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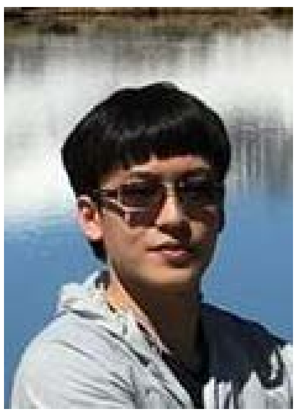

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THANKS!

Full version of this paper:
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