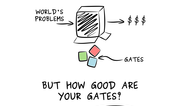
SEQUENCE REUSE IN RANDOMIZED BENCHMARKING

CHRIS GRANADE AND CHRIS FERRIE

SO YOU WANT TO BUILD A QUANTUM COMPUTER?



BUT HOW GOOD ARE YOUR GATES?

WHAT DOES RB DO?

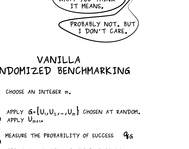
1. EFFICIENT METHOD TO ESTIMATE AVERAGE GATE FIDELITY.
2. ISOLATES GATE ERRORS FROM SPAM.
3. USED BY EXPERIMENTAL GROUPS EVERYWHERE TO PUBLISH IN NATURE REPORT NOTES-DIAGNOSE ERRORS

STATE PREPARATION AND MEASUREMENT

VANILLA RANDOMIZED BENCHMARKING

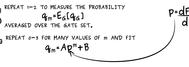
0. CHOOSE AN INTEGER m.
1. APPLY $G=\{U_0,U_1,\cdots,U_n\}$ CHOSEN AT RANDOM. APPLY U_{m+1}.
2. MEASURE THE PROBABILITY OF SUCCESS q_m
3. REPEAT $i=1$ TO MEASURE THE PROBABILITY $q_m=E(q_m)$ AVERAGED OVER THE GATE SET.
4. REPEAT $d=3$ FOR MANY VALUES OF m AND FIT $q_m=Ap^m+B$

$p=\frac{d-1}{d}$



HOW DOES RB WORK?

1. DO SOME RANDOM SHIT. TRY TO UNDO IT.
2. MEASURE THE SUCCESS RATE.
3. USE A THEOREM THAT SAYS THE SUCCESS RATE DECREASES EXPONENTIALLY WITH THE AMOUNT OF SHIT YOU DID.



NO NOISE ⟶ ALWAYS SUCCEED

NOISE ⟶ SOMETIMES FAIL

STAND BACK! I'M GOING TO DO STATISTICS!

IS IT CONSISTENT?

$$E(\hat{q}_i)=\frac{1}{N}\sum_{j=1}^{N}E(n_j)$$

$$E(n_j)=E(E(n_j|q_j))$$ (LAW OF TOTAL EXPECTATION)

$$=E(Kq_j)$$ (EACH i IS A BINOMIAL RANDOM VARIABLE)

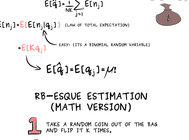
$$E(\hat{q}_i)=E(q_j)=\mu$$

RB-ESQUE ESTIMATION (MATH VERSION)

1. TAKE A RANDOM COIN OUT OF THE BAG AND FLIP IT K TIMES.
2. IF n HEADS ARE OBSERVED, ESTIMATE ITS BIAS TO BE $\hat{q}_j=\frac{n}{K}$
3. REPEAT $i=1$ TO N TIMES AND ESTIMATE THE AVERAGE BIAS TO BE

$$\hat{\mu}=\frac{1}{N}\sum_{j=1}^{N}n_j.$$

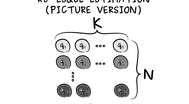
RB-ESQUE ESTIMATION (PICTURE VERSION)



BAG OF COINS ANALOGY

WHAT IS THE AVERAGE BIAS OF THE COINS?

OOPS.. SOMEONE FORGOT ABOUT ERROR BARS



IS IT CONSISTENT?

$$Var(\hat{q}_i)=\frac{1}{N}\sum_{j=1}^{N}Var(n_j)$$

$$Var(n_j)=Var(E(n_j|q_j))+E(Var(n_j|q_j))$$ (LAW OF TOTAL VARIANCE)

$$=Var(Kq_j)+E(Kq_j(1-q_j))$$

$$Var(\hat{q}_i)=\frac{1}{NK}(K^2Var(q_j)+E(q_j(1-q_j)))$$

MUST GO DEEPER..

$$Pr(n_j)=\sum_{q_j}Pr(n_j|q_j)Pr(q_j)$$ (LAW OF TOTAL PROBABILITY)

$$=\frac{1}{|B|}\sum_{q_j}\binom{K}{n_j}q_j^{n_j}(1-q_j)^{K-n_j}$$

DEPENDS ON ALL MOMENTS UP TO K!

LIKELIHOOD PRINCIPLE

PROBABILITIES SHOULD ALWAYS LOOK LIKE THIS:

$$Pr(n_j|\,)$$

LIKELIHOOD PRINCIPLE: DATA ONLY CONTAINS INFORMATION ABOUT $\,|\,$ HENCE, CAN LEARN ABOUT $\,|\,$ AND ONLY $\,|\,$.

$$Pr(\,|n_j)$$

DOES NOT MAKE SENSE!

(MAYBE)

QUICK FIX IS BEST FIX

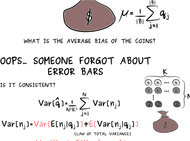
$K=1$

$$Pr(n_j=1)=\frac{1}{|B|}\sum_{q_j}q_j=\mu$$

$$Var(\hat{q}_i)=\frac{1}{N}(Var(q_j)+E(q_j(1-q_j)))$$

LESSON: DON'T FLIP THE SAME COIN MORE THAN ONCE, OR DON'T REPEAT SEQUENCES IN RB!

WHAT TO DO IF K>2



BONUS ROUND!

WE ESTIMATE (JUST AS EASILY!) THE UNITARITY WITH THE SAME DATA.

1. $q_m=E_0(q_m)=Ap^m+B$
   $E_0(q_m)=Cy^{m-1}+D$

"UNITARITY" (1 IF COHERENT, <1 IF NOT)

2. WE CALCULATE THE OPTIMAL VALUE OF K (NUMBER OF REUSES).

3. WIP: WE SHOW THAT BETA IS A GOOD MODEL FOR MANY COMMON ERROR CHANNELS.