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Analog quantum error correction with encoding a qubit into an oscillator

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arXiv : 1706.03011

Outline

I Toward a large-scale quantum computation

- Continuous variable QC
- GKP qubit
- Our work

II Analog quantum error correction

- Proposal - likelihood function -
- Error model in our work
- Three qubit bit flip code
- C4/C6 code
- Surface code
- Summary

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Toward large-scale quantum computation

- Large-scale QC requires large-scale entangled states

Trapped ions	14 qubits entangled [1]
Superconducting	10 qubits entangled [2]

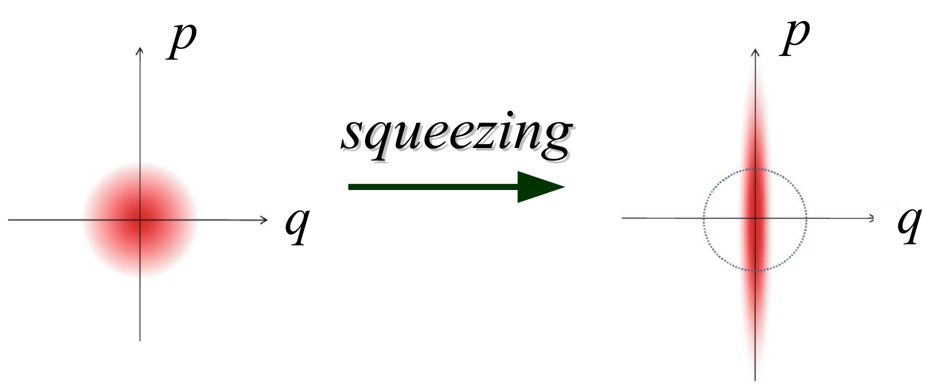
[1] T. Monz *et al.*, Phys. Rev. Lett. **106**, 130506 (2011)
[2] C. Song *et al.*, arXiv:1703.10302 (2017)
[3] J. Yoshikawa *et al.*, APLPhotonics **1**, 060801 (2016)

Toward large-scale quantum computation

- Large-scale QC requires large-scale entangled states

Trapped ions	14 qubits entangled [1]
Superconducting	10 qubits entangled [2]
Squeezed vacuum state in optical field	1,000,000 qumodes entangled [3]

Vacuum state　　　Squeezed vacuum state



Increase of the squeezing level improves measurement accuracy in q quadrature

q (p) : real (imaginary) part of optical field amplitude

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Continuous variable quantum computation

- Large-scale QC with only squeezed vacuum (SV) states is impossible because of accumulation of errors [4]
- Continuous variable (CV) state needs to be digitized using an appropriate code, such as the GKP qubit

[4] N. C. Menicucci, Phys. Rev. Lett. **112**, 120504 (2014)
[5] D. Gottesman *et al.*, Phys. Rev. A, **64**, 012310 (2001)

Continuous variable quantum computation

- Large-scale QC with only squeezed vacuum (SV) states is impossible because of accumulation of errors [4]
- Continuous variable (CV) state needs to be digitized using an appropriate code, such as the GKP qubit

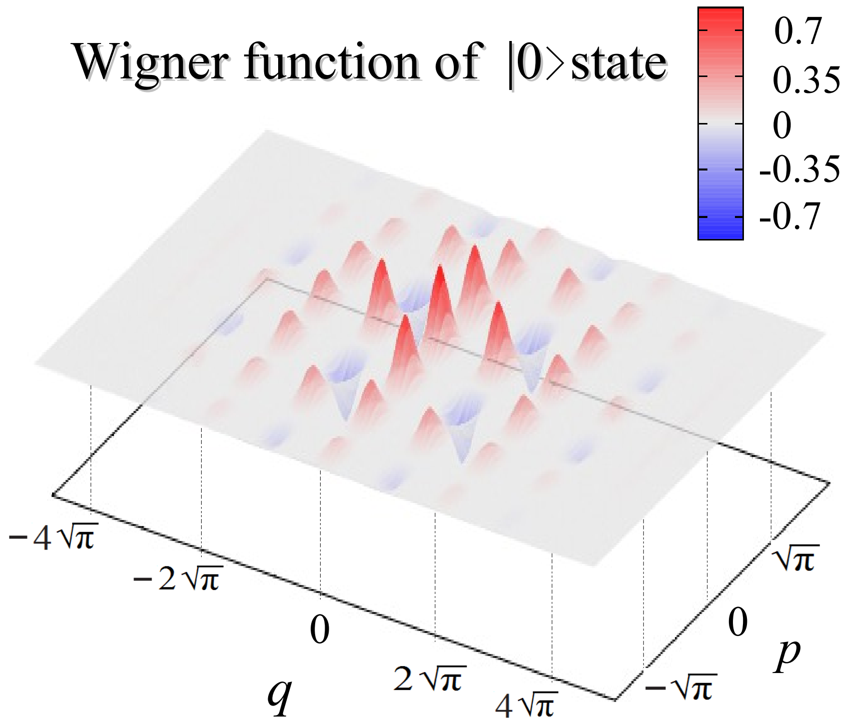
GKP qubit [5]

$$|0\rangle \propto \sum_{t=-\infty}^{\infty} \int \mathrm{e}^{-2\pi\delta^2 t^2} \mathrm{e}^{-(q-2t\sqrt{\pi})^2/(2\delta^2)} |q\rangle \, dq$$

$$|1\rangle \propto \sum_{t=-\infty}^{\infty} \int \mathrm{e}^{-\pi\delta^2(2t+1)^2/2} \mathrm{e}^{-(q-(2t+1)\sqrt{\pi})^2/(2\delta^2)} |q\rangle \, dq$$

δ^2 : variance of the GKP qubit

Wigner function of |0> state



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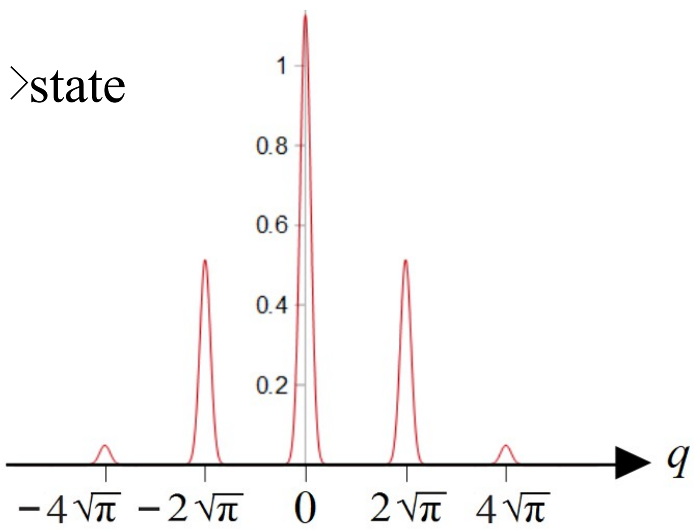
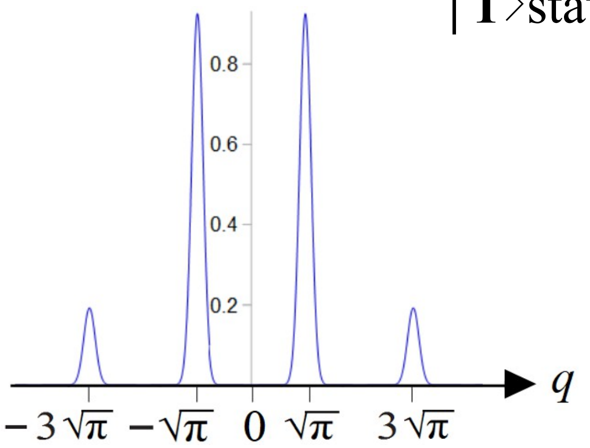
Gottesman-Kitaev-Preskill (GKP) qubit

Probability distribution

$|\mathbf{0}\rangle$state



$|\mathbf{1}\rangle$state



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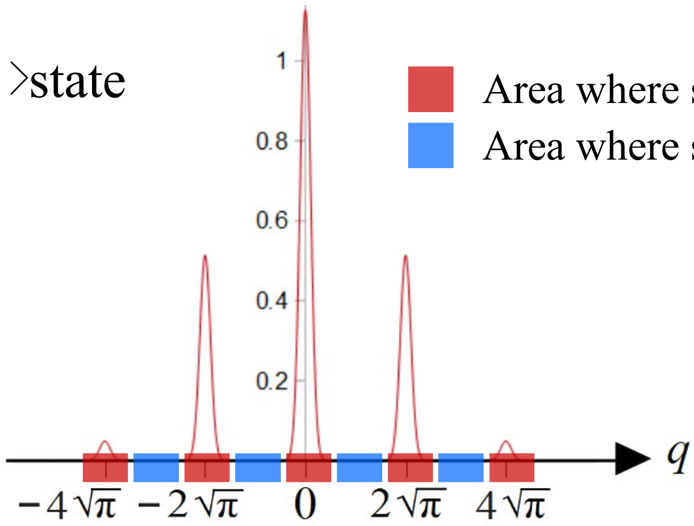
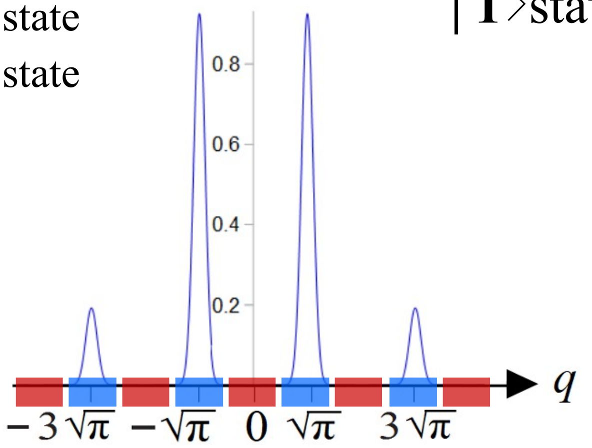


Area where state is identified as 0 state

Area where state is identified as 1 state

| 1⟩state



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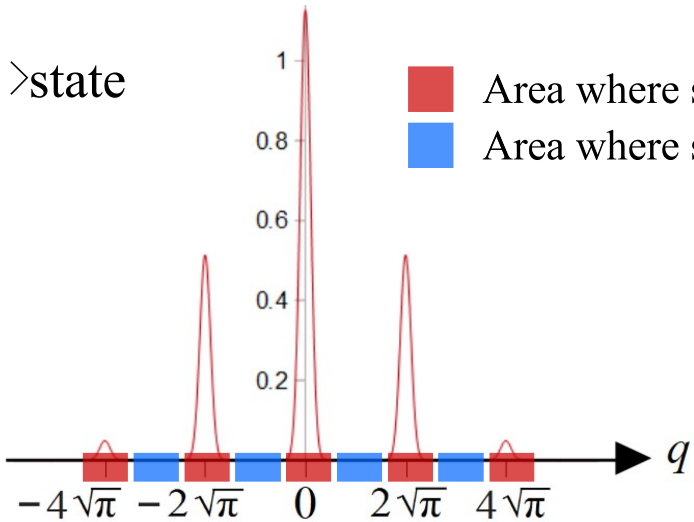

