Quantum simulation of the Rabi model in a trapped ion system

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Outline

1. Background of the Rabi model
2. Experimental setup and proposal to simulate QRM with trapped ion system
3. Experimental results

Background of the Rabi model:Why quantum Rabi model?

- Originally, Rabi model was proposed in 1936 to deal with the effect of a varying, weak magnetic field on an oriented atom possessing nuclear.

Rabi I I. On the process of space quantization[J]. Physical Review, 1936, 49(4): 324.

- With full quantized field, we get the quantum version of Rabi model, describe as

$$\hat{H}(QRM) = \frac{1}{2}\hbar\omega\hat{\sigma}_z + \hbar\omega_m\hat{a}^\dagger\hat{a} + \hbar g(\hat{\sigma}_+ + \hat{\sigma}_-)(\hat{a}^\dagger + \hat{a})$$

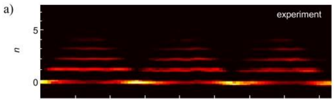
it can extend to Jaynes-Cummings model via the rotate wave approximation(RWA)($g \ll \omega_m$)

$$\hat{H}(JCM) = \frac{1}{2}\hbar\omega\hat{\sigma}_z + \hbar\omega_m\hat{a}^\dagger\hat{a} + \hbar g(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger\hat{\sigma}_-)$$

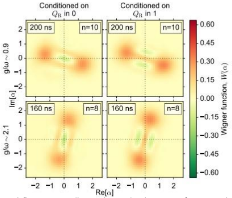
- Recently an analytic solution has been proposaled that cover all the coupling regime and experiments has push into the USC/DSC regime, where RWA, JCM is not applicable, and full QRM is required. (USC: $0.1 \leq g/\omega_m$, DSC: $1 \leq g/\omega_m$)

Braak D. Integrability of the Rabi model[J]. Physical Review Letters, 2011, 107(10):

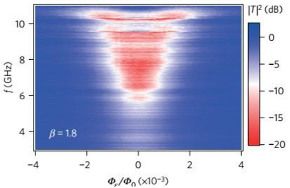
Recent achievements in USC/DSC regime



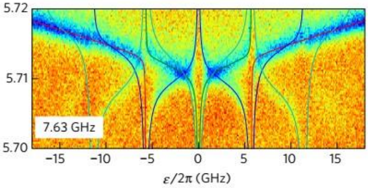
Crespi et al. "Photonic realization of the quantum Rabi model." *Physical review letters* 108.16 (2012): 163601.



Langford, N. K., et al. "Experimentally simulating the dynamics of quantum light and matter at ultrastrong coupling." arXiv preprint arXiv:1610.10065 (2016).

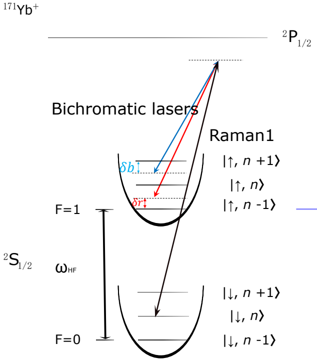


Forn-Díaz, P., et al. "Ultrastrong coupling of a single artificial atom to an electromagnetic continuum in the nonperturbative regime." *Nature Physics* 13.1 (2017): 39-43.



Yoshihara, Fumiki, et al. "Superconducting qubit-oscillator circuit beyond the ultrastrong-coupling regime." *Nature Physics* (2016).

Experimental setup and proposal to simulate QRM with trapped ion system

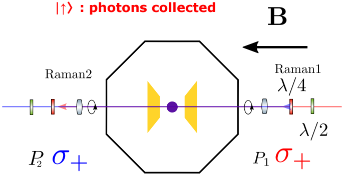


State initialization to: $|\downarrow, 0\rangle$

State detection: florensence collection

$|\downarrow\rangle$: no photons collected

$|\uparrow\rangle$: photons collected



Pedernales J S, Lizuain I, Felicetti S, et al. Quantum Rabi model with trapped ions[J]. Scientific reports, 2015, 5.

