Flow Ambiguity: A Path Towards Classically Driven Blind Quantum Computation

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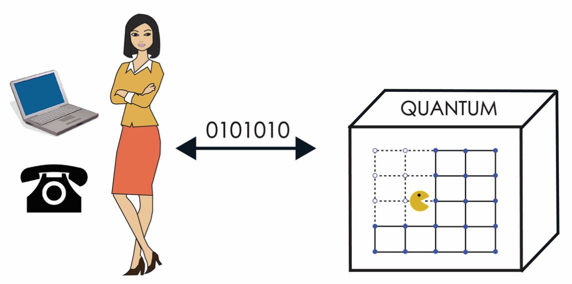
Problem Statement and Prior Work

Delegated computation

Imagine a scenario where large scale quantum computer becomes available BUT only at a few locations around the globe, however anyone can access it over the internet!



Security

What about integrity and privacy of the client's computation?

Blind quantum computation

Blind quantum computation[1]- allows a client, with weak computational power, to delegate a computation to a remote (powerful) quantum server. These protocols come mainly in two flavors:

- (*Blindness*) privacy of client's computation is preserved.
- (*Verification*) integrity of the desired computation is maintained.

[1]For more information see review article: npj Quantum Information 3, Article number: 23 (2017)

Previous works

Protocol	client's power	No. of server
BFK[2]	single qubit preparation device	1
MF[3]	measurement apparatus	1
RUV[4]	completely classical	2

A common feature among all these known protocols is that either the client requires a small quantum device on their side or there must exist at least two non-communicating quantum servers.

[2] Universal blind quantum computation. FOCS'09. 50th Annual IEEE Symposium (pp. 517-526).
[3] PRA 87(5), 050301
[4] Nature, 496(7446), 456-460. (servers are entangled but noncommunicating)

Problem statement

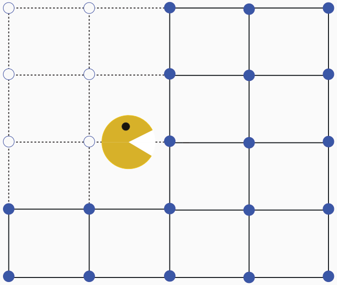
Classically Driven Blind Quantum Computation

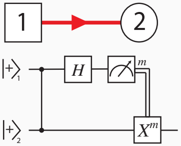
Is it possible for a completely classical client to delegate a quantum computation to a single remote quantum server while keeping the information about computation hidden?

Preliminaries

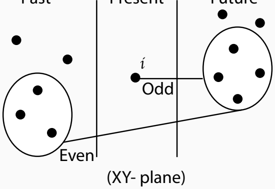
Measurement-based quantum computation

MBQC





1-bit teleportation



gflow

Our contribution

Blind computing protocol with a completely classical client

Task

Alice's target computation is given by $\Delta_A = \{\rho_I, U_A, \mathcal{M}\}$, where ρ_I[5] is the n-qubit input state, U_A is the unitary embedding that maps ρ_I to the output state $\rho_O = U_A \rho_I U_A^{\dagger}$, and $\mathcal{M}$ is the final set of measurement on the output state to get the classical output.

We propose an interactive protocol to perform this task which we call *classically driven blind quantum computation*.

[5] We take input states that can be efficiently described classically.

Classically driven blind quantum computing

Protocol steps are as follows:

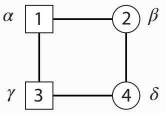
- **State preparation**: Bob prepares the graph state $|\mathcal{G}\rangle_{n\times m}$.
- **Measurements**: For $i = 1, \ldots, N$, repeat:
  1. Alice picks a bit $r_i \in_R \{0,1\}$ uniformly at random. Then, using r_i, $\boldsymbol{s}^x$, $\boldsymbol{s}^z$, she computes the angle α'_i, where

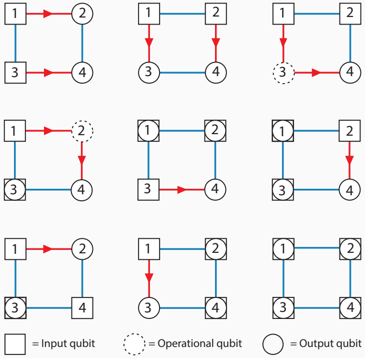
  $$\boldsymbol{\alpha}' = (-1)^{\boldsymbol{s}^x}\boldsymbol{\alpha} + (\boldsymbol{s}^z + \boldsymbol{r})\pi \mod 2\pi,$$

