Detecting metrologically useful coherence by few local measurements

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Outlines

- Brief review of quantum coherence
- the speed detection scheme
  - Relate speed to coherence
  - Relate speed to entanglement
- Demonstrate speed detection scheme in an all-optical experiment

Quantum Coherence

Quantum Coherence

- the most essential property that distinguishes quantum mechanics from classical theory
- fundamental resource for quantum information processing
- plays an important role in the fields of superfluidity, thermodynamics, and quantum biology

Quantum Coherence

- Quantifying coherence

(i) $\mathcal{C}(\rho) \geq 0$ for all states ρ, with $\mathcal{C}(\delta) = 0$ for all incoherent states $\delta \in \mathcal{I}$.

(ii a) Contractivity under incoherent channels Λ_{ICPTP}, $\mathcal{C}(\rho) \geq \mathcal{C}(\Lambda_{\text{ICPTP}}(\rho))$.

(ii b) Contractivity under selective measurements on average, $\mathcal{C}(\rho) \geq \Sigma_j p_j \mathcal{C}(\rho_j)$, where $\rho_j = K_j \rho K_j^\dagger / p_j$ and $p_j = \text{Tr}(K_j \rho K_j^\dagger)$, for any $\{K_j\}$ such that $\Sigma_j K_j^\dagger K_j = \mathbb{I}$ and $K_j \mathcal{I} K_j \subset \mathcal{I}$ for all j.

(iii) Convexity, $\mathcal{C}(q\rho + (1-q)\tau) \leq q\mathcal{C}(\rho) + (1-q)\mathcal{C}(\tau)$ for any states ρ and τ and $q \in [0, 1]$.

Quantum Coherence

- The l1-norm quantifies coherence in an intuitive way, by the off-diagonal elements of a density matrix ρ in the reference basis

$$\mathcal{C}_{l_1}(\rho) = \sum_{i \neq j} |\rho_{ij}|.$$

- Alternatively, one can quantify coherence by means of a geometric approach. Given a distance D, a generic distance-based measure of coherence is defined as

$$\mathcal{C}_D(\rho) = \min_{\delta \in \mathcal{I}} D(\rho, \delta) = D(\rho, \delta_\rho),$$

- In the quantum metrology scenario, coherence can be identified as the degree of uncertainty about the value of an observable K while performing a measurement on the state, or equivalently the sensitivity of the state to a phase shift generated by K.

Quantum Coherence

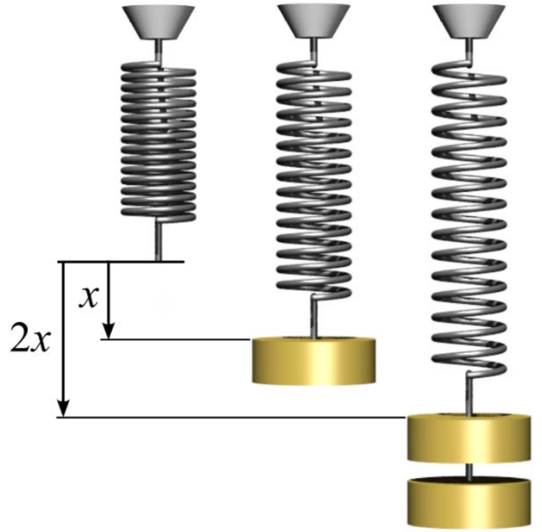
- All the methods need to perform full state tomography which require exponentially increasing resources with the system size.
- State tomography may contain redundant information, we actually do not need to know the full information about the state matrix.
- So it is important to design alternative strategies.

