Lorentz Invariant Entanglement Distribution for the Space-Based Quantum Network

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Quantum internet

One of the goals in quantum information science is the creation of a global spanning quantum network (Kimble *Nature* **453**, 1023 (2008))

Useful for potentially many tasks such as quantum teleportation, quantum distributed computing, quantum clock synchronization, quantum cryptography, quantum simulation, and super-dense coding

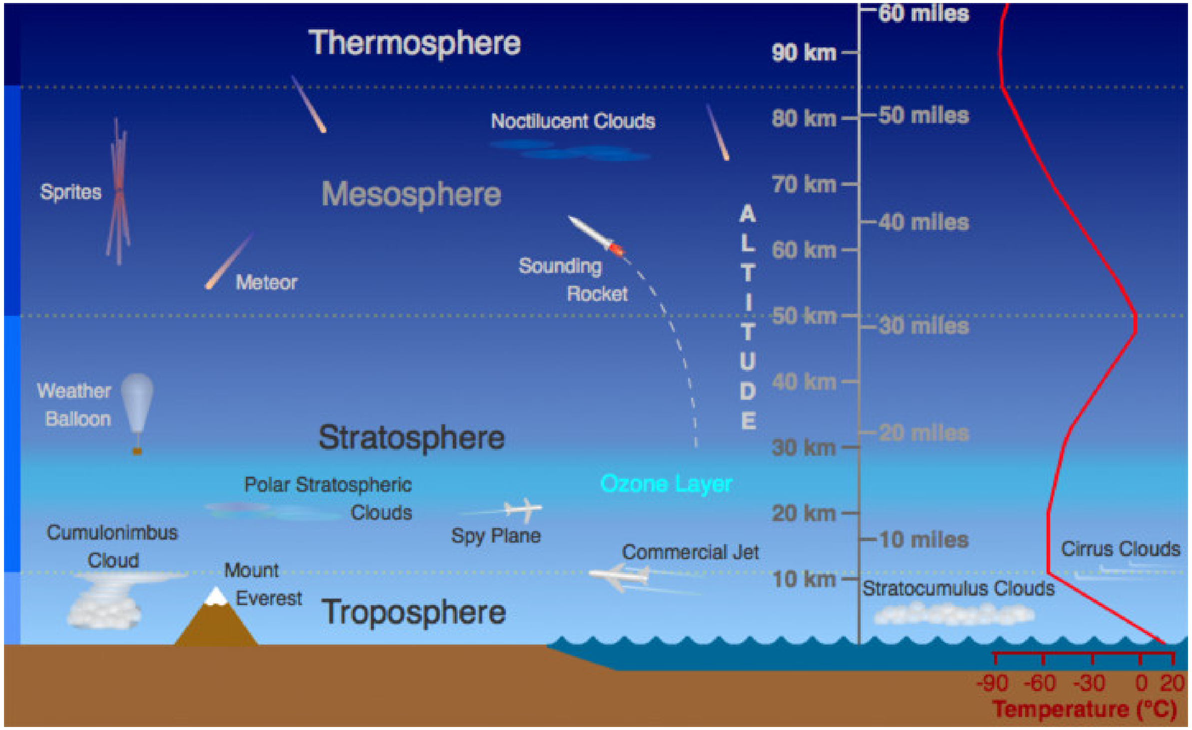
How to do entanglement distribution?

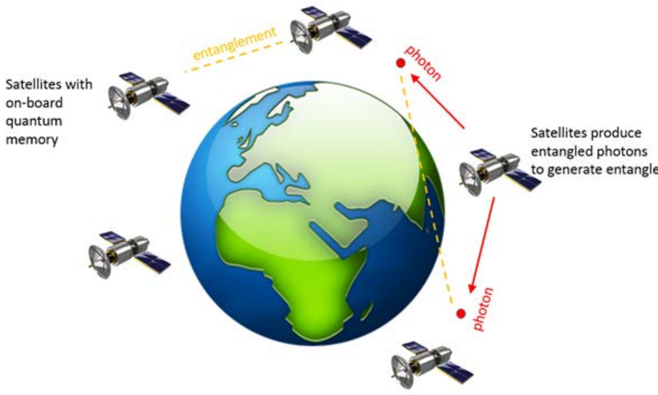
method	pros	Cons
Fiber optic cable	More reliable than open air	100 Km range Needs infrastructure,quantum repeaters
Free space (in air)	Minimal infrastructure needed	Limited by line of sight
Free Space (in space)	Ad hoc, point to point, anywhere on Earth	Quantum satellites are technically demanding

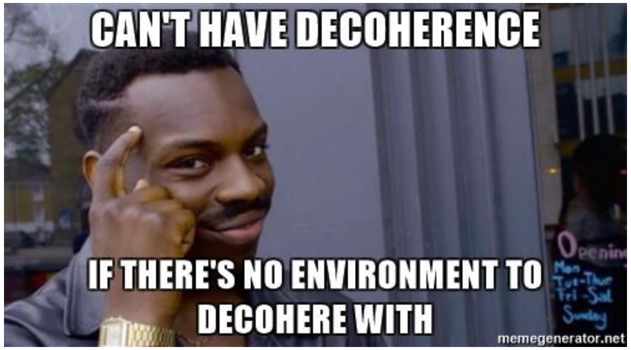
Satellite based entanglement distribution

While the technical challenges are formidable, space based entanglement distribution has some considerable advantages

- The Earth's atomosphere is quite thin (80% in the lower 12km)
- Once in space, photons can travel easily diameter of Earth and longer with no decoherence (Aspelmeyer IEEE Journal of Selected Topics in Quantum Electronics 9, 1541 (2003))
- Using a satellite network you can create ad hoc anywhere on Earth entanglement distribution