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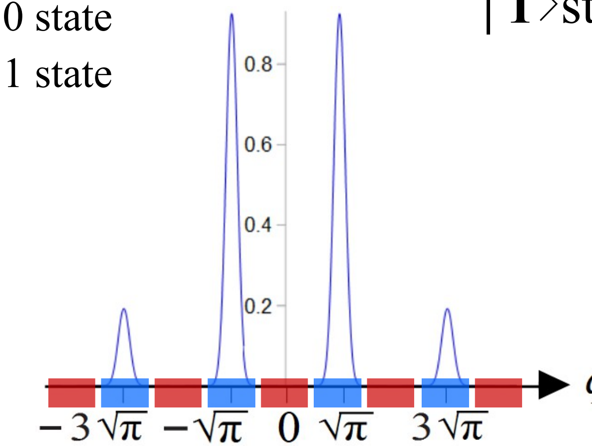
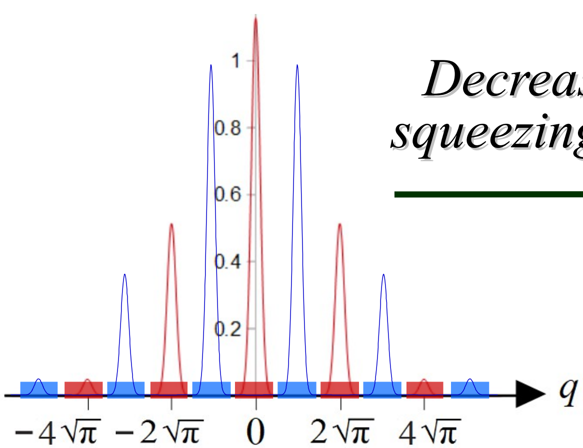
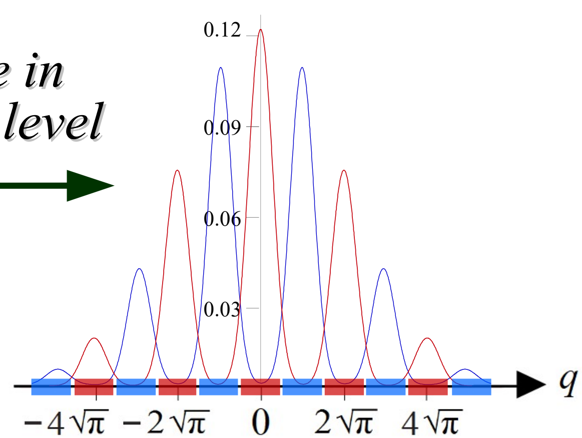


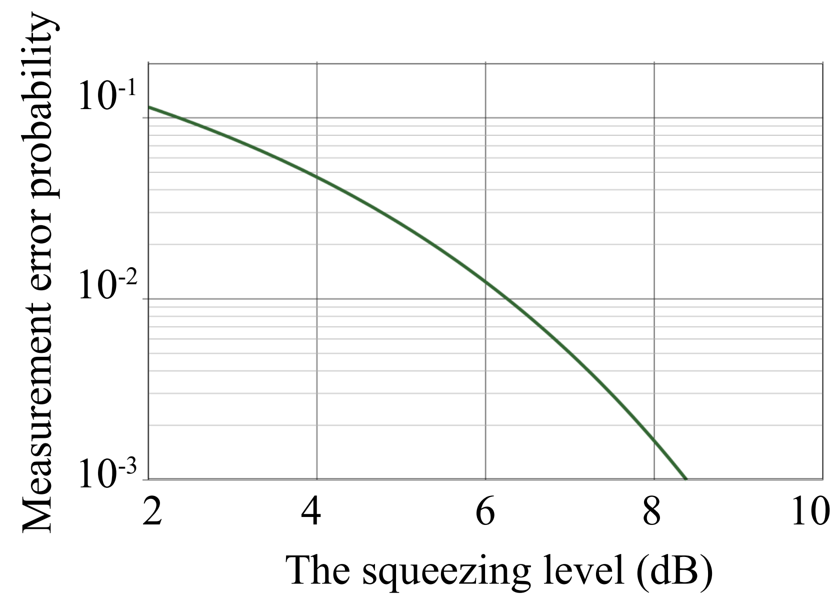
Measurement error

The more the squeezing level decreases,
the larger measurement error probability becomes



Decrease in squeezing level





Toward large-scale QC with the GKP qubit

Advantage

- Large-scale QC with the GKP qubits is possible
- GKP qubits can be entangled in the same way as SV states

Implementation

- Several methods to generate the GKP qubit are proposed [6,7]
- Achievable squeezing level of SV state is 15 dB [8]

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Problem

- Difficulty to experimentally generate the GKP qubit with the squeezing level 14.8 dB required for large-scale QC [4]

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Our work arXiv:1706.03011

Proposal

- To reduce the required squeezing level, we have focused on analog information contained in the GKP qubit

- We propose a maximum-likelihood method which harnesses the analog information and improves QEC performance

Main results [9]

- The first proposal to achieve the hashing bound for the quantum capacity of the Gaussian quantum channel

- The required squeezing level can be reduced by ~ 1 dB

[9] K F *et al.*, arXiv:1706.03011 (2017)

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The true deviation $\bar{\Delta}$ obeys the Gaussian distribution $f(x)$

We regard the Gaussian distribution as a likelihood function $f(|\bar{\Delta}|)$

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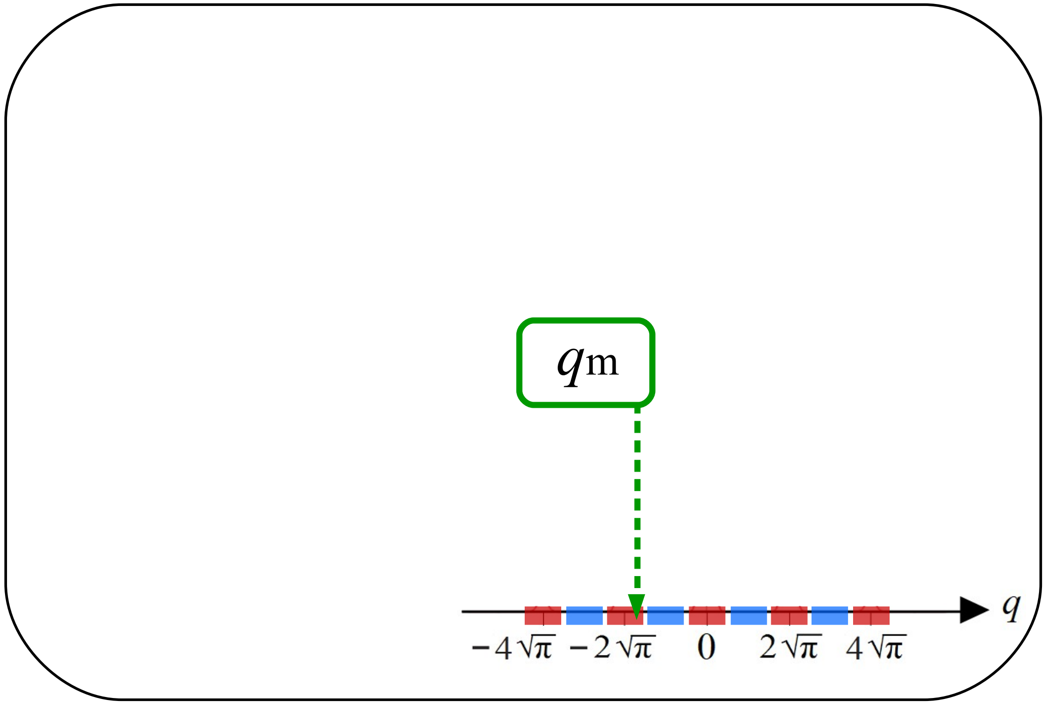

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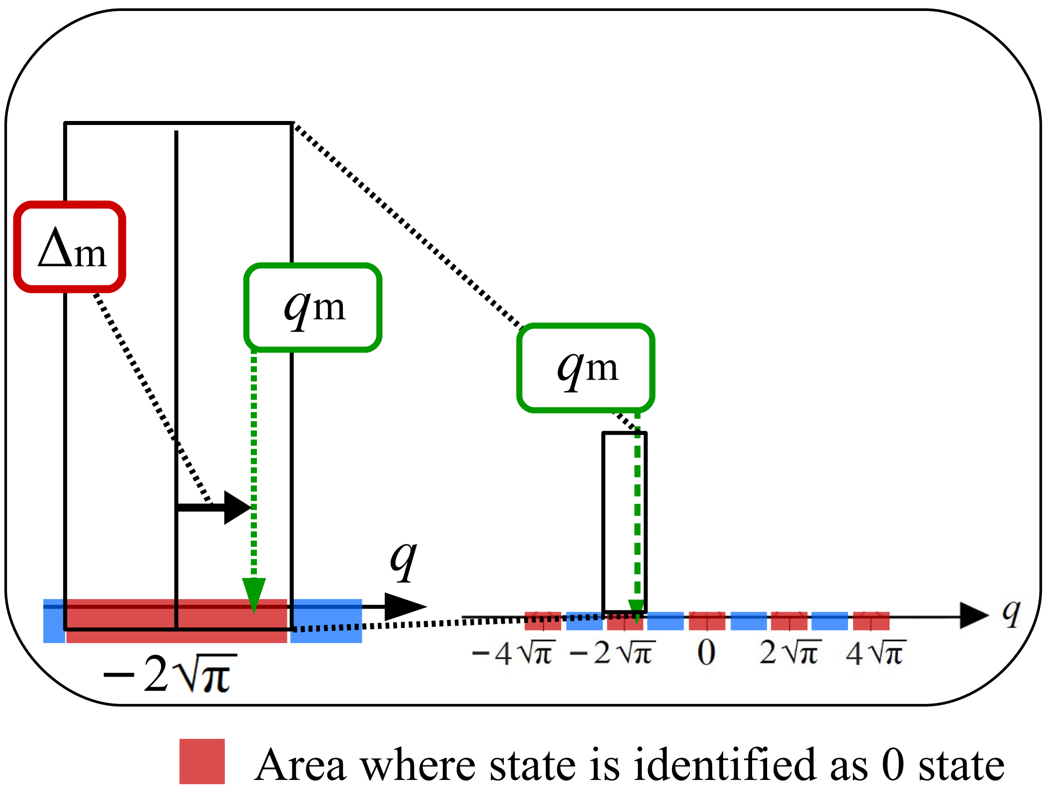

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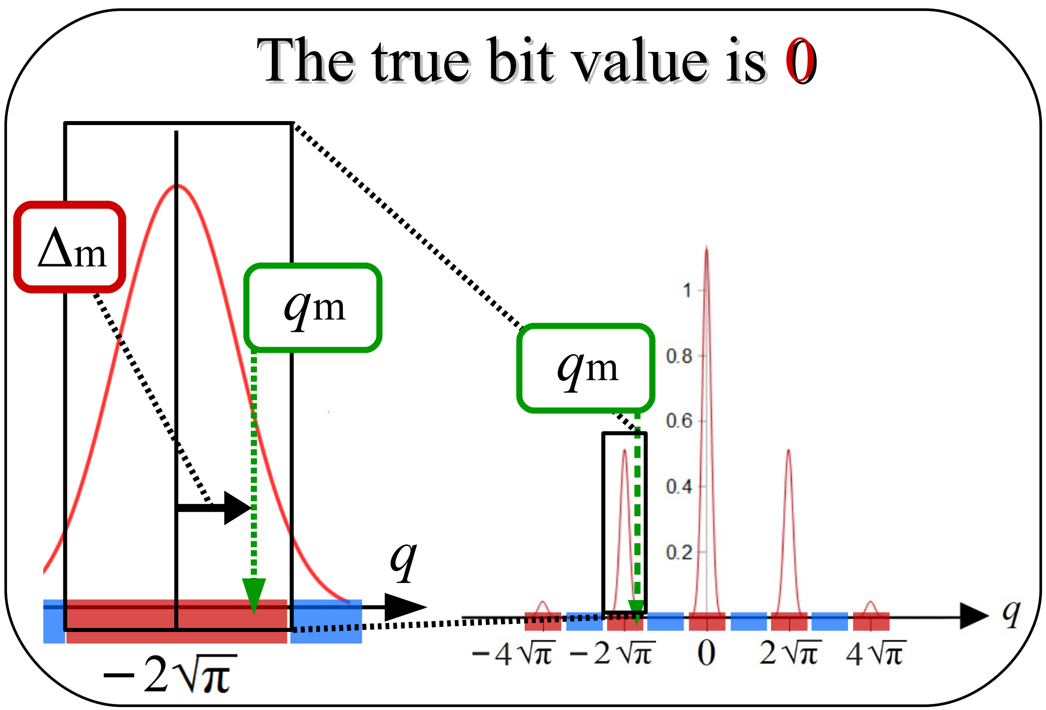