Speed detection

Speed of evolution

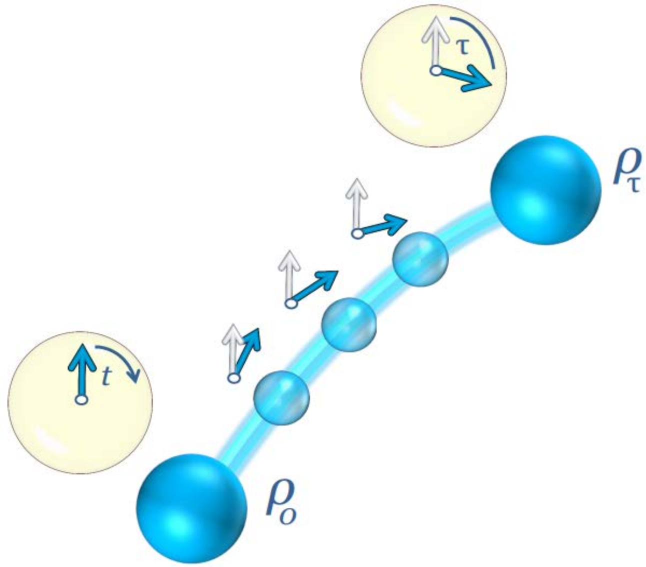
Classical

- Exerting forces to test material strength
- Evaluating reflexes to assess neuromuscular efficiency



Quantum

- Add a unitary gate to test the speed of evolution of the system



Speed of evolution

- Definition

We quantify the system speed over an interval $0 \leq t \leq \tau$ by the average rate of change of the state, which is given by mean values of quantum operators $\langle \cdot \rangle_{\rho_t} = \mathrm{Tr}(\cdot \rho_t)$:

$$s_\tau(\rho_t) := \frac{||\rho_\tau - \rho_0||_2}{\tau} = \frac{(\langle \rho_\tau \rangle_{\rho_\tau} + \langle \rho_0 \rangle_{\rho_0} - 2\langle \rho_\tau \rangle_{\rho_0})^{1/2}}{\tau},$$

where the Euclidean distance is employed

Speed of evolution

- The state overlaps can be quantified by measuring the swap operator on two system copies.

$$\langle\sigma\rangle_\rho = \langle V\rangle_{\rho\otimes\sigma}$$
$$V(|\phi_1\rangle \otimes |\phi_2\rangle) = |\phi_2\rangle \otimes |\phi_1\rangle, \forall|\phi_{1,2}\rangle$$

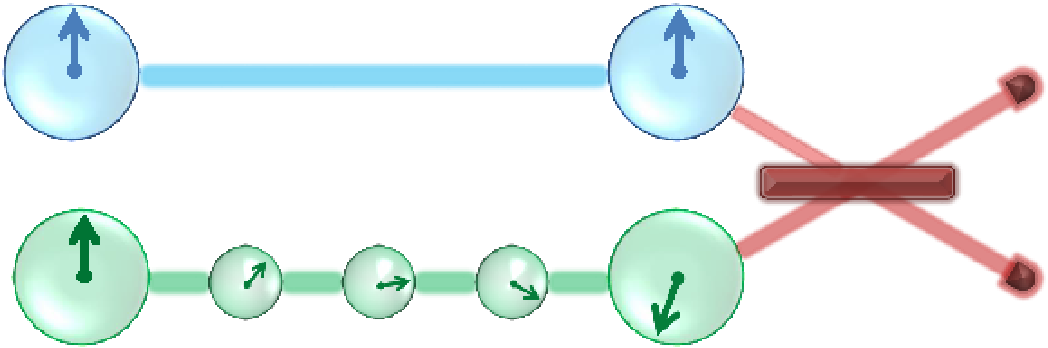
- For two qubit swaps

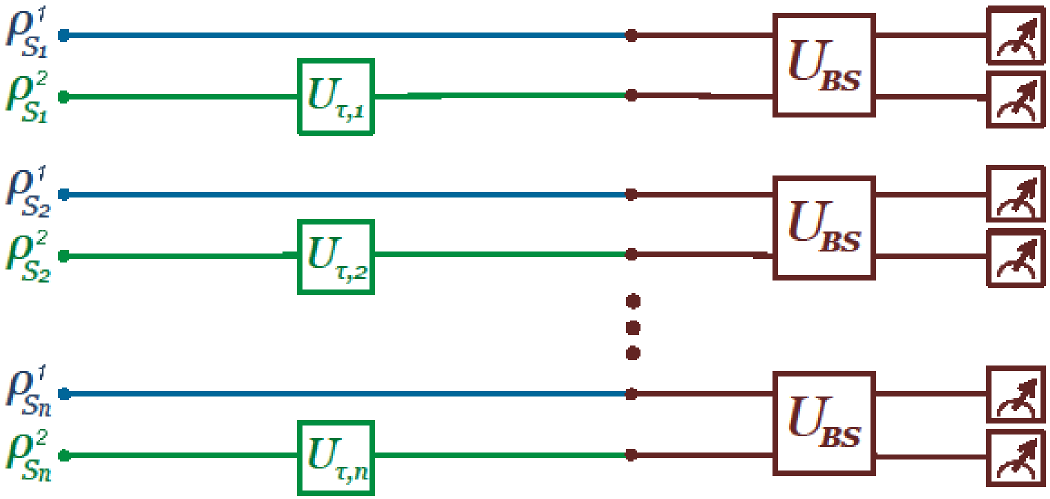
$$V_{12} = \mathbb{I} - 2P^-_{12}, P^-_{12} = |\psi\rangle\langle\psi|_{12}, |\psi\rangle = 1/\sqrt{2}(|01\rangle - |10\rangle).$$

