Detecting metrologically useful coherence by few local measurements

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Outlines

- Brief review of quantum coherence
- The speed detection scheme
  - Relate speed to coherence
  - Relate speed to entanglement
- Demonstrate speed detection scheme in an all-optical experiment
Quantum Coherence
Quantum Coherence

• the most essential property that distinguishes quantum mechanics from classical theory

• fundamental resource for quantum information processing

• plays an important role in the fields of superfluidity, thermodynamics, and quantum biology
Quantum Coherence

- Quantifying coherence

(i) \( C(\rho) \geq 0 \) for all states \( \rho \), with \( C(\delta) = 0 \) for all incoherent states \( \delta \in \mathcal{I} \).

(ii a) Contractivity under incoherent channels \( \Lambda_{\text{ICPTP}} \),

\[ C(\rho) \geq C(\Lambda_{\text{ICPTP}}(\rho)) \]

(ii b) Contractivity under selective measurements on average,

\[ C(\rho) \geq \sum_j p_j C(\rho_j) \]

where \( \rho_j = K_j \rho K_j^\dagger / p_j \) and \( p_j = \text{Tr}(K_j \rho K_j^\dagger) \), for any \( \{K_j\} \) such that \( \sum_j K_j^\dagger K_j = \mathbb{1} \) and \( K_j \mathcal{I} K_j \subset \mathcal{I} \) for all \( j \).

(iii) Convexity,

\[ C(q\rho + (1-q)\tau) \leq qC(\rho) + (1-q)C(\tau) \]

for any states \( \rho \) and \( \tau \) and \( q \in [0, 1] \).
The $l_1$-norm quantifies coherence in an intuitive way, by the off-diagonal elements of a density matrix $\rho$ in the reference basis

$$C_{l_1}(\rho) = \sum_{i \neq j} |\rho_{ij}|.$$ 

Alternatively, one can quantify coherence by means of a geometric approach. Given a distance $D$, a generic distance-based measure of coherence is defined as

$$C_D(\rho) = \min_{\delta \in \mathcal{I}} D(\rho, \delta) = D(\rho, \delta_\rho),$$

In the quantum metrology scenario, coherence can be identified as the degree of uncertainty about the value of an observable $K$ while performing a measurement on the state, or equivalently the sensitivity of the state to a phase shift generated by $K$. 

Quantum Coherence
All the methods need to perform full state tomography which require exponentially increasing resources with the system size.

State tomography may contain redundant information, we actually do not need to know the full information about the state matrix.

So it is important to design alternative strategies.
Speed detection
Speed of evolution

Classical

- Exerting forces to test material strength
- Evaluating reflexes to assess neuromuscular efficiency

Quantum

- Add a unitary gate to test the speed of evolution of the system
**Speed of evolution**

- **Definition**

We quantify the system speed over an interval $0 \leq t \leq \tau$ by the average rate of change of the state, which is given by mean values of quantum operators $\langle \cdot \rangle_{\rho_t} = \text{Tr}(\cdot \rho_t)$:

$$s_\tau(\rho_t) := \frac{\|\rho_\tau - \rho_0\|_2}{\tau} = \frac{(\langle \rho_\tau \rangle_{\rho_\tau} + \langle \rho_0 \rangle_{\rho_0} - 2\langle \rho_\tau \rangle_{\rho_0})^{1/2}}{\tau},$$

where the Euclidean distance is employed.
Speed of evolution

• The state overlaps can be quantified by measuring the swap operator on two system copies.

$$\langle \sigma^- \rangle_{\rho} = \langle V \rangle_{\rho \otimes \sigma^-}$$

$$V(|\phi_1 \rangle \otimes |\phi_2 \rangle) = |\phi_2 \rangle \otimes |\phi_1 \rangle, \forall |\phi_{1,2} \rangle$$

• For two qubit swaps

$$V_{12} = I - 2P_{12}^-, P_{12}^- = |\psi \rangle \langle \psi |_{12}, |\psi \rangle = 1 / \sqrt{2}(|01 \rangle - |10 \rangle)$$

• Each local swap can be recast in terms of projections on the Bell singlet
Speed of evolution