The future is now

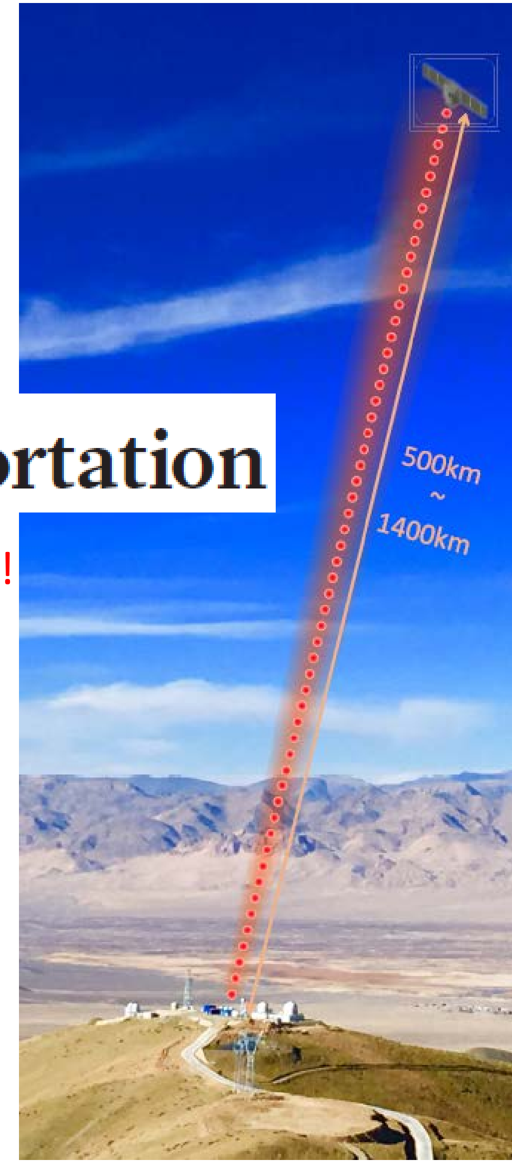
Satellite-based entanglement distribution over 1200 kilometers

Yin et al., Science 356, 1140 (2017)

Ground-to-satellite quantum teleportation

Ren et al., Nature (2017)

Teleportation up to 1400 km!!!



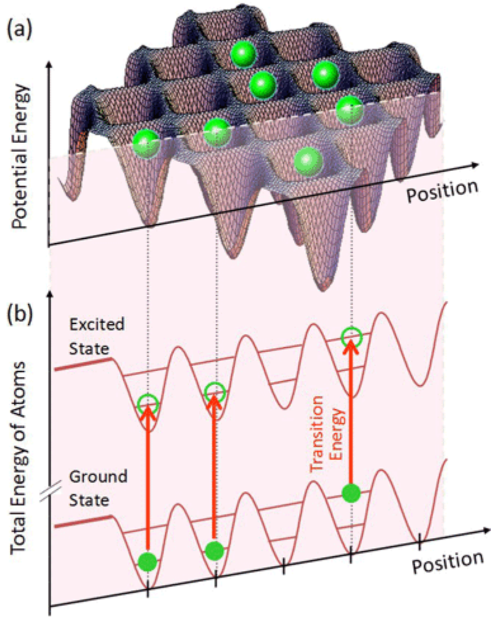
Main applications of space quantum network

If we try to anticipate the most important applications of a global quantum network, we can guess that they might be

- QKD
- Clock synchronization

Why? Because these are both already huge industries. Cryptography is very important in the internet age, and synchronizing clocks has a huge impact on fields such as

- data transfer networks
- financial trading
- airport traffic control
- rail transportation networks
- telecommunication networks
- the global positioning system (GPS)
- long baseline interferometry



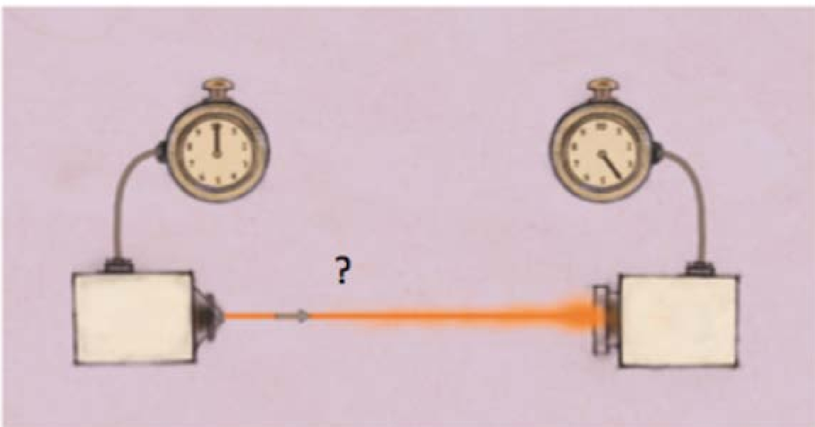
With optical atomic clocks reaching accuracies 10^-18, and If these require synchronization at this level, this may require quantum mechanics

Ludlow et al, RMP 87, 637 (2015)

Clock synchronization

Classically, including relativity, there are 2 ways to synchronize a clock:

Einstein clock synchronization



Eddington slow clock transport

In either case, either light or matter has to be exchanged between the two parties

Susceptible to interference along the way (i.e. noise, properties of the medium, etc.)

How to synchronize clocks using a quantum network?

Quantum clock synchronization

In quantum clock synchronization, the synchronization procedure only depends upon the initial creation of entanglement. During the synchronization protocol itself, only some timing-irrelevant information is exchanged:

Protocol

1) Alice and Bob initially prepare many entangled singlet pairs

$$|0\rangle|1\rangle - |1\rangle|0\rangle$$
$$= |+\rangle|-\rangle - |-\rangle|+\rangle$$

There is the always-on Hamiltonian $H = \omega(\sigma_1^z + \sigma_2^z)$

But this state has no time evolution under this state

2) Alice measures her qubits in the $|+\rangle / |-\rangle$ basis. Now the qubits start to evolve as

Alice: $|0\rangle \pm e^{i\omega t}|1\rangle$ Bob: $|0\rangle \mp e^{i\omega t}|1\rangle$

3) For all the cases that Alice obtained $|+\rangle$, Bob applies Z gate to phase flip spin. Now Bob has an ensemble of

$$|0\rangle + e^{i\omega t}|1\rangle$$

This is a clock that is synchronized with Alice's initial measurement!