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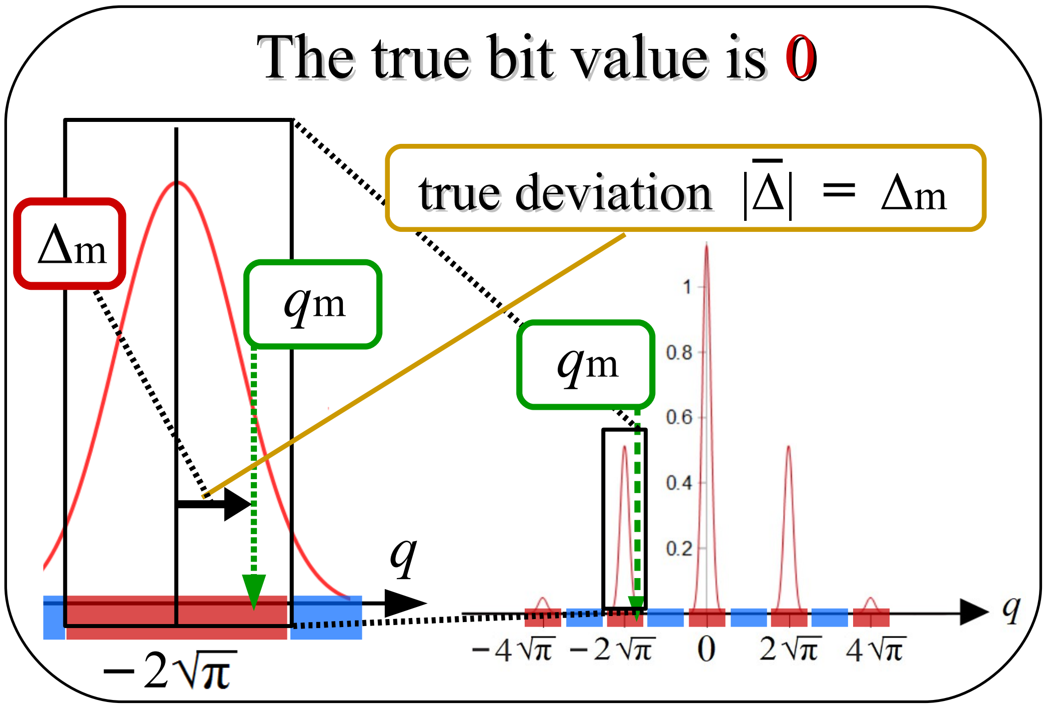

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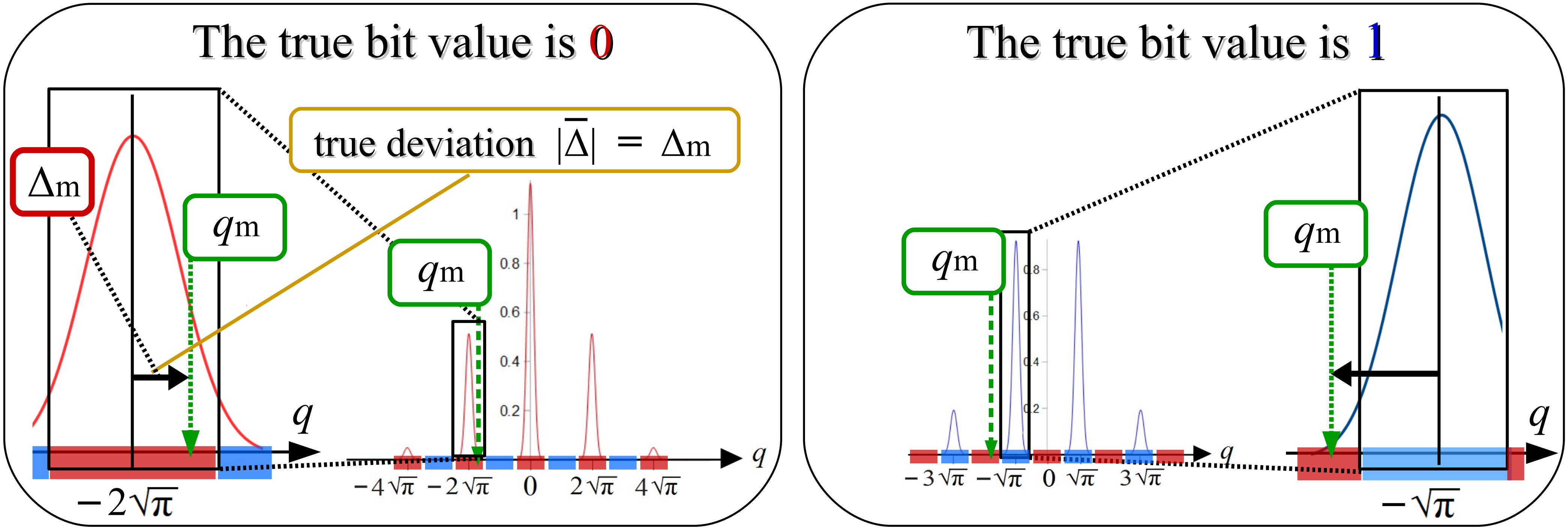

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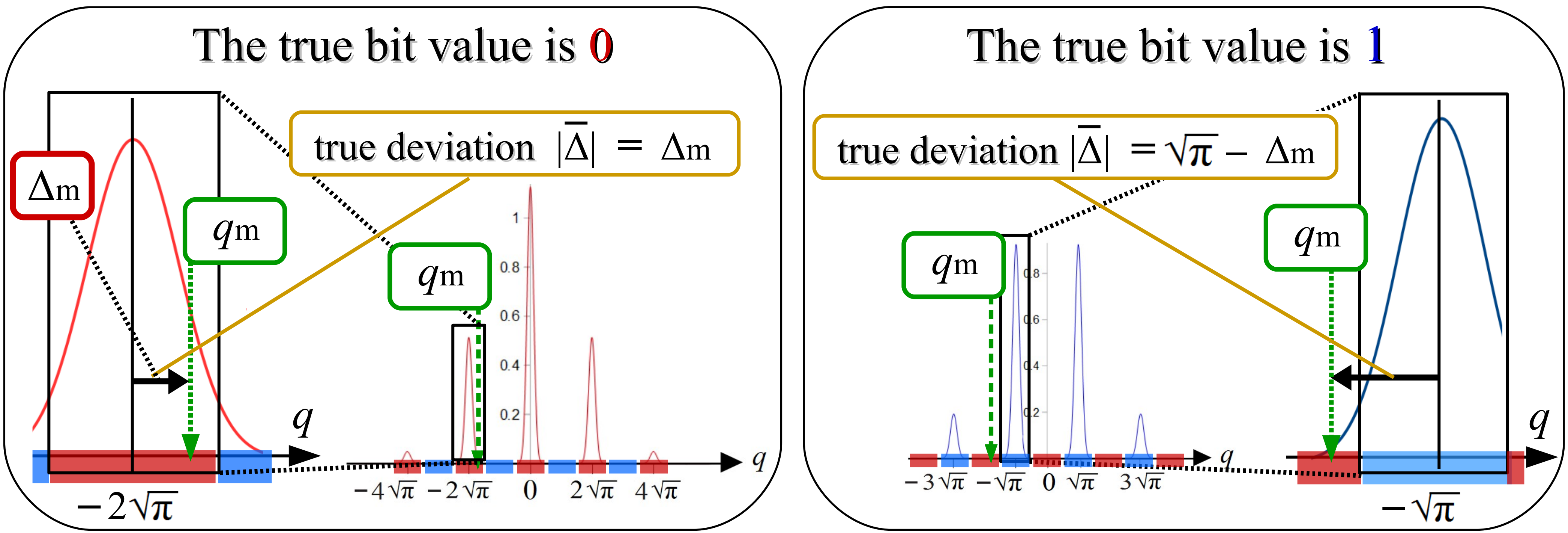

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Error model in this work

The Gaussian quantum channel

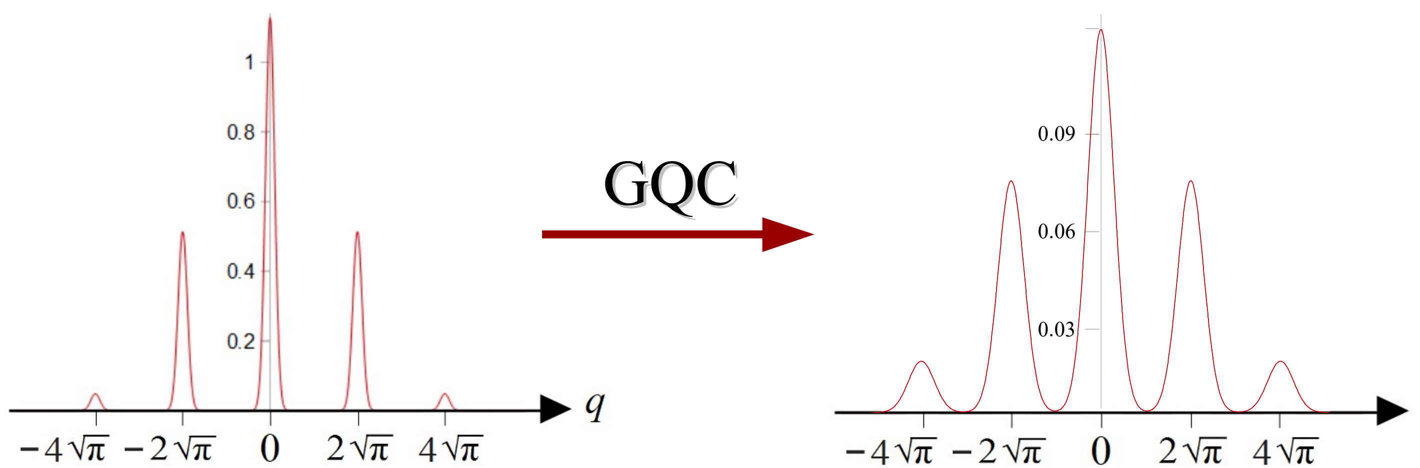
The Gaussian quantum channel (GQC) leads to a displacement in the quadrature by a complex Gaussian random variable [5]

Described by superoperator ζ acting on density operator ρ as

$$\rho \to \zeta(\rho) = \frac{1}{\pi\xi^2}\int d^2\alpha e^{-|\alpha|^2/\xi^2} D(\alpha)\rho D(\alpha)^\dagger$$

$D(\alpha)$ is a displacement operator in the phase space

The GQC conserves the position of the Gaussian peak, but increases the variance by ξ^2



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Ex.) Three-qubit bit-flip code

- Single logical qubit is encoded into three qubits

$$\alpha\,|0\rangle + \beta\,|1\rangle \rightarrow \alpha\,|000\rangle_{123} + \beta\,|111\rangle_{123} \quad (\,|\alpha|^2 + |\beta|^2 = 1\,)$$

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$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle_{123} + \beta|111\rangle_{123} \qquad (|\alpha|^2 + |\beta|^2 = 1)$$

Bit flip error	*Error pattern*
No error	$\alpha|000\rangle_{123} + \beta|111\rangle_{123}$
on qubit 1	$\alpha|100\rangle_{123} + \beta|011\rangle_{123}$
on qubit 2&3	$\alpha|011\rangle_{123} + \beta|100\rangle_{123}$
on qubit 2	$\alpha|010\rangle_{123} + \beta|101\rangle_{123}$
on qubit 1&3	$\alpha|101\rangle_{123} + \beta|010\rangle_{123}$
⋮	⋮