- Each local swap can be recast in terms of projections on the Bell singlet

Speed of evolution

- Overlap detection network





Relate speed to coherence

- Proof that speed bounds any QFI

The system speed can express in terms of the Hilbert-Schmidt distance $D_{\mathrm{HS}}(\rho,\sigma) = \sqrt{\mathrm{Tr}((\rho-\sigma)^2)}$ and the related norm

$$\mathcal{S}_\tau(\rho, H) := s_\tau(\rho)^2/2 = D^2_{\mathrm{HS}}(\rho, U_\tau \rho U_\tau^\dagger)/(2\tau^2) = ||U_\tau \rho U_\tau^\dagger - \rho||_2^2/(2\tau^2).$$

The zero limit is

$$\mathcal{S}_0(\rho, H) := \lim_{\tau->0} \mathcal{S}_\tau(\rho, H) = -1/2\mathrm{Tr}([\rho, H]^2).$$

By expanding the quantity in terms of the state spectrum and eigenbasis, one has

$$\mathcal{S}_0(\rho, H) = \sum_{i\neq j}(\lambda_i - \lambda_j)^2/2|\langle i|H|j\rangle|^2.$$

The expression of the quantum fisher information

$$\mathcal{I}_F(\rho, H) = \sum_{i\neq j}(\lambda_i - \lambda_j)^2/(2(\lambda_i + \lambda_j))|\langle i|H|j\rangle|^2.$$

Since $\lambda_i + \lambda_j \leq 1, \forall i, j$,

$$\mathcal{S}_0(\rho, H) \leq \mathcal{I}_F(\rho, H), \forall \rho, H.$$

Relate speed to entanglement

- Proof that a non-linear scaling of speed witnesses entanglement

In the quantum metrology scenario, the quantum fisher information of separable states achieve at best (with the adopted normalization)

$$\mathcal{I}_F(\rho, H_n) = n/4$$

while entanglement enables up to a quadratic improvement

$$\mathcal{I}_F(\rho, H_n) = n^2$$

So $\mathcal{I}_F(\rho, H_n) > n/4$ witnesses entanglement.

Thus, the speed funtion also witnesses entanglement

$$\mathcal{S}_\tau(\rho, H_n) > n/4.$$

Experimental Demonstration

Experimental demostration

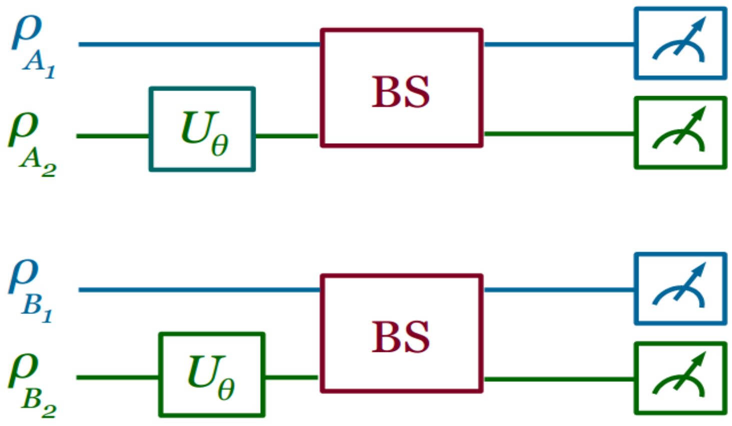
- We experimentally demonstrate the scheme of a two-qubit system AB.