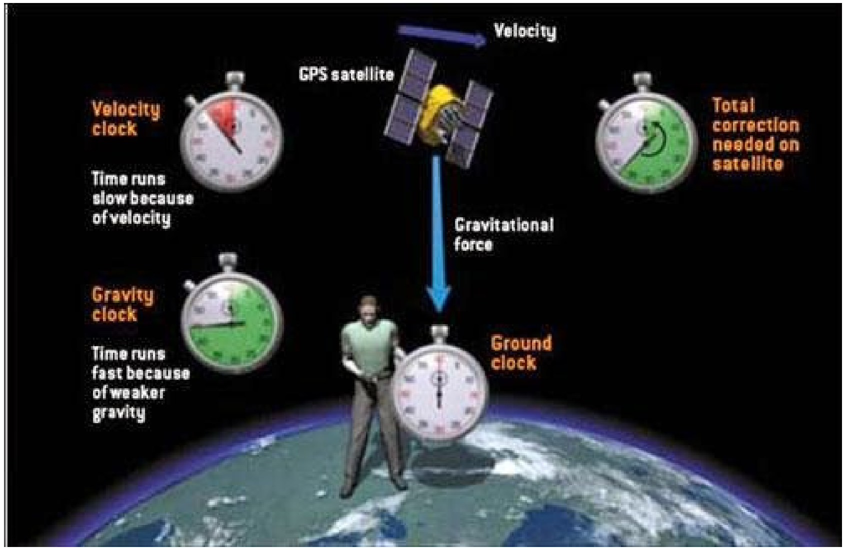
Error sources in quantum clock synchronization

If quantum clock synchronization is done using a space-based network, extremely high demands on the error will be necessary.

Current atomic clock synchronizations are already at 10^-14. GPS satellites are synchronized to atomic clocks to an error of 10ns.

Sources of error

- Gates fidelities in the protocol
- Fidelity of initial entangled state
  (entanglement purification)
- Relativistic effects in entanglement distribution

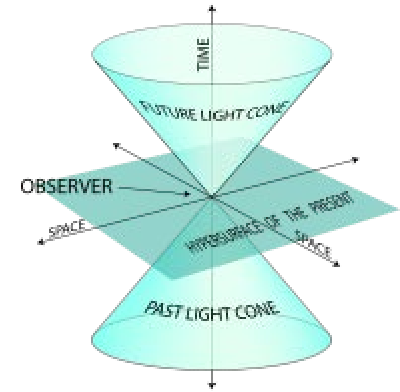


As GPS satellites today already need to take into account of relativistic effects, it is not surprising that relativistic effects might be important when comparing to atomic clock accuracies.

Satellites in LEO have $\beta = v/c = 10^{-5}$

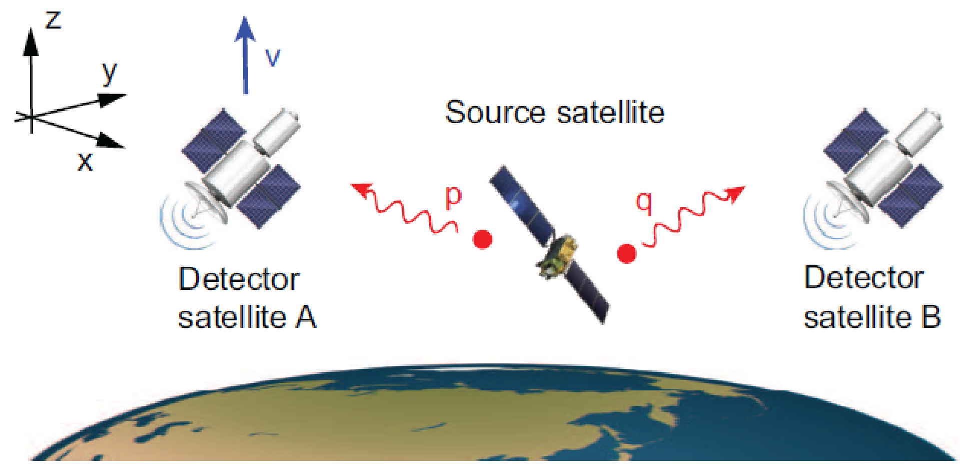
Outline for the rest of this talk

I) Effects of relativity on entanglement distribution
1) Single photon relativistic transformation
2) Different ways of distributing entanglement
3) Estimates of error for various states



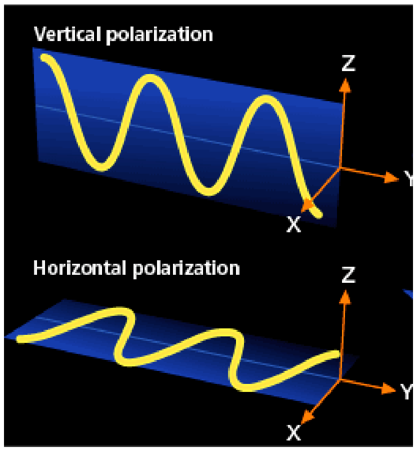
II) Entanglement purification for imperfect photon sources
1) Entanglement purification including vacuum states
2) Generalized entanglement purification criterion
3) Minimum fidelities

III) Conclusions



Single photon polarization

Polarization is NOT a relativistic invariant. It appears different according to different observers.



p

$$|p,v\rangle = R(\hat{p})(0,\sin\phi,\cos\phi,0)^T$$

$$= (0,(\cos\theta - 1)\sin 2\phi/2, \cos\theta\sin^2\phi + \cos^2\phi, -\sin\theta\sin\phi)$$

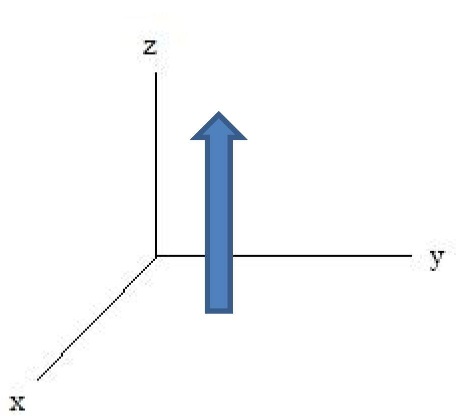
p

$$|p,h\rangle = R(\hat{p})(0,\cos\phi,-\sin\phi,0)^T$$

$$= (0,\cos\theta\cos^2\phi + \sin^2\phi, (\cos\theta - 1)\sin 2\phi/2, -\sin\theta\cos\phi)$$