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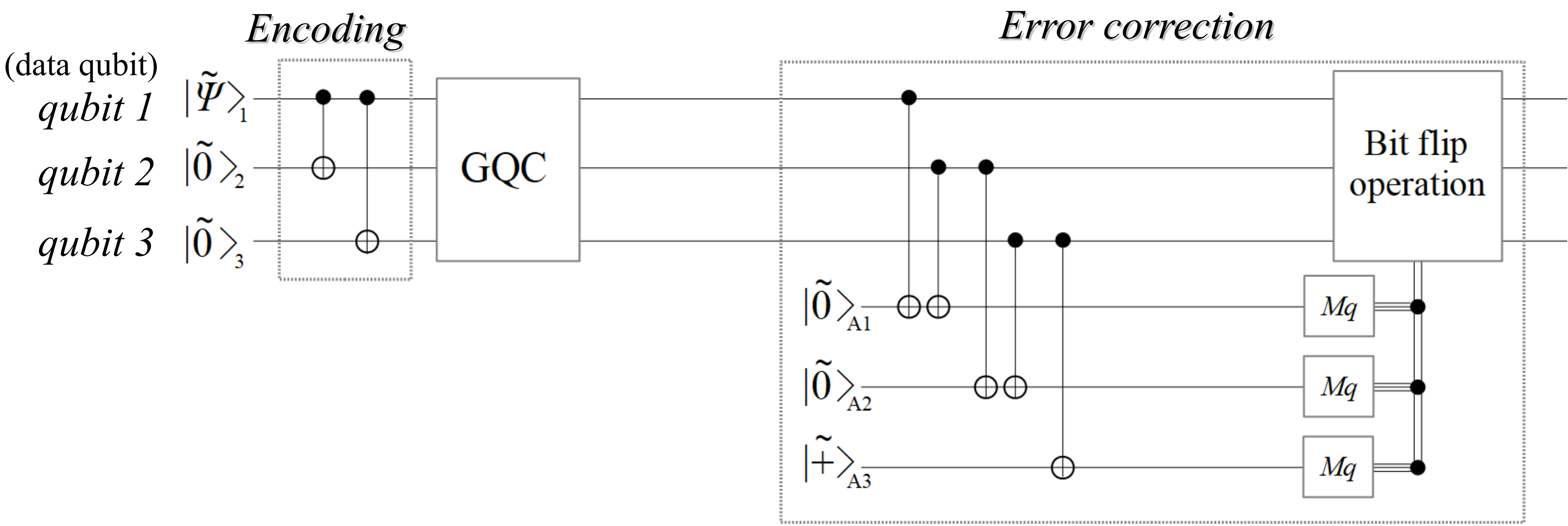
on qubit 2&3	$\alpha|011\rangle_{123} + \beta|100\rangle_{123}$

- The patterns of error on qubit 1 and 2&3 have same syndrome
- In conventional method based on majority voting, the pattern of error on qubit 1 is selected

Analog QEC for three-qubit bit-flip code

A quantum circuit for three-qubit bit-flip code



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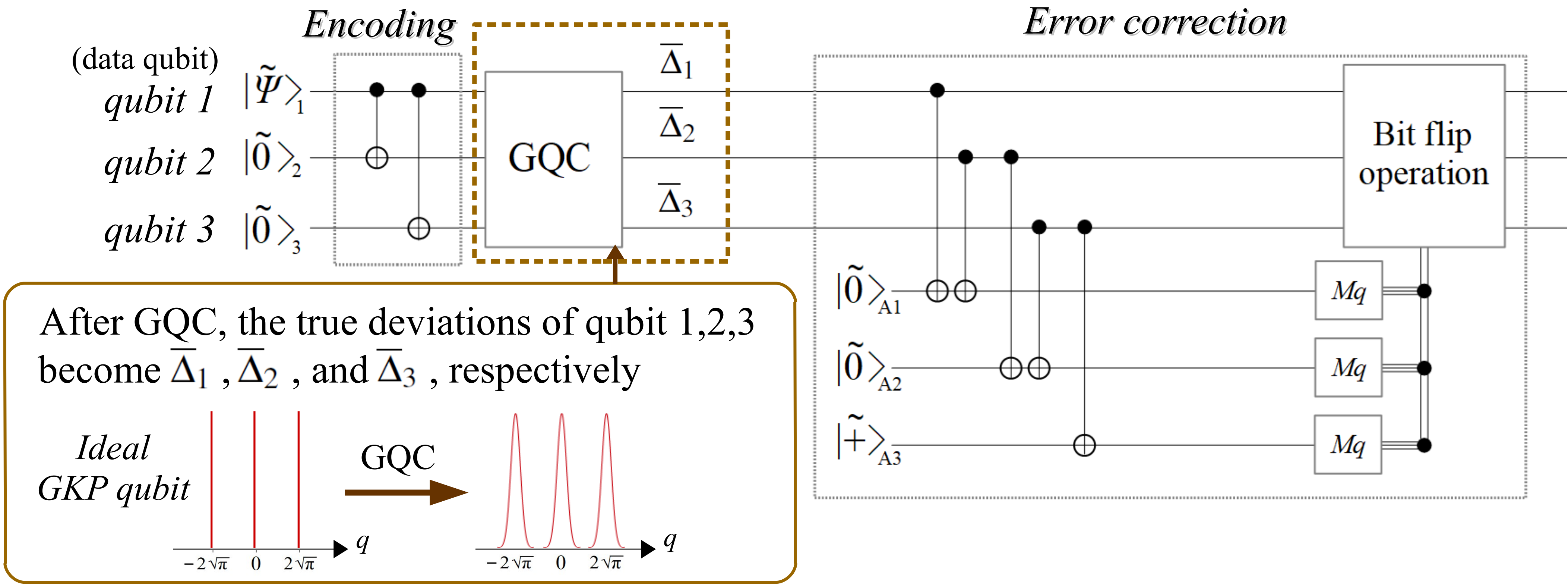

After GQC, the true deviations of qubit 1,2,3 become $\bar{\Delta}_1$, $\bar{\Delta}_2$, and $\bar{\Delta}_3$, respectively

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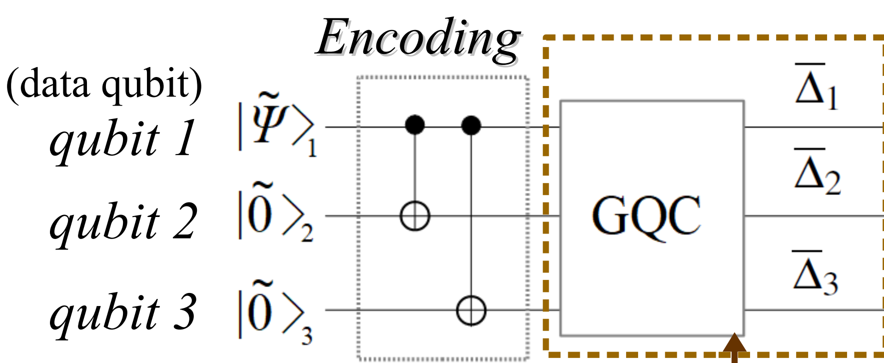

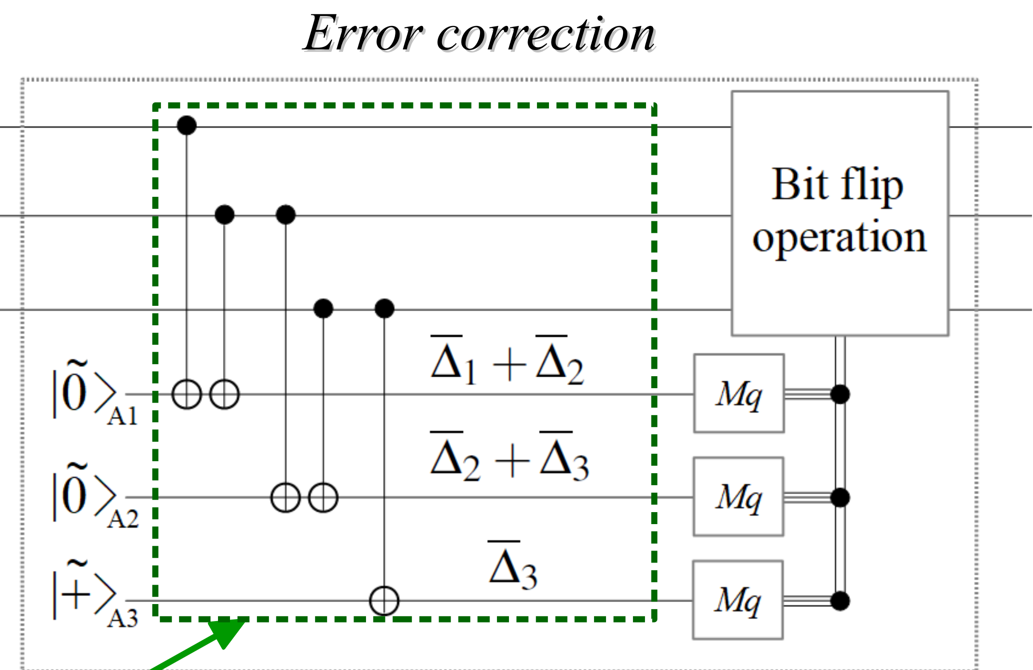

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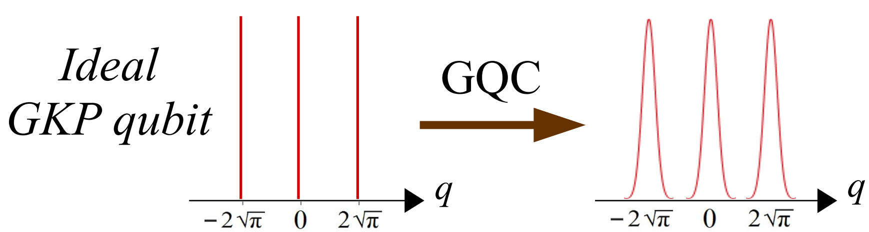

After CNOT, the true deviation of

ancilla 1 is $\bar{\Delta}_1 + \bar{\Delta}_2$,

ancilla 2 is $\bar{\Delta}_2 + \bar{\Delta}_3$,

and ancilla 3 is $\bar{\Delta}_3$,

assuming ancilla qubits are ideal

Analog QEC for three-qubit bit-flip code

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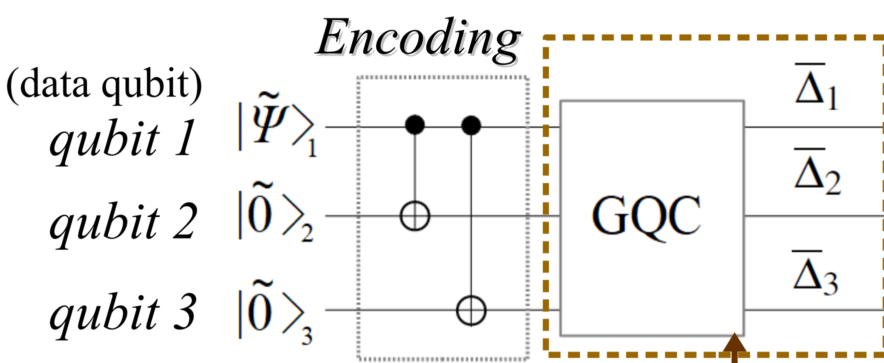

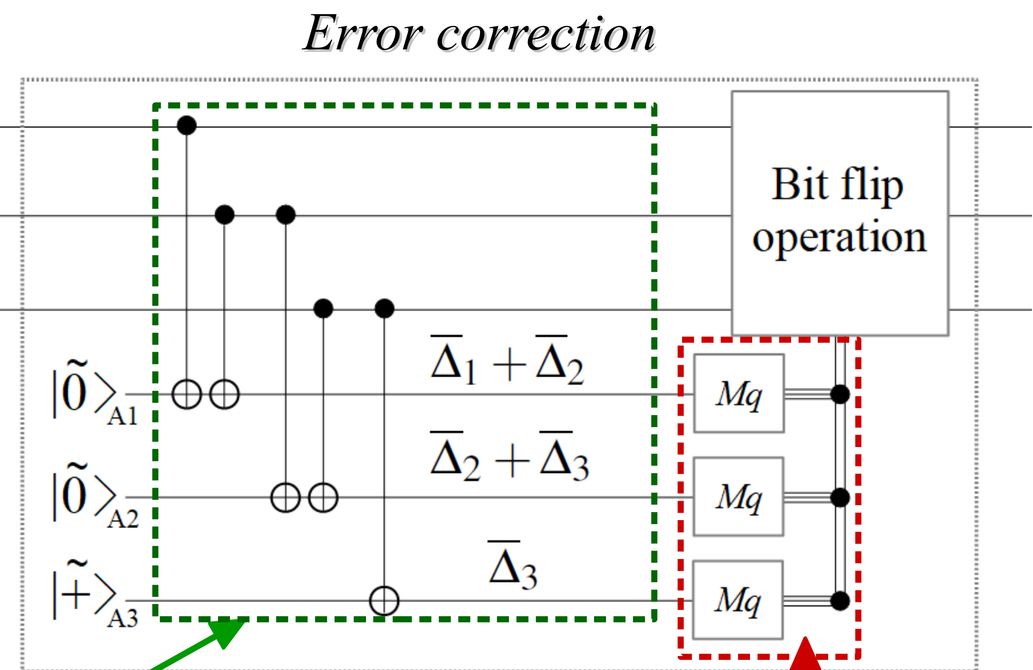

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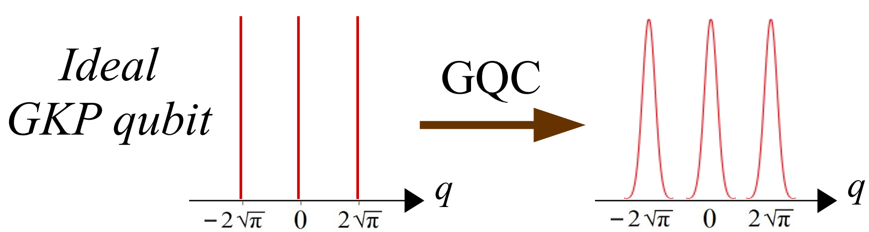