Ion-light interaction

The total Hamiltonian of the ion-light matter interaction include three parts, written as

$$\hat{H} = \hat{H}_m + \hat{H}_e + \hat{H}_i \tag{1}$$

Here $\hat{H}_m$ of the motional model X can be written as

$$\hat{H}_m = \hbar\omega_X(\hat{a}^\dagger\hat{a} + \frac{1}{2}) \tag{2}$$

$\hat{H}_e$ describes the internal electronic level structure of spin down$|\downarrow\rangle$ and spin up $|\uparrow\rangle$, written as

$$\hat{H}_e = \hbar\frac{\omega_{HF}}{2}\sigma_Z \tag{3}$$

the ion-light interaction part

$$\hat{H}_i = \hbar\frac{\Omega}{2}(\sigma_+ + \sigma_-)(e^{i(\vec{k}\vec{x}-\omega t+\phi)} + e^{-i(\vec{k}\vec{x}-\omega t+\phi)}), \tag{4}$$

Ion-light interaction

In the interaction picture $\hat{H}_0 = \hat{H}_m + \hat{H}_e$, $\Delta = \omega - \omega_{HF}$

- If $\Delta = 0$, we get the carrier transition Hamiltonian,

$$\hat{H}_{car}(\phi) = \frac{\hbar\Omega}{2}(\hat{\sigma}_+ e^{-i\phi} + \hat{\sigma}_- e^{i\phi}). \tag{5}$$

- If $\Delta = -\omega_X$, we get the red sideband transition Hamiltonian or the Jaynes-Cummings(JC) coupling,

$$\hat{H}_{JC}(\phi) = \frac{i\hbar\eta\Omega}{2}(\hat{\sigma}_- \hat{a}^\dagger e^{i\phi} - \hat{\sigma}_+ \hat{a} e^{-i\phi}), \tag{6}$$

- If $\Delta = \omega_X$, we get the blue sideband transition Hamiltonian or the anti-Jaynes-Cummings(aJC)coupling

$$\hat{H}_{aJC}(\phi) = \frac{i\hbar\eta\Omega}{2}(\hat{\sigma}_+ \hat{a}^\dagger e^{-i\phi} - \hat{\sigma}_- \hat{a} e^{i\phi}). \tag{7}$$

Simulate Quantum Rabi model

- Remind that the Quantum Rabi Model(QRM) is written as

$$\hat{H}_R = \frac{1}{2}\hbar\omega\hat{\sigma}_z + \hbar\omega_m \hat{a}^\dagger\hat{a} + \hbar g\hat{\sigma}_x(\hat{a} + \hat{a}^\dagger) \tag{8}$$

- In our system, if we employ a new transformation $\hat{H}_v = \hat{V}^\dagger\hat{H}\hat{V} + i\hbar\frac{d\hat{V}^\dagger}{dt}\hat{V}$ under $\hat{V} = exp(-i\hat{H}_{0v}t/\hbar)$ where $\hat{H}_{0v} = \frac{1}{2}\hbar\omega_v\hat{\sigma}_z + \hbar\nu_v\hat{a}^\dagger\hat{a}$ is the free energy of the virtual trapped ion system. The resulting Hamiltonian in the interaction picture reads

$$\begin{aligned}\hat{H}_v = {} & \frac{1}{2}\hbar\omega_v\hat{\sigma}_z + \hbar\nu_v\hat{a}^\dagger\hat{a} \\ & + \hbar\Omega[e^{i\psi}e^{i(\omega_l-\omega+\delta\omega)t}\hat{\sigma}_- + e^{-i\psi}e^{-i(\omega_l-\omega+\delta\omega)t}\hat{\sigma}_+] \\ & - i\hbar\frac{\Omega\eta}{2}[\hat{a}e^{-i(\nu-\delta\nu)t} + \hat{a}^\dagger e^{i(\nu-\delta\nu)t}] \\ & \times [e^{i\psi}e^{i(\omega_l-\omega+\delta\omega)t}\hat{\sigma}_- + e^{-i\psi}e^{-i(\omega_l-\omega+\delta\omega)t}\hat{\sigma}_+]\end{aligned}$$

Here, the virtual detuning parameters are detuned as $\delta\omega = \omega - \omega_v$ and $\delta\nu = \nu - \nu_v$. For simplicity, we choose $\psi = \frac{\pi}{2}$ from now on.