- The system is prepared in a mixture of Bell states

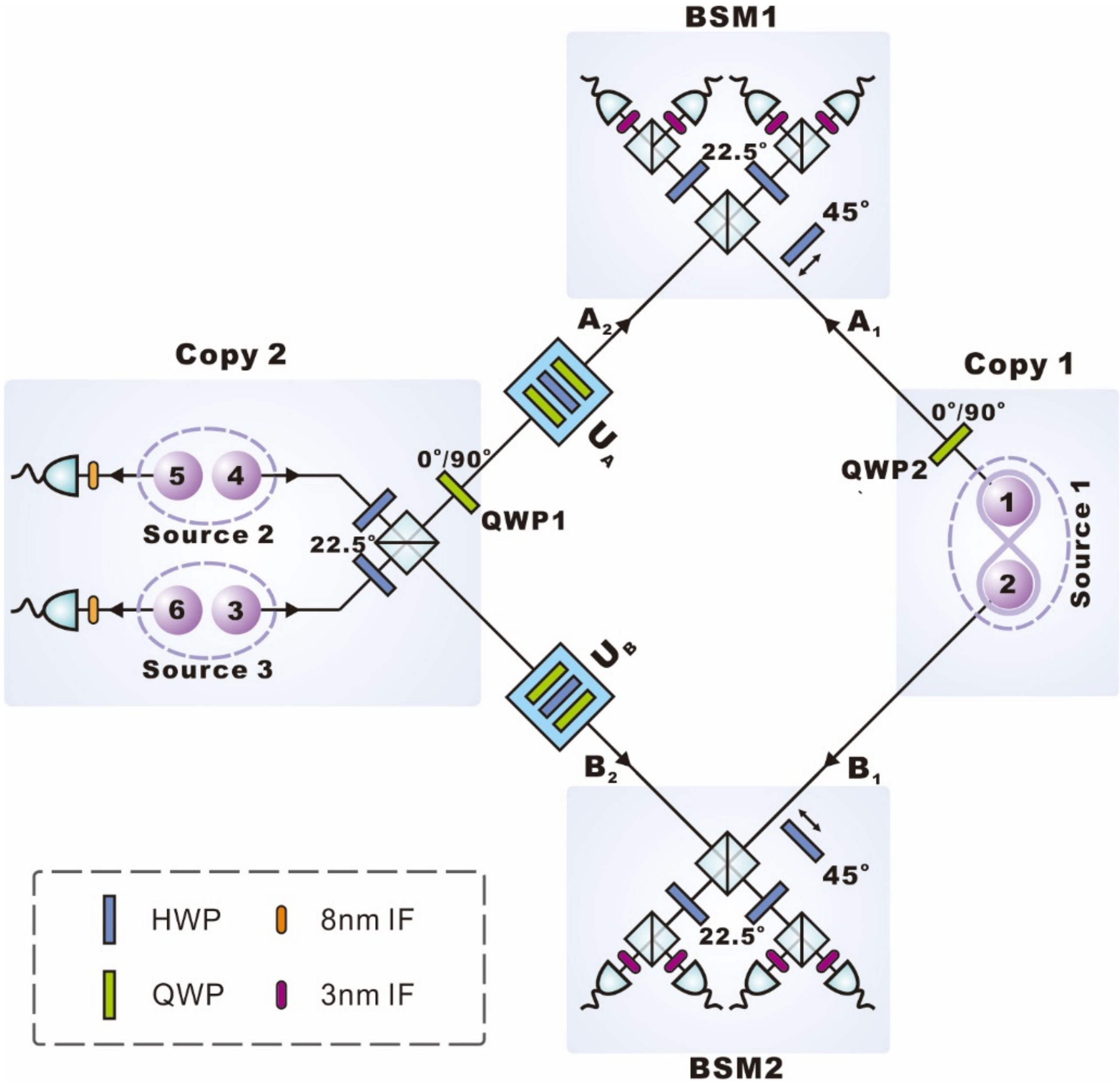
$$\rho_{p,AB} = p|\phi^+\rangle\langle\phi^+| + (1-p)|\phi^-\rangle\langle\phi^-|, |\phi^\pm\rangle = 1/\sqrt{2}(|00\rangle \pm |11\rangle),$$