- Overlap detection network
Proof that speed bounds any QFI

The system speed can express in terms of the Hilbert-Schmidt distance $D_{HS}(\rho, \sigma) = \sqrt{\text{Tr}((\rho - \sigma)^2)}$ and the related norm

$$S_\tau(\rho, H) := \frac{s_\tau(\rho)^2}{2} = \frac{D_{HS}^2(\rho, U_\tau \rho U_\tau^\dagger)}{(2\tau^2)} = \frac{\|U_\tau \rho U_\tau^\dagger - \rho\|_2^2}{(2\tau^2)}.$$ 

The zero limit is

$$S_0(\rho, H) := \lim_{\tau \to 0} S_\tau(\rho, H) = -\frac{1}{2}\text{Tr}([\rho, H]^2).$$

By expanding the quantity in terms of the state spectrum and eigenbasis, one has

$$S_0(\rho, H) = \sum_{i \neq j} (\lambda_i - \lambda_j)^2 / 2|\langle i | H | j \rangle|^2.$$ 

The expression of the quantum fisher information

$$\mathcal{I}_F(\rho, H) = \sum_{i \neq j} (\lambda_i - \lambda_j)^2 / (2(\lambda_i + \lambda_j))|\langle i | H | j \rangle|^2.$$ 

Since $\lambda_i + \lambda_j \leq 1$, $\forall i, j,$

$$S_0(\rho, H) \leq \mathcal{I}_F(\rho, H), \forall \rho, H.$$
Proof that a non-linear scaling of speed witnesses entanglement

In the quantum metrology scenario, the quantum fisher information of separable states achieve at best (with the adopted normalization)

\[ \mathcal{I}_F(\rho, H_n) = n/4 \]

while entanglement enables up to a quadratic improvement

\[ \mathcal{I}_F(\rho, H_n) = n^2 \]

So \( \mathcal{I}_F(\rho, H_n) > n/4 \) witnesses entanglement. Thus, the speed function also witnesses entanglement

\[ S_\tau(\rho, H_n) > n/4. \]
Experimental Demonstration
We experimentally demonstrate the scheme of a two-qubit system AB.

The system is prepared in a mixture of Bell states

\[ \rho_{p,AB} = p|\phi^+\rangle\langle\phi^+| + (1-p)|\phi^-\rangle\langle\phi^-|, |\phi^\pm\rangle = 1/\sqrt{2}(|00\rangle \pm |11\rangle) , \]

We run a series of experiments with equally stepped values of the mixing parameter \( p=0,0.1,0.2,\ldots,0.9,1 \)

We choose the perturbation as the phase shift induced by three pauli matrix, and we choose the phase shift \( \theta=\pi/6 \)
Experimental demonstration
We employ a sandwich-like EPR source.

The two BBO crystals are identically cut, with one true zero order HWP in the middle.

The source has high brightness, high collection efficiency, high fidelity at the same time. It is extremely suitable for multiphoton experiments.

The detailed description can be found in PRL 115, 260402.
• We prepare copy 2 from two SPDC sources by post-selection.

• Reason: The four photons interfering into the BSMs form a closed-loop network. This poses the problem to rule out the same-order noise. We generate Copy 2 with two trigger photons which guarantee to generate the two copies from different sources.
We employ standard linear optical Bell state measurement scheme

The HWPs after the first PBS are set to be 22.5, which can measure the photons in the +/- basis. When the input state is $|\phi^+\rangle$, the measurement result will be $|+ +\rangle$ or $|- -\rangle$; When the input state is $|\phi^-\rangle$, the measurement result will be $|+ -\rangle$ or $|- +\rangle$; If we want to discriminate $|\psi^{\pm}\rangle$, we need to insert a 45 HWP in one of the input port of the PBS.
We performed tomographies of the input Bell states and of the BSMs. The fidelity of the input states are respectively $0.9889 (\phi_1^+)$, $0.9901 (\phi_1^-)$, $0.9279 (\phi_2^+)$, $0.9319 (\phi_2^-)$. The average fidelities of BSM1 and BSM2 are $0.9389 \pm 0.0030$ and $0.9360 \pm 0.0034$.

The speed measurement results for three directions:
Conclusion

• We propose efficient speed detection scheme by measuring a set of local observables increasing linearly with the number of qubits.

• We show the speed of evolution of a quantum system can reveal its key properties including metrologically useful coherence and entanglement.

• We demonstrate the scheme in an all-optical experiment.

• arXiv: 1611.02004
Thank you!