Standard SO(3) rotations

$$R(\hat{p}) = R_z(\phi)R_y(\theta)$$



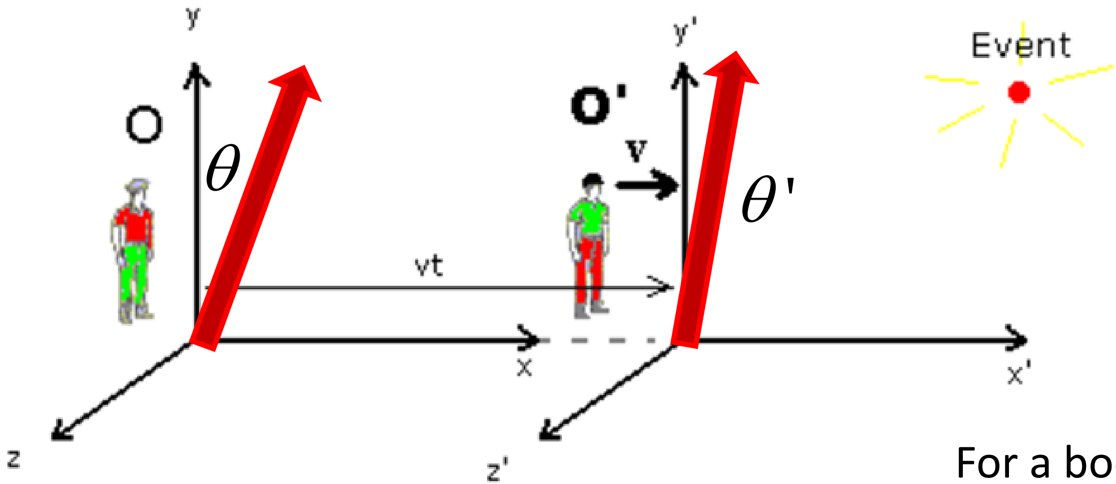
i.e. for a photon in the z-direction $\theta = 0$

$$|p,v\rangle = (0,0,1,0)$$

$$|p,h\rangle = (0,1,0,0)$$

Relativistic transformation

Transformation from Alice's from to Bob:



For a boost in the z-direction:

$$\sin\theta \to \sin\theta' = \frac{\sin\theta}{\sqrt{\sin^2\theta + \gamma^2(\cos\theta - \beta)^2}}$$

$$\phi \to \phi' = \phi.$$

$$U(\Lambda)|p,\lambda\rangle = e^{-i\lambda\Theta(\Lambda,p)}|\Lambda p,\lambda\rangle$$

Λ is the Lorentz transform
Θ is the Wigner phase

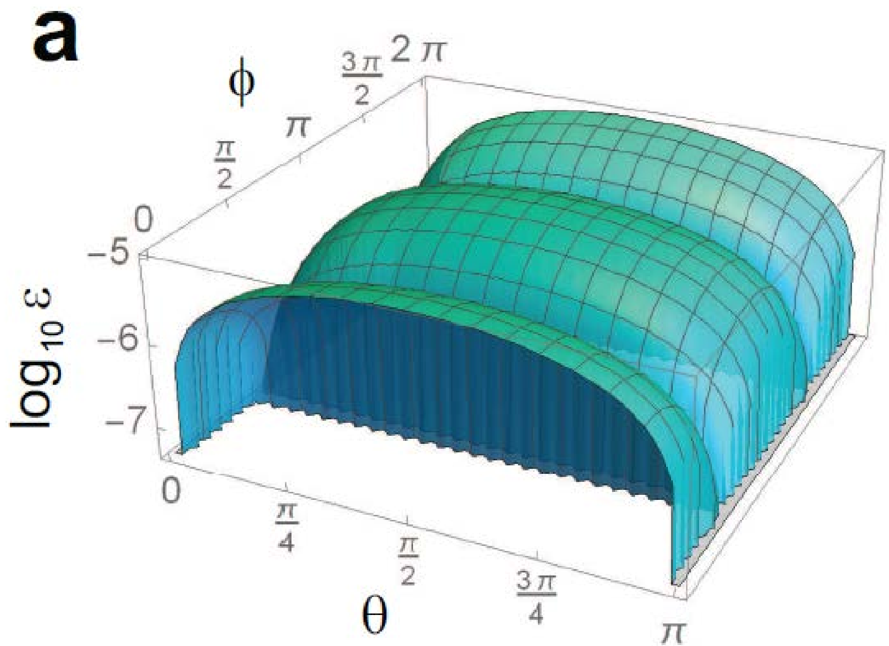
Or approximately $\theta' \approx \pi\left(\frac{\theta}{\pi}\right)^{1-\frac{2}{\pi\ln 2}\beta}$

Basic effect: compression/expansion of θ

Error for polarized photons

Evaluate trace distance of original horizontally polarized photon with boost

$$\varepsilon = \mathrm{Tr}(\sqrt{(\rho^{(S)} - \rho^{(A)})^2})/2$$



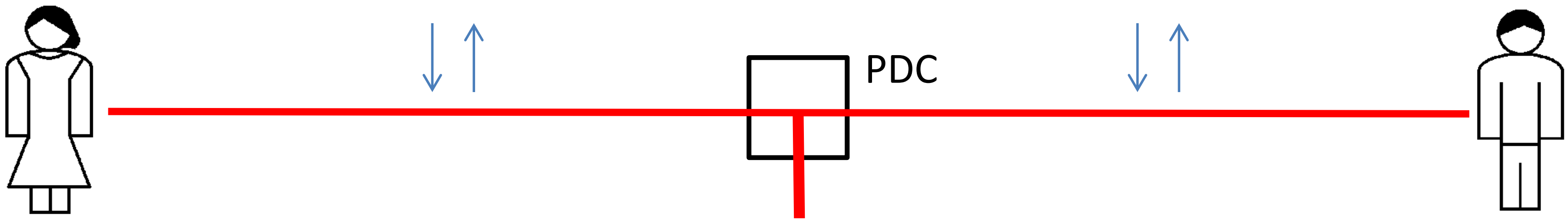
$$\varepsilon_h \approx \beta \sin\theta \cos\phi$$