- We run a series of experiments with equally stepped values of the mixing parameter p=0,0.1,0.2,…,0.9,1

- We choose the perturbation as the phase shift induced by three pauli matrix, and we choose the phase shift theta=pi/6

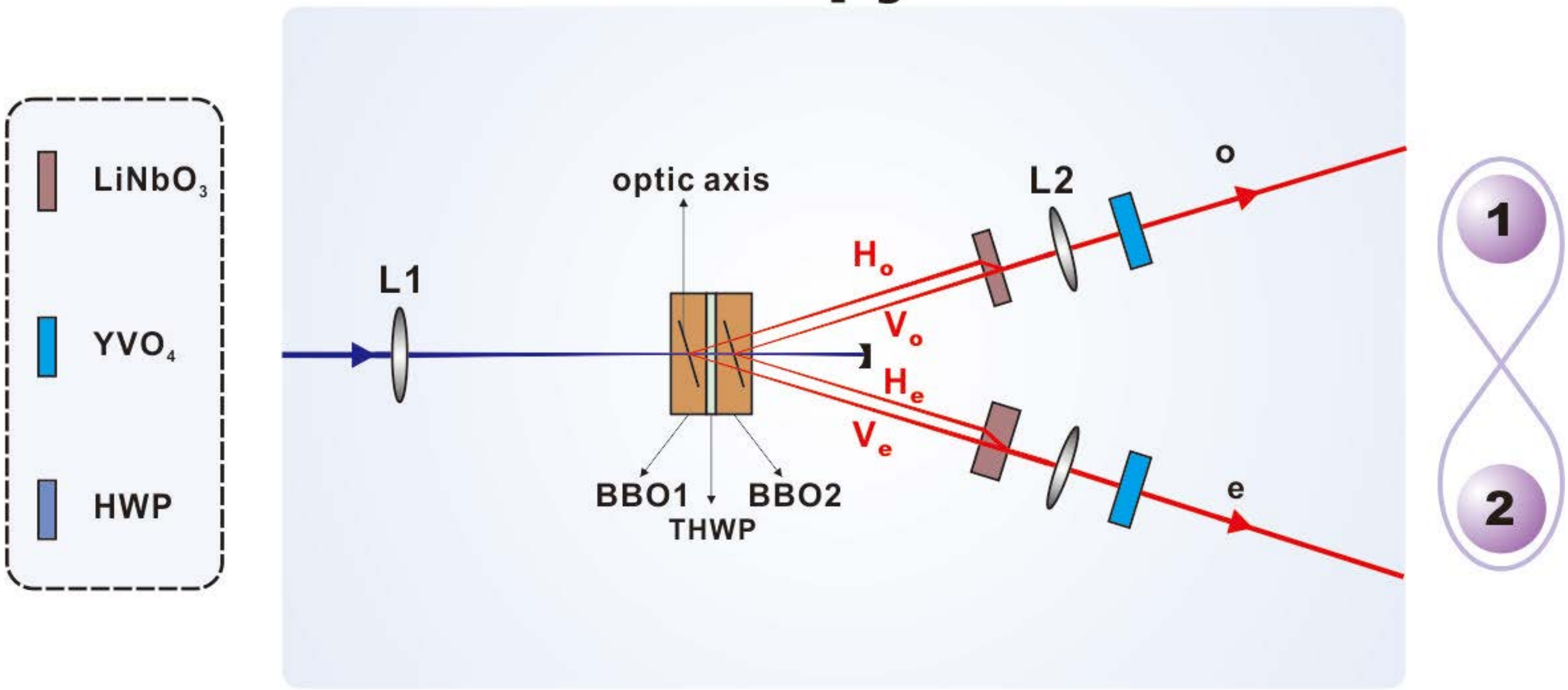


Experimental demostration



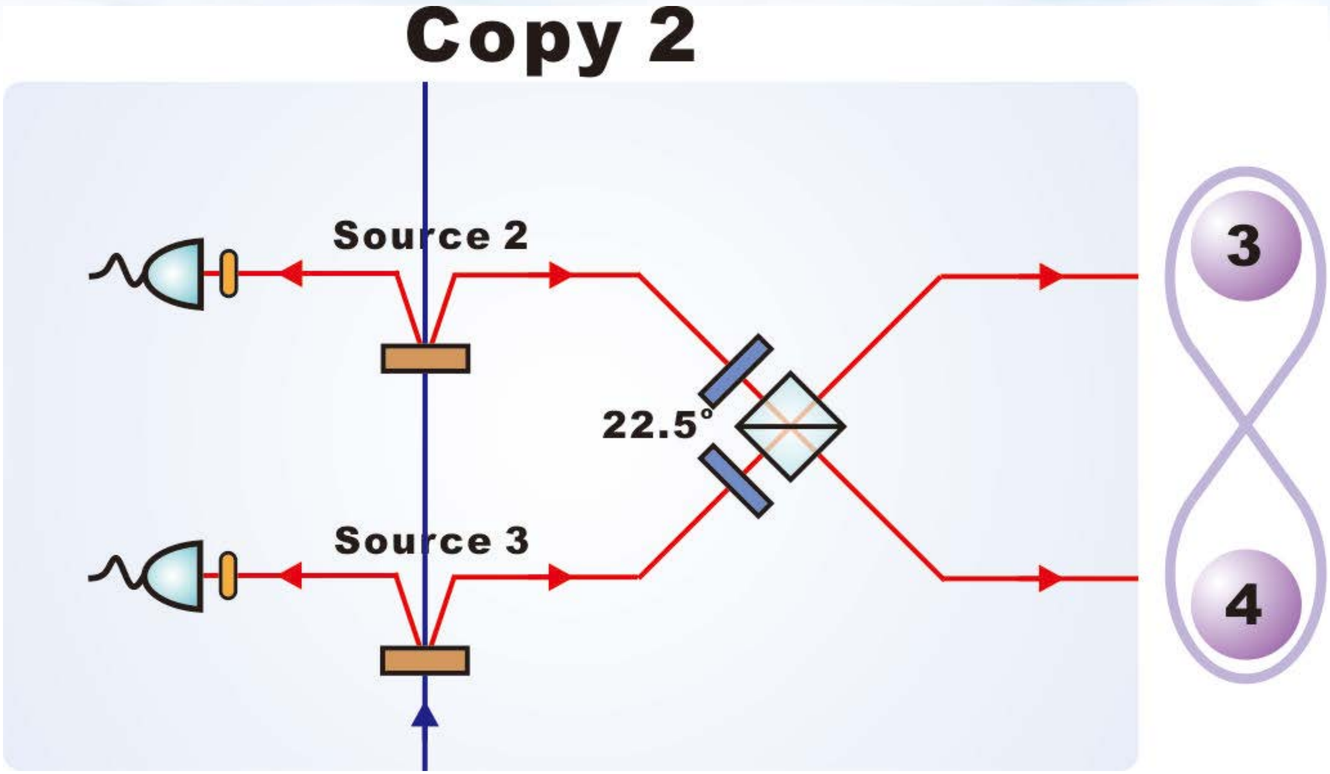
Experimental demostration

Copy 1



- We employ a sandwichlik EPR source.
- The two BBO crystals are identically cut, with one true zero order HWP in the middle
- The source has high brightness, high collection efficiency, high fidelity at the same time. It is extremely suitable for multiphoton experiments.
- The detailed description can be found in PRL 115, 260402