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From the measurement of ancillae, we obtain the measurement deviations Δ_{mi} (i=1,2,3)

Ex.) <u>No error</u>
we obtain the $\bar{\Delta}_i$ (i=1,2,3) correctly

Ex.) <u>Double errors on qubit 2 and 3</u>
we need to decide between single error on qubit 1 and double errors on qubit 2&3

Syndrome with analog information

Ex.) If double errors on qubit 2 and 3, and we obtain the measurement deviation of three qubits $\Delta m1$, $\Delta m2$, and $\Delta m3$, there are the two possibilities as follows:

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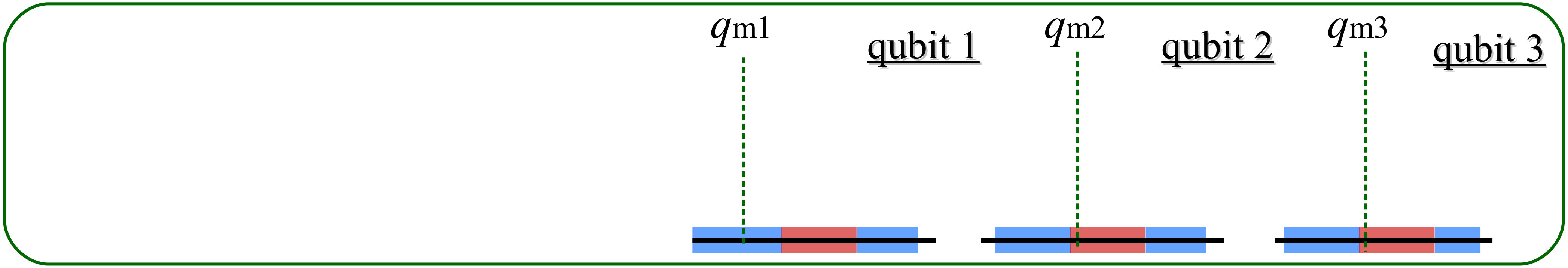

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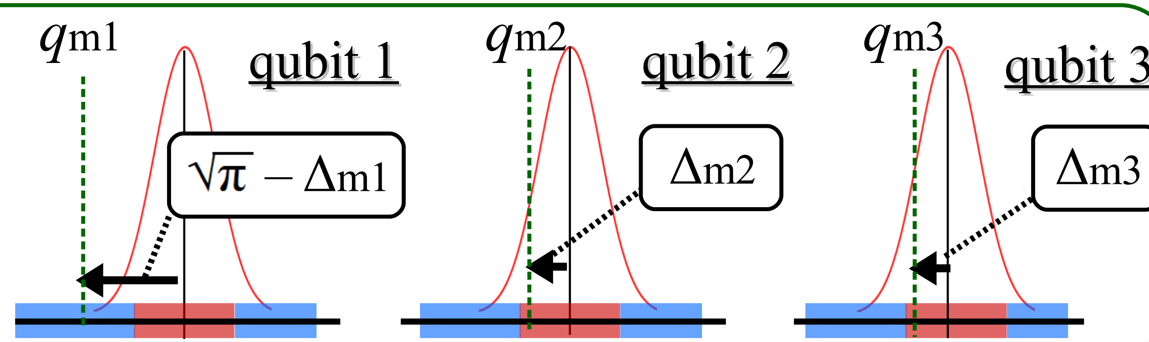

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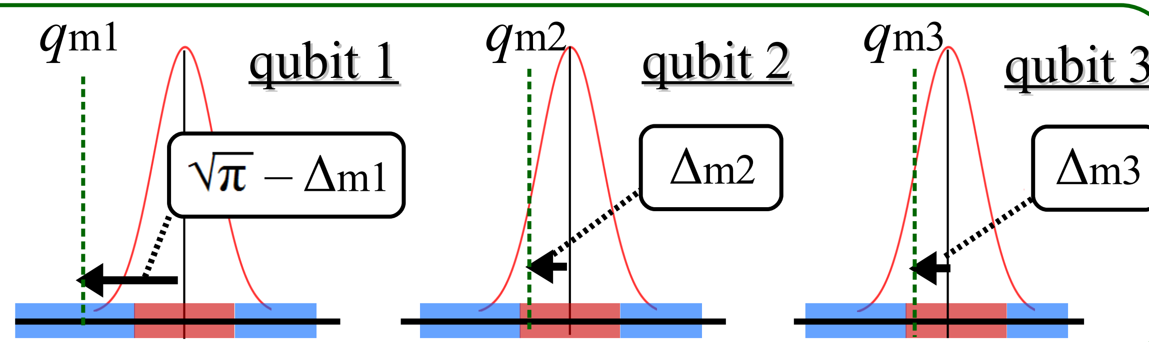
Error on qubit 1

Likelihood for error on qubit 1

$$f(\sqrt{\pi}-\Delta m1)\times f(\Delta m2)\times f(\Delta m3)$$



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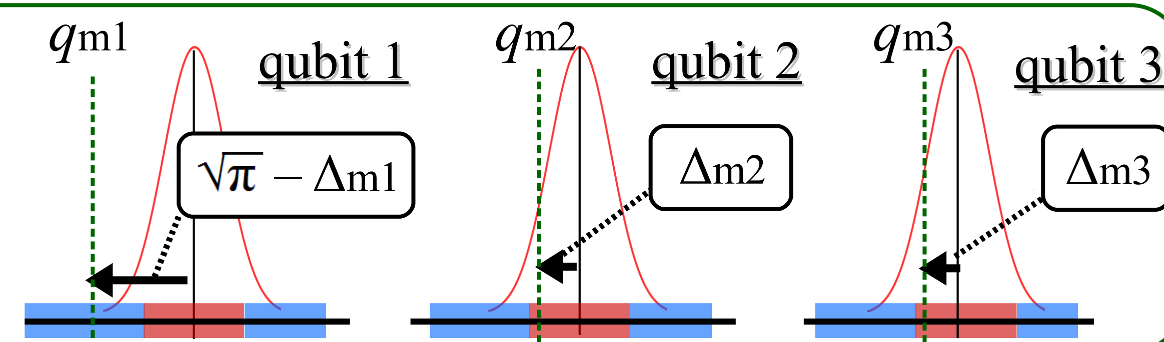

Errors on qubit 2 & 3

Likelihood for errors on qubit 2 & 3

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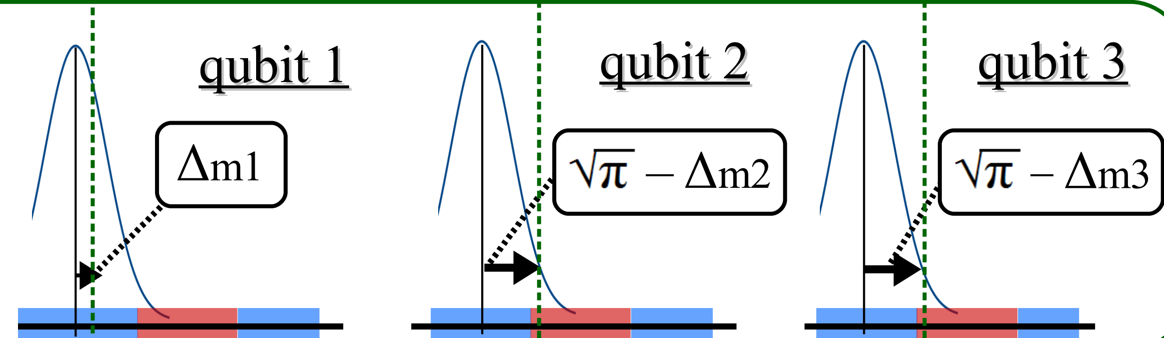

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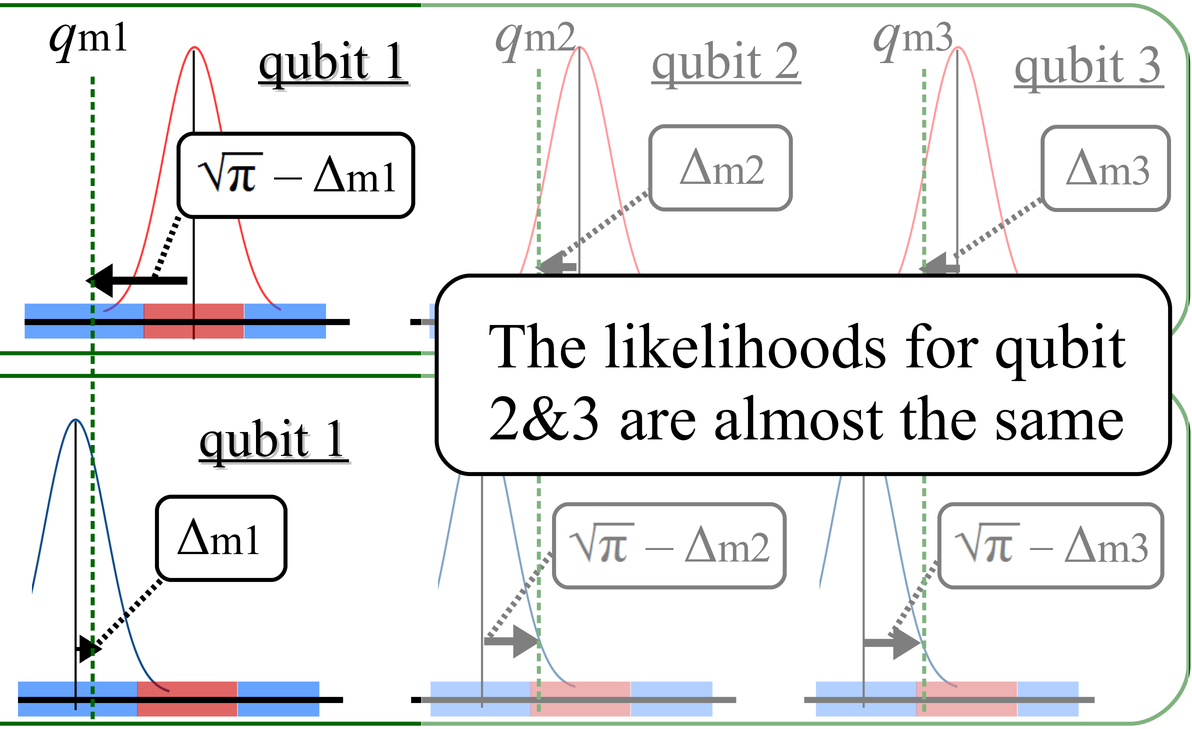