Errors on order of $\beta = v / c = 10^{-5}$

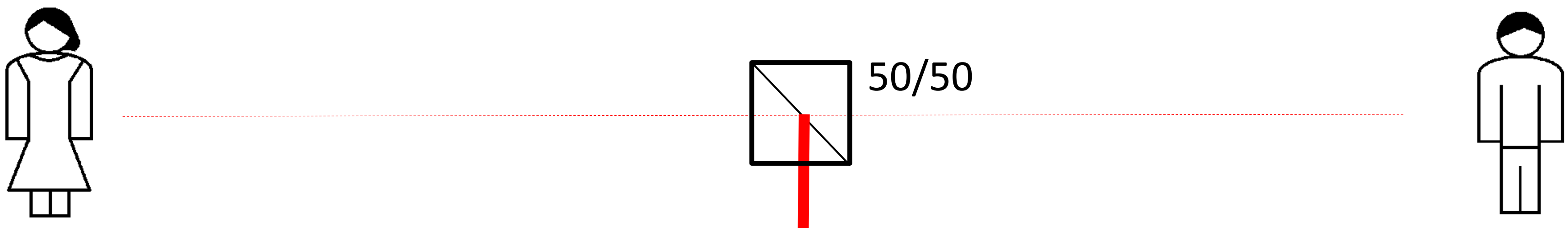
Small, but significant in comparison to clock accuracies

Entangled states with photons

Type I: Polarization encoding
$$|\Psi_{\rm I}^{(S)}\rangle = \frac{1}{\sqrt{2}}\left(|\boldsymbol{p},h\rangle_A|\boldsymbol{q},h\rangle_B - |\boldsymbol{p},v\rangle_A|\boldsymbol{q},v\rangle_B\right),$$

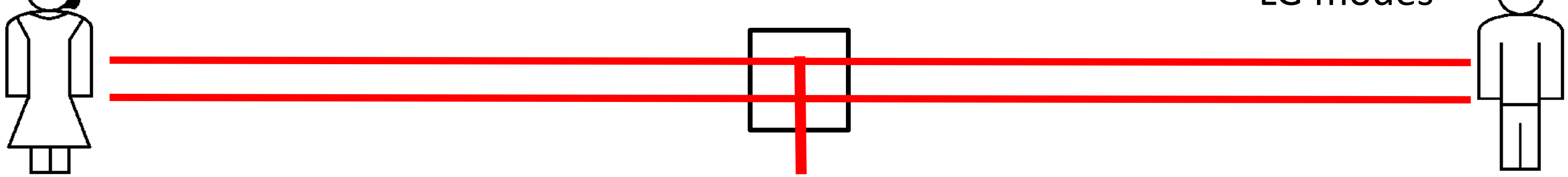


Type II: Single photon
$$|\Psi_{\rm II}^{(S)}\rangle = \frac{1}{\sqrt{2}}(|\boldsymbol{p},\lambda\rangle_A|0\rangle_B - |0\rangle_A|\boldsymbol{q},\lambda\rangle_B).$$



Type III: Dual rail
$$|\Psi_{\rm III}^{(S)}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_{A1}|\boldsymbol{p},\lambda\rangle_{A2}|0\rangle_{B1}|\boldsymbol{q},\lambda\rangle_{B2} - |\boldsymbol{p},\lambda\rangle_{A1}|0\rangle_{A2}|\boldsymbol{q},\lambda\rangle_{B1}|0\rangle_{B2}).$$

This could be literally two spatial modes, or LG modes

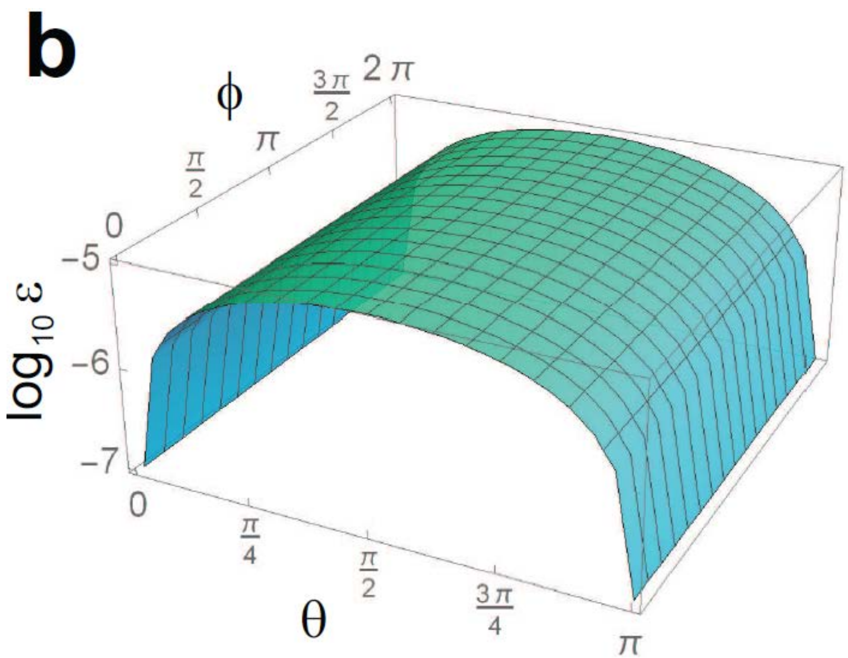


Type I: Lorentz transform induced error

Since polarization isn't a Lorentz invariant, the polarization entangled will be susceptible to relativistic effects