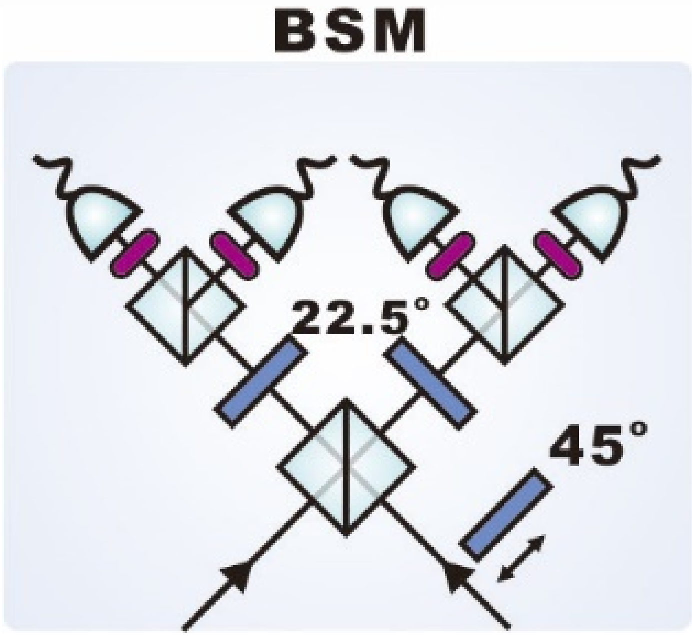
Experimental demostration



- We prepare copy 2 from two SPDC sources by post-selection.

- Reason: The four photons interfering into the BSMs form a closed-loop network. This poses the problem to rule out the same-order noise. We generate Copy 2 with two trigger photons which guarantee to generate the two copies from different sources.

Experimental demostration



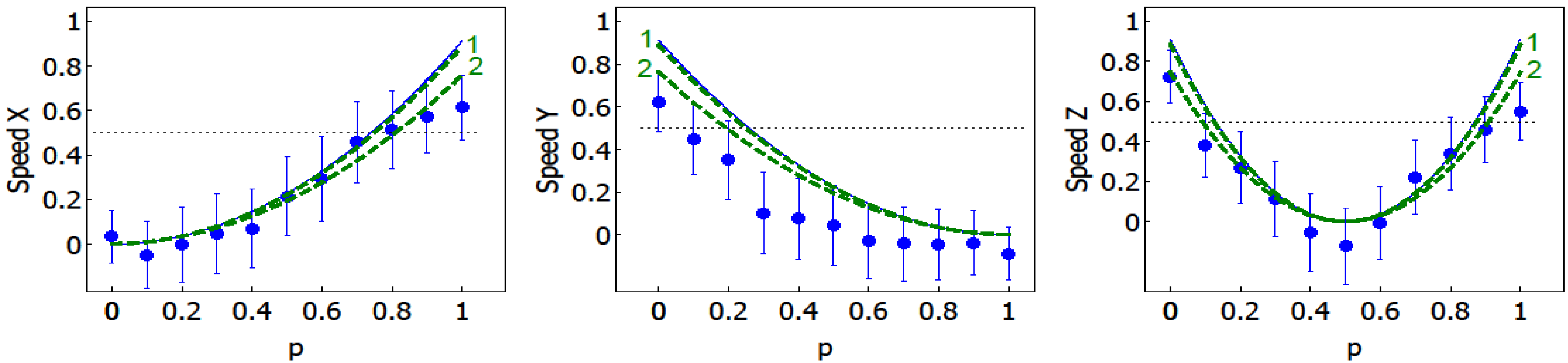
- We employ standard linear optical Bell state measurement scheme

- The HWPs after the first PBS are set to be 22.5, which can measure the photons in the +/- basis. When the input state is $|\phi^+\rangle$, the measurement result will be $|+\,+\rangle$ or $|-\,-\rangle$; When the input state is $|\phi^-\rangle$, the measurement result will be $|+\,-\rangle$ or $|-\,+\rangle$; If we want to discriminate $|\psi^\pm\rangle$, we need to insert a 45 HWP in one of the input port of the PBS.

Results

- We performed tomographies of the input Bell states and of the BSMs. The fidelity of the input states are respectively $0.9889\ (\phi_1^+), 0.9901\ (\phi_1^-),\ 0.9279\ (\phi_2^+)$, $0.9319\ (\phi_2^-)$. The average fidelities of BSM1 and BSM2 are 0.9389 ± 0.0030 and 0.9360 ± 0.0034.

- The speed measurement results for three directions:



Conclusion

- We propose efficient speed detection scheme by measuring a set of local observables increasing linearly with the number of qubits.
- We show the speed of evolution of a quantum system can reveal its key properties including metrologically useful coherence and entanglement.
- We demonstrate the scheme in an all-optical experiment.
- arXiv: 1611.02004

Thank you!