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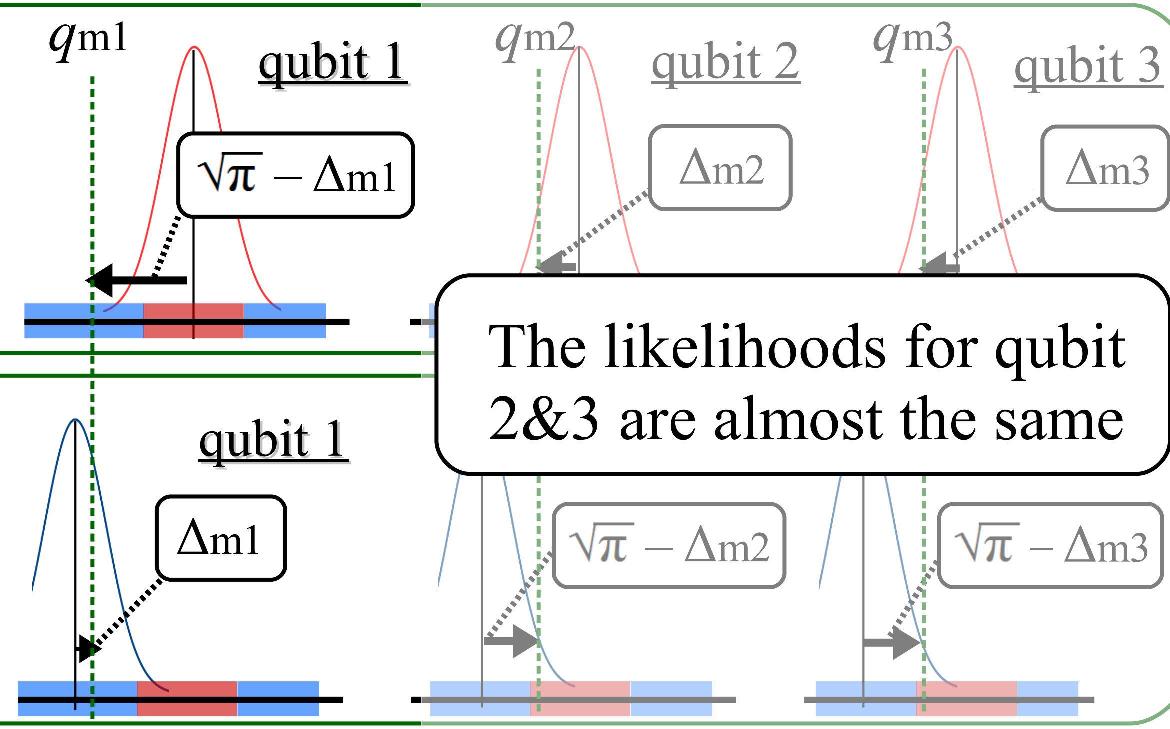
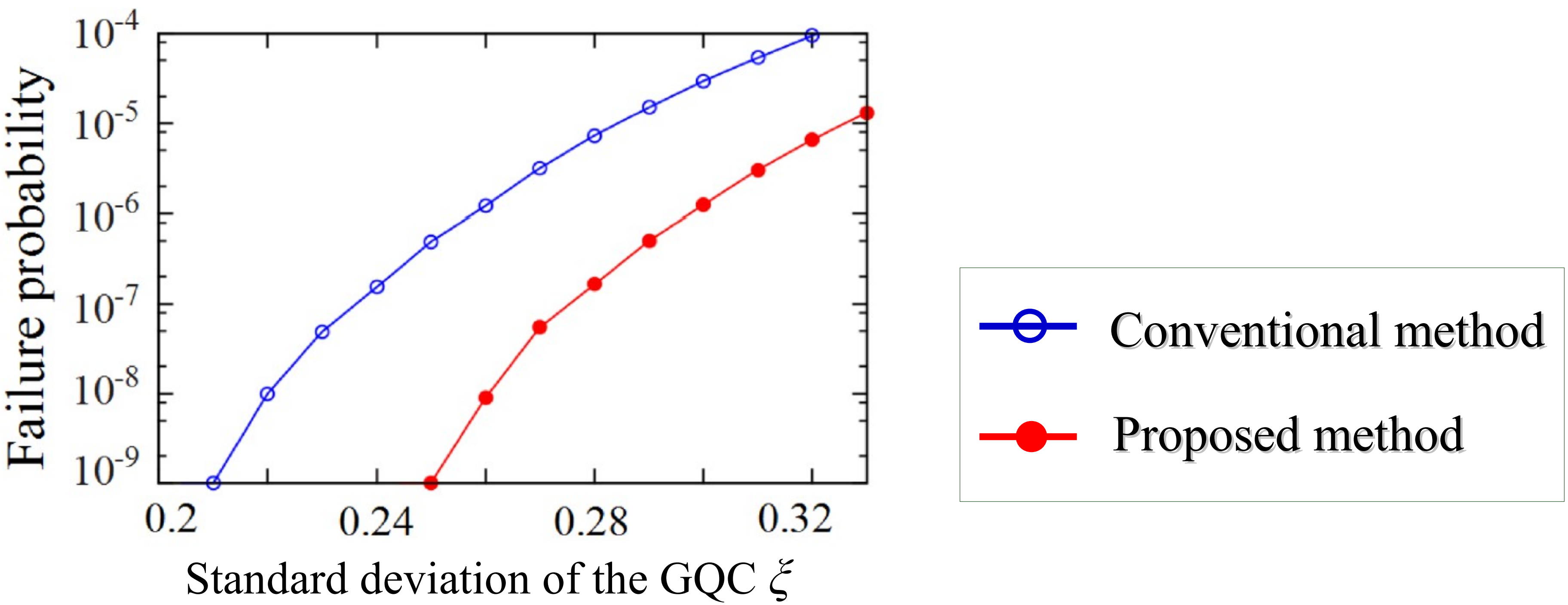


By comparing the likelihoods for the error patterns, we can correct the double-error one

Results for the three-qubit bit-flip code

- Our method can improve the QEC performance and reduce the squeezing level required for the failure probability 10^{-9} by 1.5 dB
- Our method can correct double errors on the three qubits



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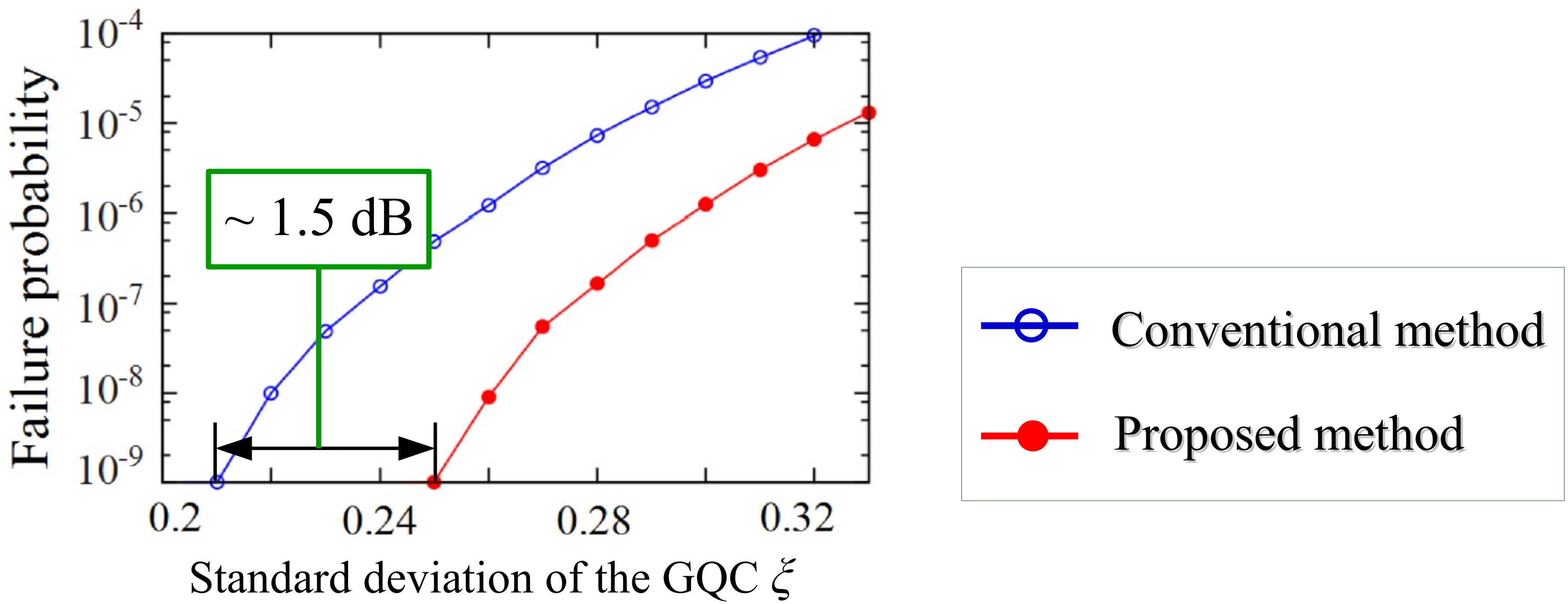

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- Our work

II Analog quantum error correction

- Proposal - likelihood function -
- Error model in our work
- Three qubit bit flip code
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Examination of the required squeezing

- To examine required squeezing level for large-scale QC, we numerically calculated the hashing bound for the GQC

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Hashing bound ξ_{hb}

The hashing bound is the maximum value of the condition that yields the non-zero quantum capacity

Encoding → GQC → Decoding

Increasing the variance by ξ^2

- The GQC has nonvanishing quantum capacity for $\xi < \xi_{hb}$

Achievable hashing bound

Achievable hashing bound for the GQC [5][10]

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The GQC has nonvanishing quantum capacity for $\xi < \xi_{hb}$

GKP qubit concatenated with CSS code + Binary information → $\xi_{hb} \sim .555$

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Optimal method (*Open problem*) → $\xi_{max} \sim .607$

- $\sim .607$ has been conjectured as the lower bound of quantum capacity for the GQC

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GKP qubit concatenated with CSS code + Binary information + Analog information (This work) → $\xi_{max} \sim .607$

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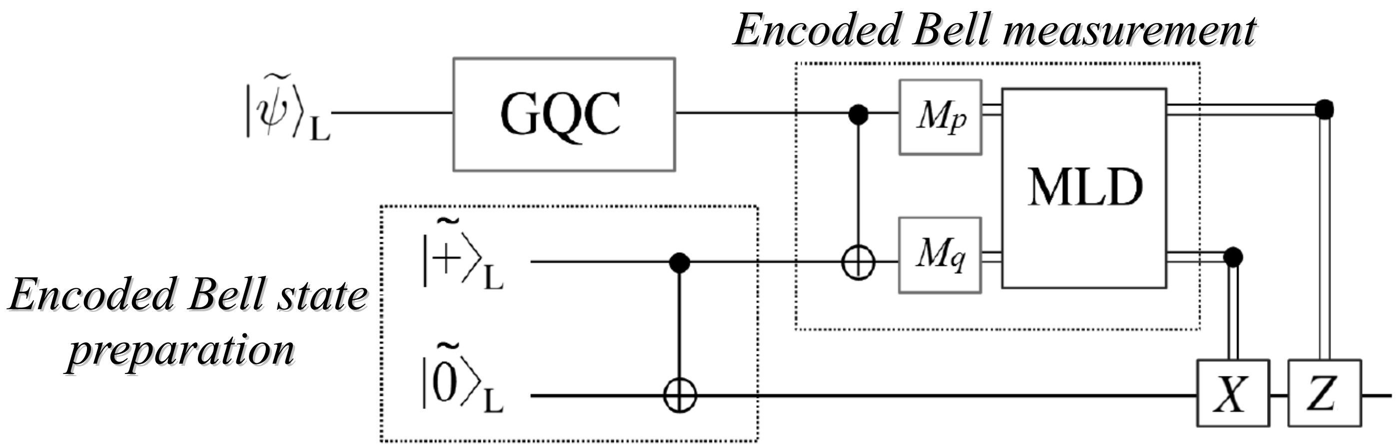
Analog QEC for the C4/C6 code

- We applied our method to the Knill's C4 /C6 code [11] using a message passing algorithm proposed by Poulin [12,13]