Again evaluate

$$\varepsilon = \mathrm{Tr}(\sqrt{(\rho^{(S)} - \rho^{(A)})^2})/2$$



$$\varepsilon_{\mathrm{I}} \approx \beta \sin\theta$$

Again, errors of order

$$\beta = v / c = 10^{-5}$$

Type II and III: Lorentz transform induced errors

Transforming the single photon and dual rail entangled states

$$|\Psi_{\mathrm{II}}^{(\Lambda)}\rangle = \frac{1}{\sqrt{2}}(e^{-i\lambda\Theta(\Lambda,\boldsymbol{p})}|-\Lambda\boldsymbol{p},\lambda\rangle_A|0\rangle_B - e^{-i\lambda\Theta(\Lambda,\boldsymbol{q})}|0\rangle_A|\Lambda\boldsymbol{q},\lambda\rangle_B).$$

$$\begin{aligned}|\Psi_{\mathrm{III}}^{(\Lambda)}\rangle = & e^{-i\lambda(\Theta(\Lambda,\boldsymbol{p})+\Theta(\Lambda,\boldsymbol{q}))}(|0\rangle_{A1}|\mathbf{p},\lambda\rangle_{A2}|0\rangle_{B1}|\mathbf{q},\lambda\rangle_{B2} \\ & -|\mathbf{p},\lambda\rangle_{A1}|0\rangle_{A2}|\mathbf{q},\lambda\rangle_{B1}|0\rangle_{B2}), \qquad (13)\end{aligned}$$

Since the encoding here is in terms of photon **number**, and not polarization, the amount of entanglement is automatically Lorentz invariant. (Avron et al., Eur. J. Phys 20, 153 (1999))

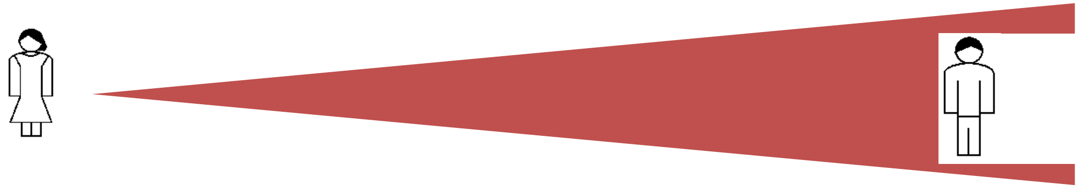
For the single photon case, the Wigner phase is different, hence there can be a phase error which does not affect the entanglement, but can be an error on QCS

For dual rail case, the state is preserved. But the fidelity will be affected by the integrity of the dual rail modes under a Lorentz boost.

Diffraction effects

When a photon is transmitted in space it will diffract

Alice



Bob

This affects the entanglement under Lorentz transforms because it affects the tightness of the cone

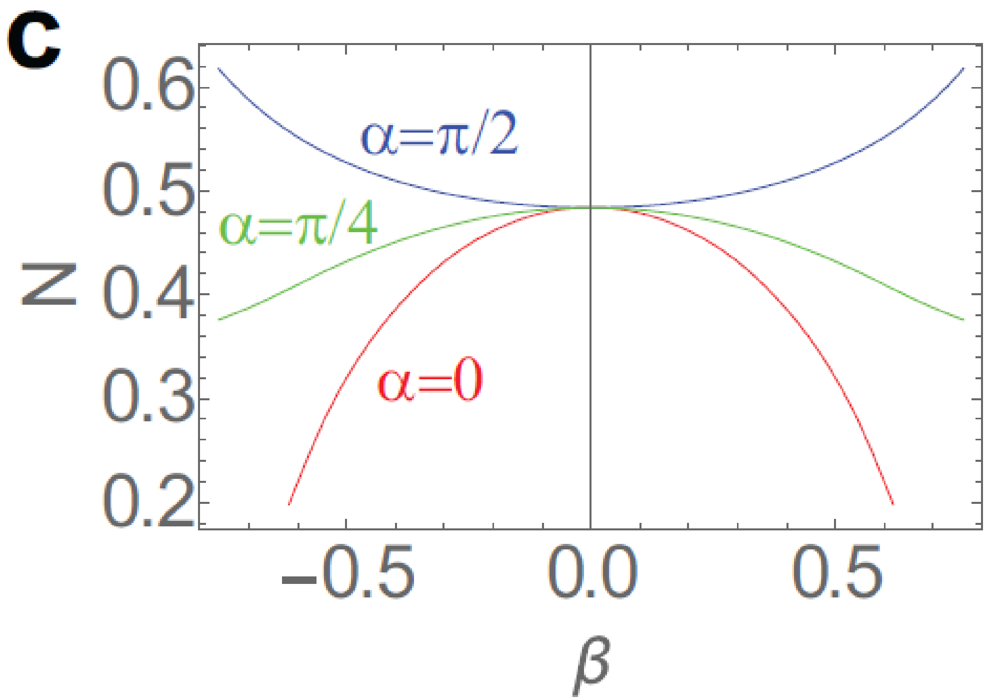
$$f(p) = \frac{1}{\sqrt{M}} e^{-\frac{\theta^2}{2\sigma^2}} \delta(|p| - p_0)$$

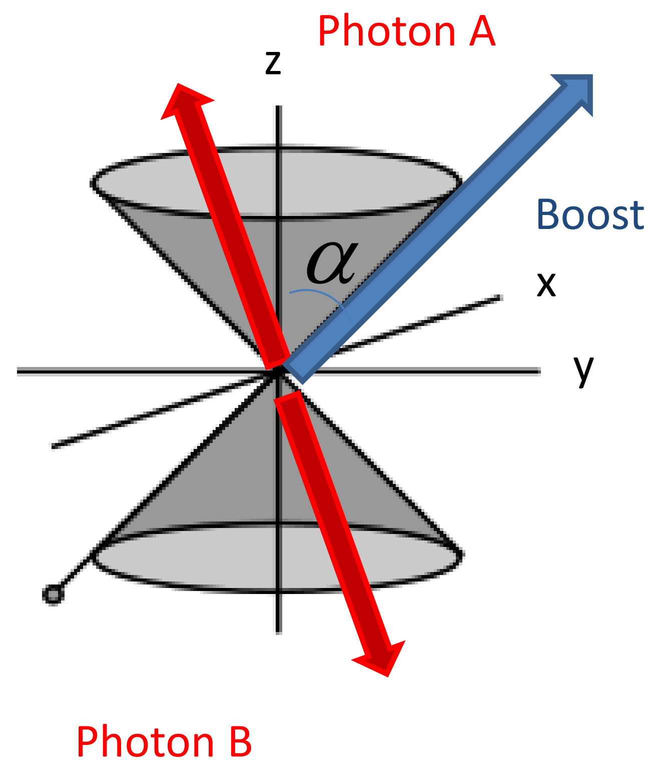
$$|\tilde{\Psi}\rangle = \int \tilde{d}\boldsymbol{p}\tilde{d}\boldsymbol{q} f_A(\boldsymbol{p}) f_B(\boldsymbol{q}) |\Psi(\boldsymbol{p},\boldsymbol{q})\rangle$$