  2. Alice transmits α'_i to Bob.
  3. Bob measures the ith qubit in the basis $\{|\pm_{\alpha'_i}\rangle\}$ and transmits the measurement outcome $b'_i \in \{0,1\}$ to Alice.
  4. Alice records $b_i = b'_i \oplus r_i$ in $\mathbf{b}$ and then updates the dependency sets $(\boldsymbol{s}^x, \boldsymbol{s}^z)$. If $i \in O$, then she also records b_i in $\mathbf{p}^C_{\mathcal{B}}$
- **Post Processing**: Alice performs final (classical) operations on the set of output qubits by calculating $\mathbf{p} = \mathbf{p}^C_{\mathcal{B}} \oplus \mathbf{s}^z_O$, where $\mathbf{s}^z_O$ is used to represent the final set of Z corrections .

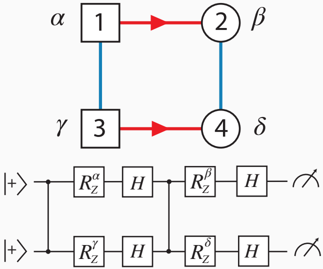
Example: 2×2 cluster state

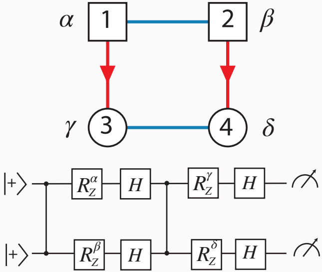
To demonstrate this we take a simple example of 2×2 cluster state $G(I, O)_{(2\times 2)}$. The figure shows 9 possible open graphs compatible with all the flow conditions.





Flow - circuit mapping





In general different flows correspond to different computations.

Security

- Description of Alice's computation - $\Delta_A^{\mathrm{MBQC}} := (G_{n\times m}, \boldsymbol{\alpha}, f)$
- Information Bob receives $:= (\boldsymbol{b'}, \boldsymbol{\alpha'})$

Aim (Formal)

$$H(A, F|B', A') = ?$$

We observe the following relation:

$$H(A, F|B', A') \geq H(F) - N + H(B', A'|A, F)$$

Therefore, our task reduces to calculate:

1. $H(B', A'|A, F)$
2. $H(F)$

1. Conditional entropy: $H(B', A'|A, F)$

Claim

$H(B', A'|A, F) \geq N$ regardless of Bob's strategy.

Outline of the proof:

- Construct full joint probability distribution for all the variables in the protocol - $\Pr(b', \alpha', \alpha, f, b, r)$
- Use dependencies between different variables to calculate the full joint probability.
- Marginalizing over B and using the joint probability distribution $\Pr(b', \alpha', \alpha, f, r)$ to compute $\Pr(b', \alpha'|\alpha, f)$

This in turn gives a lower bound on the conditional entropy

$$H(A, F|B', A') \geq H(F) = \log_2 N_F$$

Side result

$I(B', A'; A, F) \leq H(A')$

2. Flow counting argument: H_F

Theorem

$$\#\mathcal{G}(I,O)_{n,m} = F_{2\min(n,m)+1}^{|n-m|} \prod_{\mu=2}^{\min(n,m)} F_{2\mu}^2.$$

where F_i is the ith Fibonacci number.

Upon simplification: $\#\mathcal{G}(I,O)_{n,m} = 2^{2N\log_2\phi + O(N^\epsilon)}$ for $\epsilon < 1$, $N = nm$, and $m = \text{poly}(n)$

Combining $N_F \geq \#\mathcal{G}(I,O)_{n,m}$ with the previous result, we get $\log_2 N_F \geq \log_2 \#\mathcal{G}(I,O)_{n,m} \approx 1.388N$.

Final Result

$$H(A,F|B',A') \geq 1.388N$$

Conclusion and Future Directions

To sum up

- We explore the possibility of classically driven blind quantum computation.

- This is shown by observing that multiple non-equivalent computations in the MBQC model can yield the same transcript of measurement angles and results, even when the resource state and order of measurements are fixed.

- We also show that, in a single run of the protocol, the amount of information obtained by the server about client's computation is bounded.

Open problems

- Is it possible to exploit this novel cryptographic tool, flow ambiguity, to achieve *universal* classically driven blind quantum computation?

- More importantly, can such a technique be used as a building block for *verification* of quantum computers by completely *classical client* or in other words to prove if $\mathrm{BQP} = \mathrm{IP}_{BQP}$?

Thank you for your attention!

Paper reference: Phys. Rev. X 7, 031004 (2017).