[11] E. Knill, Nature, **434**, 39-44 (2005)
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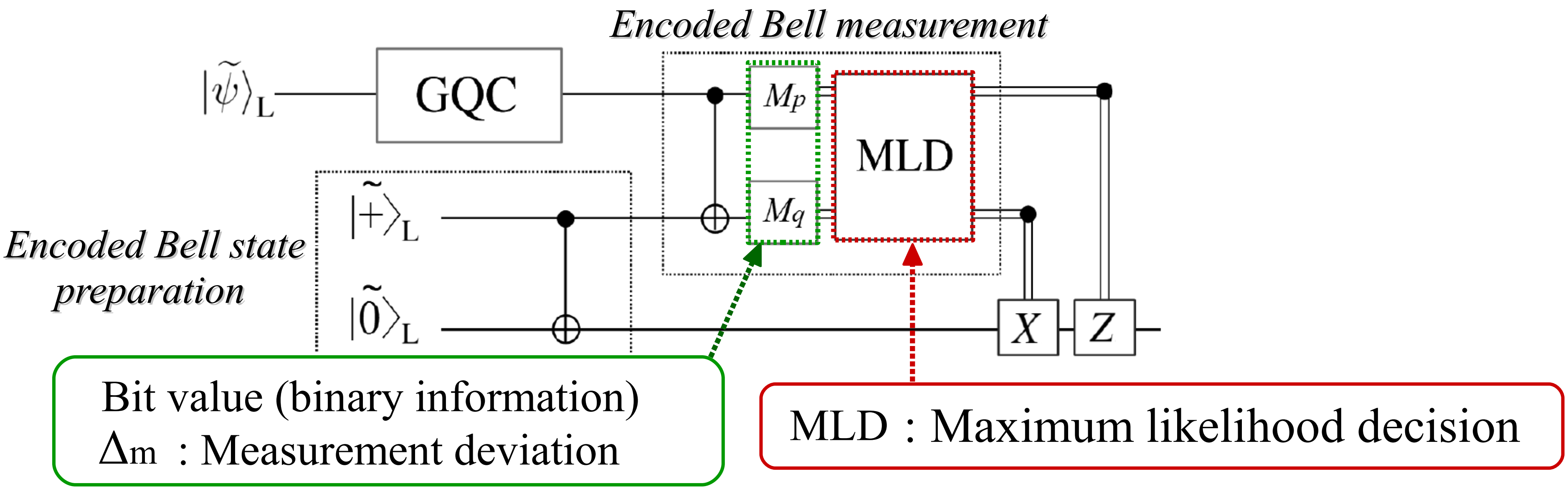

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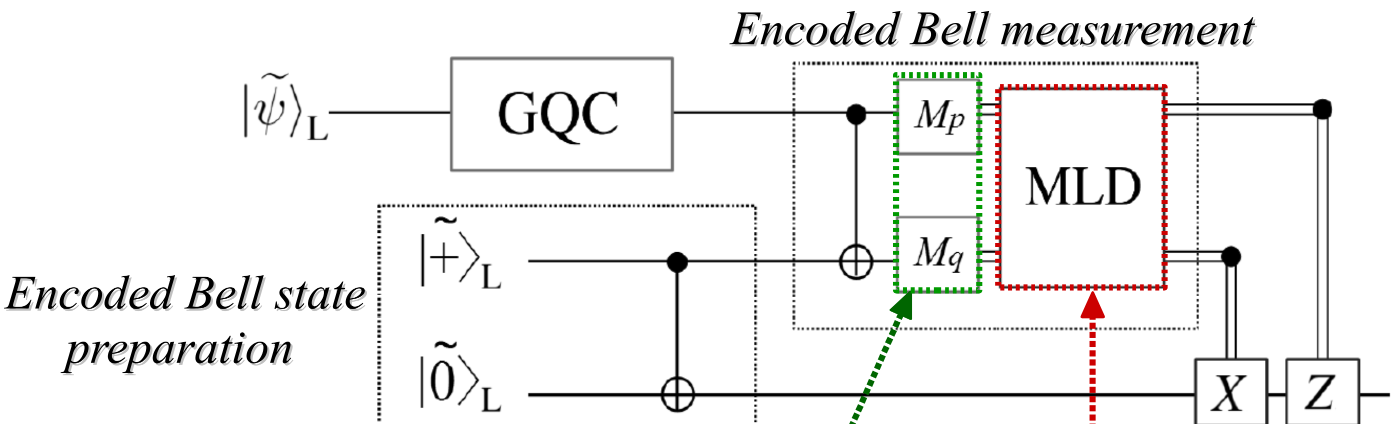

Bit value (binary information)
Δm : Measurement deviation

MLD : Maximum likelihood decision

	Convention method	Proposal
Likelihood for correct bit value	$p = \int_{-\frac{\sqrt{\pi}}{2}}^{\frac{\sqrt{\pi}}{2}} dx \frac{1}{\sqrt{2\pi\xi^2}} \exp(-x^2/2\xi^2)$	$f(\Delta_m) = \frac{1}{\sqrt{2\pi\xi^2}} e^{-\Delta_m^2/(2\xi^2)}$
Likelihood for incorrect bit value	$1 - p$	$f(\sqrt{\pi} - \lvert\Delta_m\rvert)$

ξ^2 : Noise level of GQC

[11] E. Knill, Nature, **434**, 39-44 (2005)
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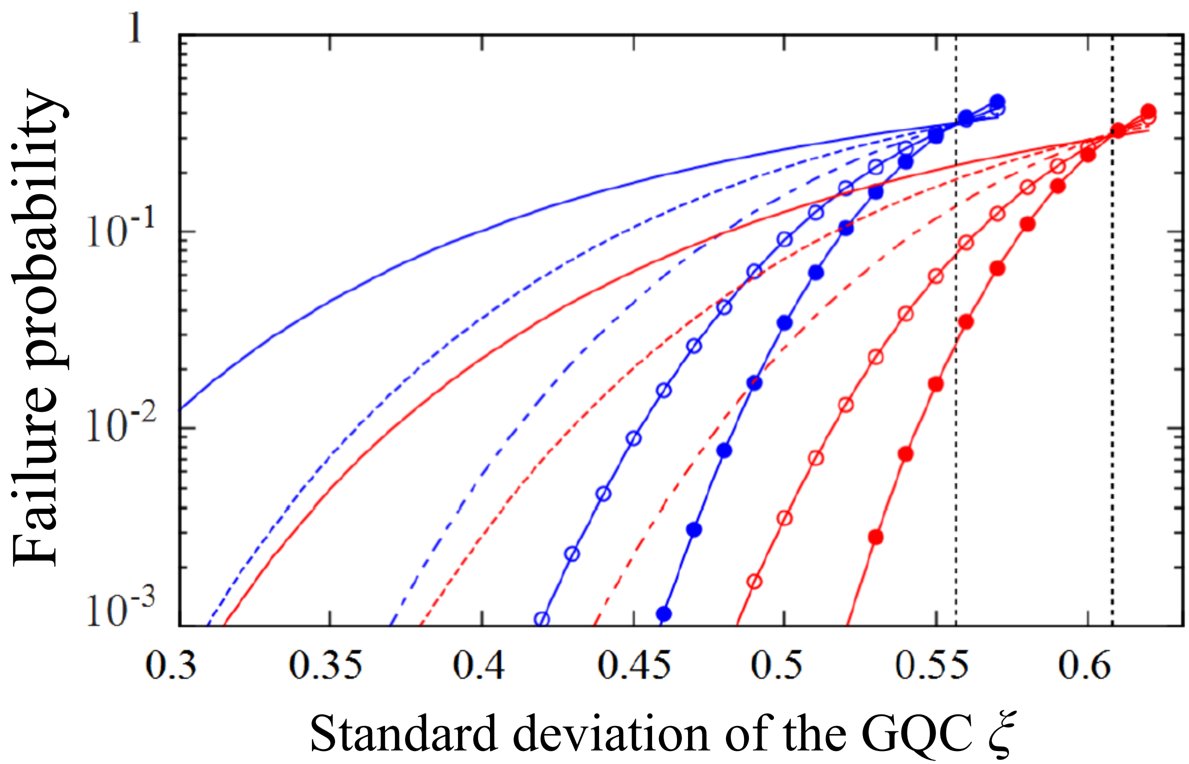
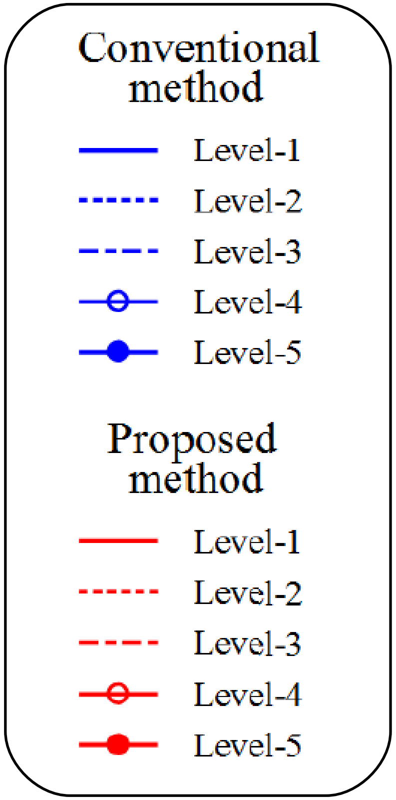
Results for the C4/C6 code

- Our method can improve the QEC performance and reduce the squeezing level required for fault-tolerant QC





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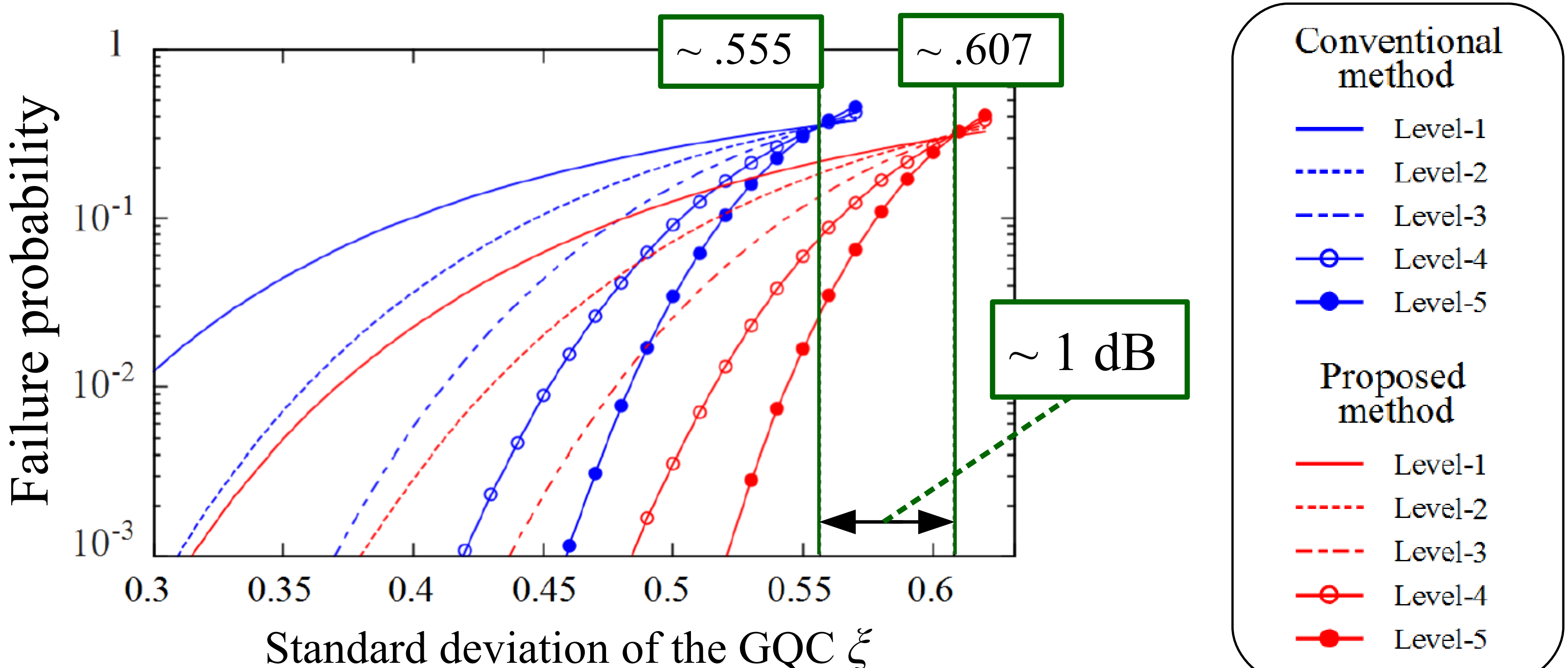
- Our method can achieve the hashing bound ~ .607 [5][10]