$|\Psi(\boldsymbol{p},\boldsymbol{q})\rangle$ =type I, II, III states

Effect on entanglement

For two photons propagating in opposite directions mainly in the z-direction, including diffraction effects, calculate the entanglement (using negativity N)

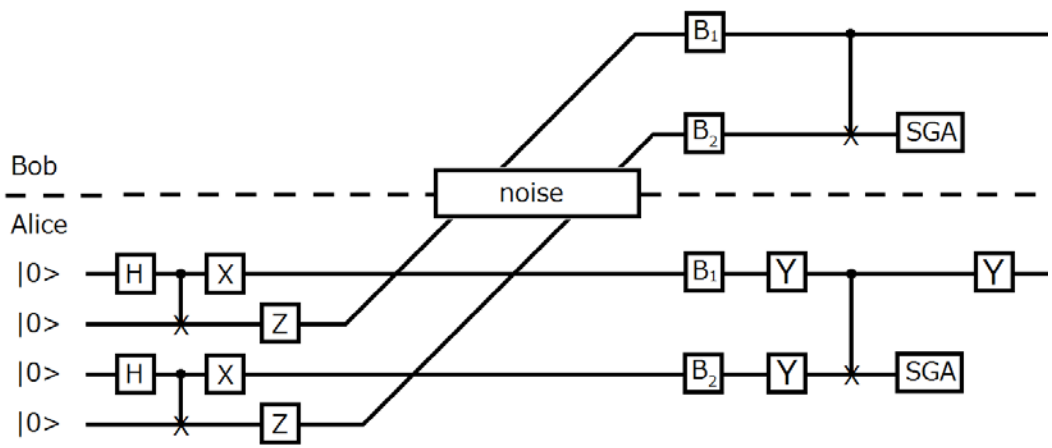




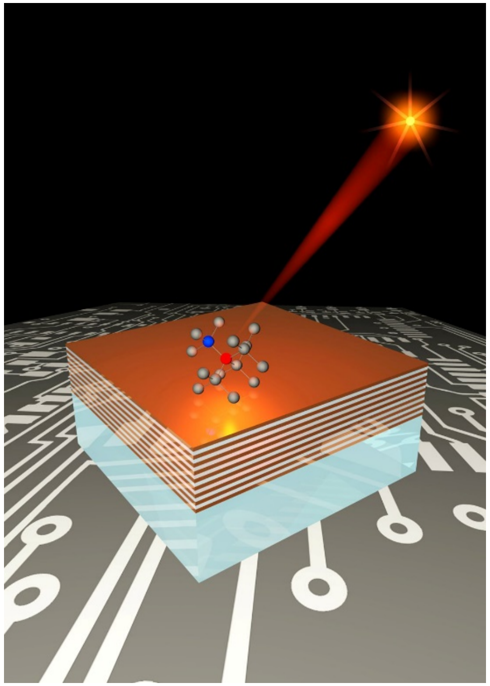
For a=0, one of the cones is tightened, but the other is broadened -> loss of entanglement
For a=pi/2 there is an overall tightening of the cones -> increased entanglement

Purification of entanglement from imperfect sources

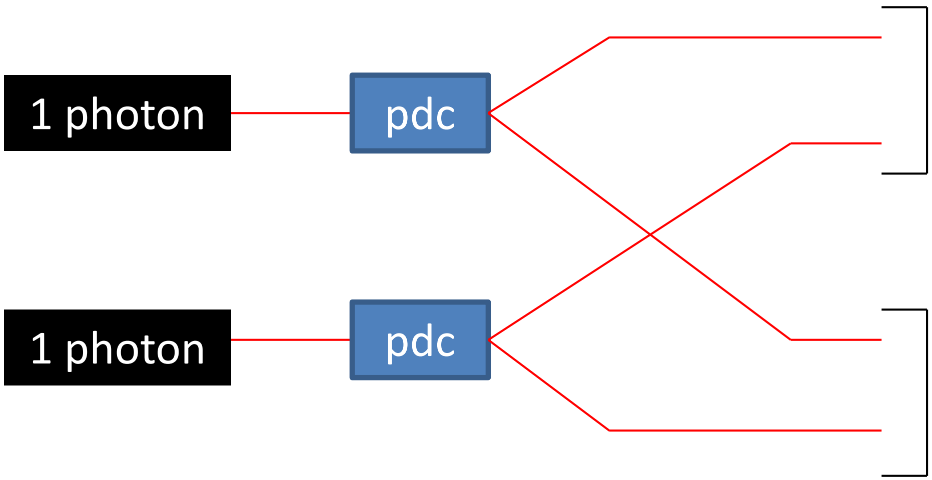
- Photon sources are not perfect. Sometimes they do not fire, sometimes the photon are lost.
- Standard purification assumes that entangled pairs always arrive.
- What if sometimes the photons are not there and they are vacuum?



https://doi.org/10.1103/PhysRevA.53.2046



Purification of coherent vacuum states



$$|\psi_1> = \epsilon|00> \ +\gamma(\alpha|HH> \ +\beta|VV>)$$

$$|\psi_2> = \epsilon|00> \ +\gamma(\alpha|HH> \ +\beta|VV>)$$

Suppose we replace the first few steps of the purification circuit with this LOQC setup.
Now suppose our photon source is imperfect and produces the state $\epsilon|0> \ + \gamma|1>$
The resultant density matrix would be given by $\rho = \ |\ \psi_1 > |\psi_2 > < \psi_2| < \psi_1|$
This is different than what you would get from plain old photon loss.

(not going to show the matrices explicitly because they are big and tedious)

Purification of coherent vacuum states (how to)

1) Include the vacuum state as a 3rd (placeholder) logical value.
2) Derive the action of single qubit and Cz gates on the vacuum
3) Generalize Werner states to include the 3rd logical value
4) Run the purification circuit on the generalized werner state

CNOT $\lvert a^x\rangle\lvert b^y\rangle$	$\lvert\psi^x\rangle$	$\lvert\phi^x\rangle$	$\lvert B_0^x\rangle$	$\lvert B_1^x\rangle$	$\lvert B_2\rangle$
$\lvert\psi^y\rangle$	$\lvert\psi^{xy}\rangle\lvert\phi^y\rangle$	$\lvert\phi^{xy}\rangle\lvert\psi^y\rangle$			$\lvert B_2\rangle\lvert\psi^y\rangle$
$\lvert\phi^y\rangle$	$\lvert\psi^{xy}\rangle\lvert\psi^y\rangle$	$\lvert\phi^{xy}\rangle\lvert\phi^y\rangle$			$\lvert B_2\rangle\lvert\phi^y\rangle$
$\lvert B_0^y\rangle$	$\lvert\psi^{xy}\rangle\lvert B_0^y\rangle$	$\lvert\phi^{xy}\rangle\lvert B_0^y\rangle$	$\lvert B_0^{xy}\rangle\lvert B_0^y\rangle$	$\lvert B_1^x\rangle\lvert B_0^y\rangle$	$\lvert B_2\rangle\lvert B_0^y\rangle$
$\lvert B_1^y\rangle$	$y\lvert\psi^{xy}\rangle\lvert B_1^y\rangle$	$\lvert\phi^{xy}\rangle\lvert B_1^y\rangle$	$\lvert B_0^x\rangle\lvert B_1^y\rangle$	$\lvert B_1^{xy}\rangle\lvert B_1^y\rangle$	$\lvert B_2\rangle\lvert B_1^y\rangle$
$\lvert B_2\rangle$	$\lvert\psi^x\rangle\lvert B_2\rangle$	$\lvert\phi^x\rangle\lvert B_2\rangle$	$\lvert B_0^x\rangle\lvert B_2\rangle$	$\lvert B_1^x\rangle\lvert B_2\rangle$	$\lvert B_2\rangle\lvert B_2\rangle$

TABLE I. CNOT on original basis states

CNOT $\lvert a^x\rangle\lvert b^y\rangle$	$\lvert 0\rangle\lvert 2\rangle$	$\lvert 2\rangle\lvert 0\rangle$	$\lvert 1\rangle\lvert 2\rangle$	$\lvert 2\rangle\lvert 1\rangle$
$\lvert\psi^y\rangle$	$\lvert 0\rangle\lvert 2\rangle\lvert\psi^y\rangle$	$\lvert 2\rangle\lvert 0\rangle\lvert\psi^y\rangle$	$y\lvert 1\rangle\lvert 2\rangle\lvert\phi^y\rangle$	$\lvert 2\rangle\lvert 1\rangle\lvert\phi^y\rangle$
$\lvert\phi^y\rangle$	$\lvert 0\rangle\lvert 2\rangle\lvert\phi^y\rangle$	$\lvert 2\rangle\lvert 0\rangle\lvert\phi^y\rangle$	$y\lvert 1\rangle\lvert 2\rangle\lvert\psi^y\rangle$	$\lvert 2\rangle\lvert 1\rangle\lvert\psi^y\rangle$

TABLE II. CNOT on slightly different basis states

input state	result
$Y\lvert\psi^\pm\rangle$	$\lvert\phi^\mp\rangle$
$Y\lvert\phi^\pm\rangle$	$\lvert\psi^\mp\rangle$
$Y\lvert B_2\rangle$	$\lvert B_2\rangle$

$$\lvert B_0^\pm\rangle = \lvert 0\rangle\lvert 2\rangle \pm \lvert 1\rangle\lvert 2\rangle$$
$$\lvert B_1^\pm\rangle = \lvert 2\rangle\lvert 0\rangle \pm \lvert 2\rangle\lvert 1\rangle$$
$$\lvert B_2\rangle = \lvert 2\rangle\lvert 2\rangle$$

$$W = f\psi^- + \frac{1-f-\epsilon}{3}(\psi^+ + \phi^- + \phi^+) + \epsilon_0(B_0^+ + B_0^-) + \epsilon_1(B_1^+ + B_1^-) + \epsilon_2 B_2$$
$$\epsilon = \epsilon_0 + \epsilon_1 + \epsilon_2$$

Purification of coherent vacuum states (conclusions)

- When these states are run through the purification circuit, a multi-dimensional recurrence relation is produced
- This can be numerically solved to find the fidelity needed for purification
- The correction is slight
  - At worst f>=.65 compared to the usual f>=.5

$$f' = \frac{f^2 + x^2}{r}$$

$$\epsilon_0' = \frac{\epsilon_0(1-\epsilon)}{2r}$$

$$\epsilon_1' = \frac{\epsilon_1(1-\epsilon)}{2r}$$

$$\epsilon_2' = \frac{\epsilon_2(f+x)}{r}$$

$$x = \frac{1-f-\epsilon}{3}$$

$$r = x(\epsilon_0 + \epsilon_1 - \epsilon_2 + 2 - x) + f(f+\epsilon)$$

Conclusions

- Polarization states are sufficiently degraded by relativistic effects to not be useful for clock synchronization or other time sensitive tasks. This is true even if you know the satellite velocity (up to an error around 10^-5)
- Dual rail and single photon states are good for these applications because photon number is Lorentz invariant.
- In principle a large amount of diffraction could cause the dual rails to overlap. This problem can be remedied with Gauss Laguerre modes
- For more details see

Byrnes, Ilyas, Tessler, Takeoka, Jambulingam, and Jonathan P. Dowling arXiv: 1704.04774

Thank you for your time
Don't forget to like and subscribe and be sure to smash that like button.

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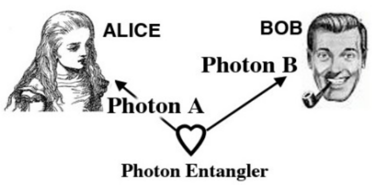

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