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Outline

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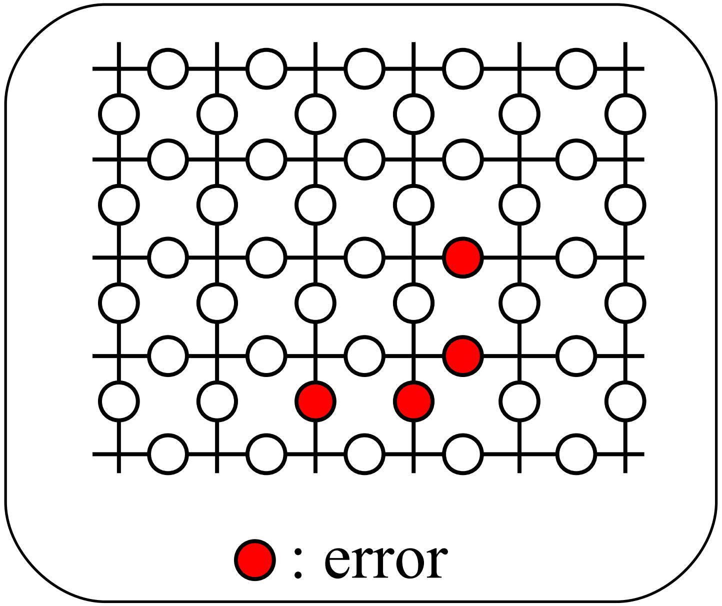
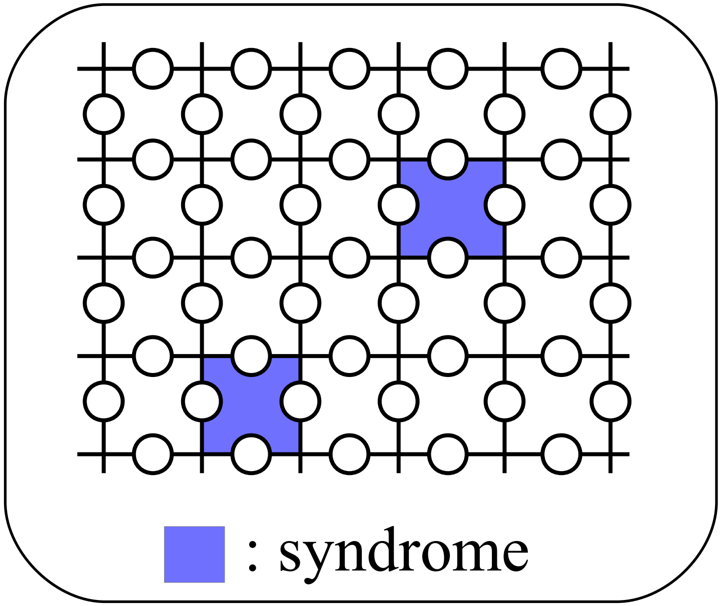
Summary

Analog QEC for the surface code

- We applied our method to a surface code which is used to implement topological QC [14,15]





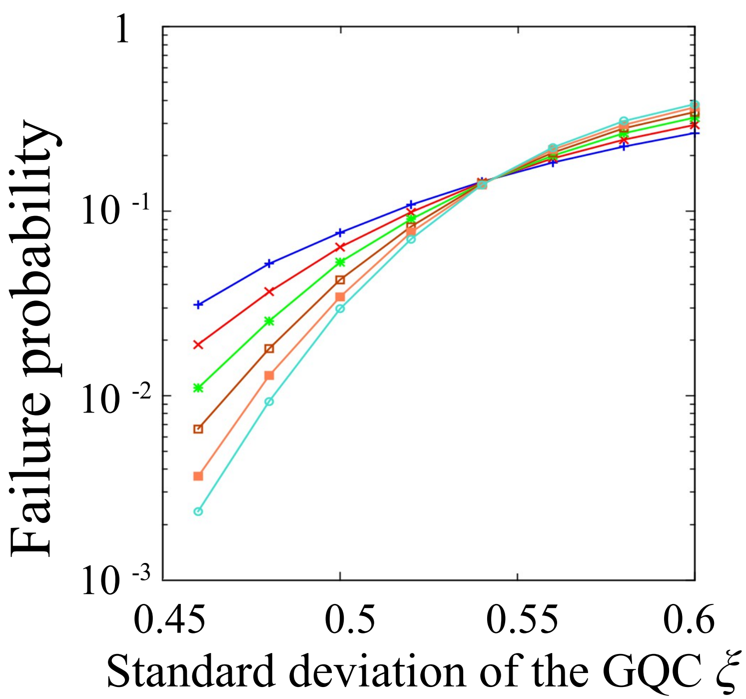
- Errors are detected at the boundary of the error chain
- From the boundary information, we need to decide the most likely error chain by using minimum-weight perfect match algorithm

[14] A. Y. Kitaev, Ann. Phys. **303**, 2 (2003)
[15] R. Raussendorf *et al.*, Phys. Rev. Lett. **98**, 190504 (2007)

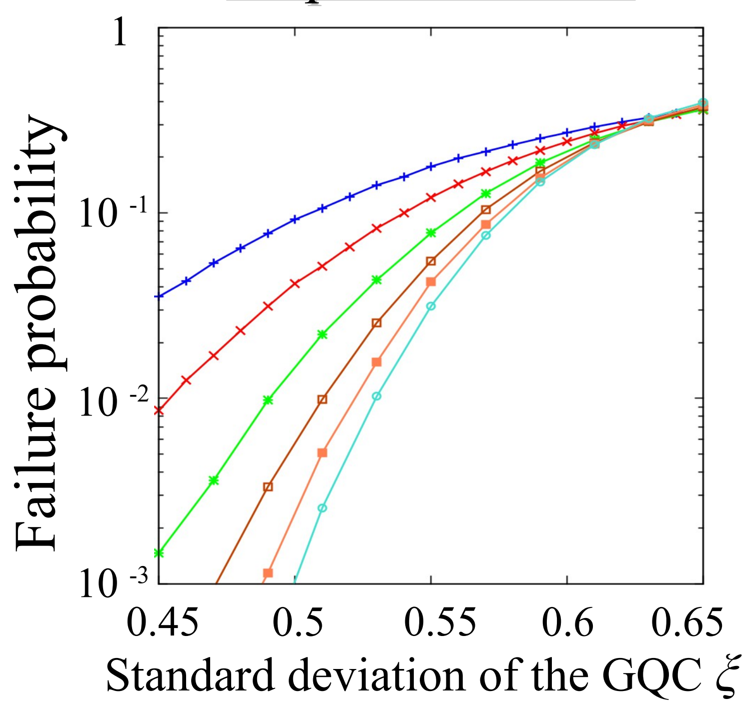
Results for the surface code

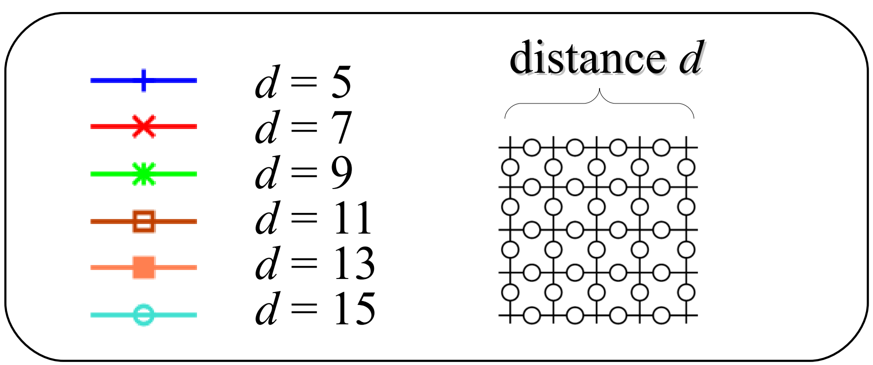
- Our method can also improve the QEC performance and reduce the squeezing level required for Topological QC [16]

Conventional method



Proposed method





[16] K.F, K. Fujii, A.Tomita, and A. Okamoto (*in preparation*)

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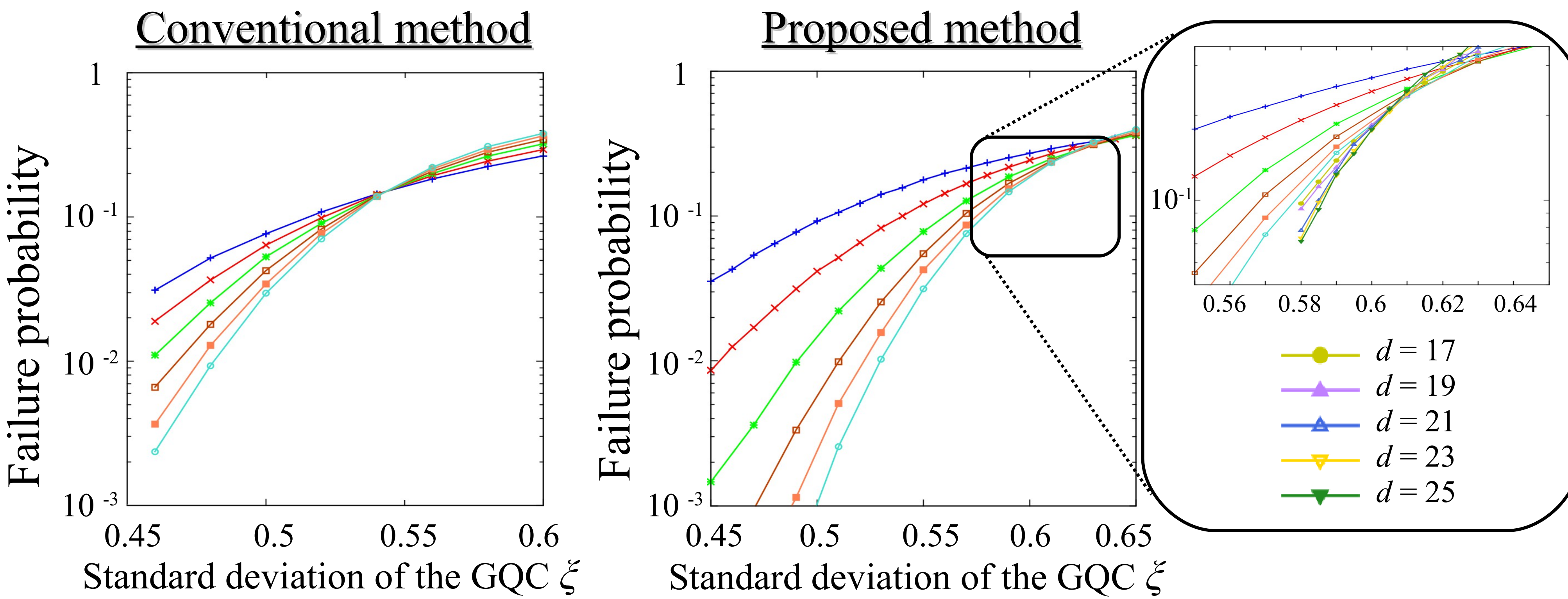

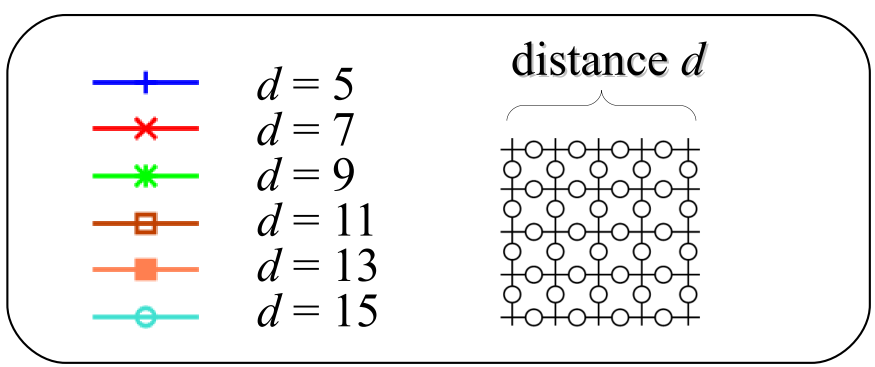

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Summary

- The GKP qubit is a promising element toward large-scale QC

- Proposal to harness analog information contained in the GKP qubit to reduce the requirement for large-scale QC

- Proposal can achieve the hashing bound for the optimal method against the GQC

- Our method can be applied to various QEC codes such as, concatenated code, non-concatenated code, and surface code

Thank you for your attention !