For $\omega_l = (\omega - \delta\omega) + (\nu - \delta\nu_v)$, we again get the anti-JCM-type Hamiltonian

$$\hat{H}_1 = \frac{1}{2}\hbar\delta\omega\hat{\sigma}_z + \hbar\delta\nu\hat{a}^\dagger\hat{a} + \hbar\frac{\Omega\eta}{2}(\hat{a}\hat{\sigma}_- + \hat{a}^\dagger\hat{\sigma}_+) \tag{9}$$

For $\omega_l = (\omega - \delta\omega) - (\nu - \delta\nu_v)$, we again get the JCM-type Hamiltonian

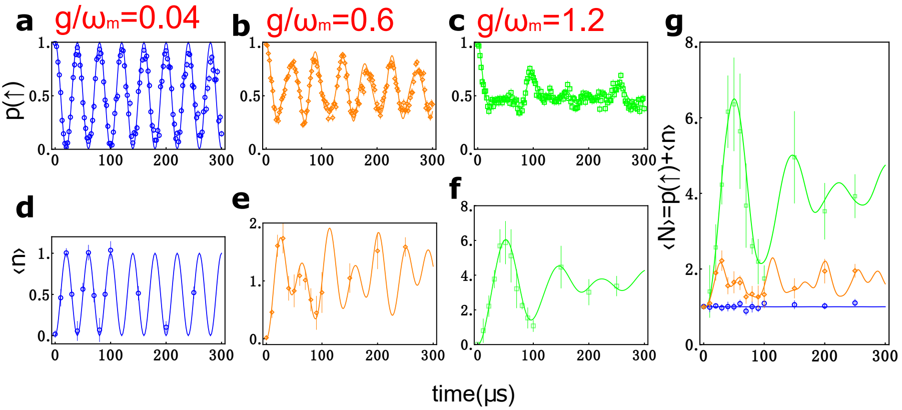
$$\hat{H}_2 = \frac{1}{2}\hbar\delta\omega\hat{\sigma}_z + \hbar\delta\nu\hat{a}^\dagger\hat{a} + \hbar\frac{\Omega\eta}{2}(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger\hat{\sigma}_-) \tag{10}$$

Now we come to the time-evolution of a quantum state under the sum

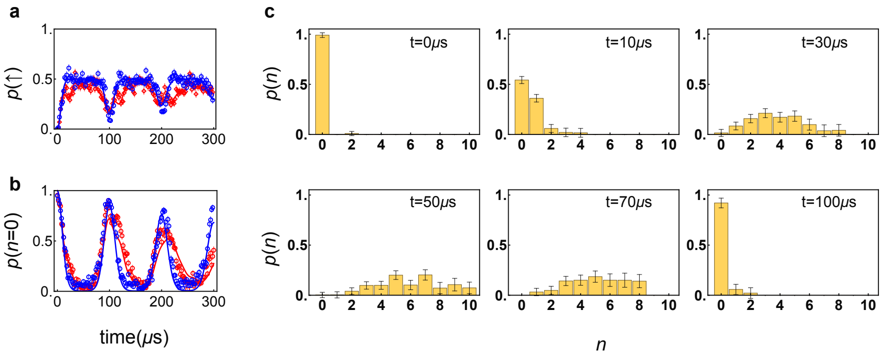
$$\hat{H}_1 + \hat{H}_2 = \hbar\delta\omega\hat{\sigma}_z + 2\hbar\delta\nu\hat{a}^\dagger\hat{a} + \hbar\frac{\Omega\eta}{2}\hat{\sigma}_x(\hat{a} + \hat{a}^\dagger) \tag{11}$$

The striking similarity between the QRM Hamiltonian 8 and Eq. 11 leads to the interpretation of s pesudo effective atomic and trap frequency given by $\omega = 2\delta\omega$, $\omega_m = 2\delta\nu$ and $g = \frac{\eta\Omega}{2}$

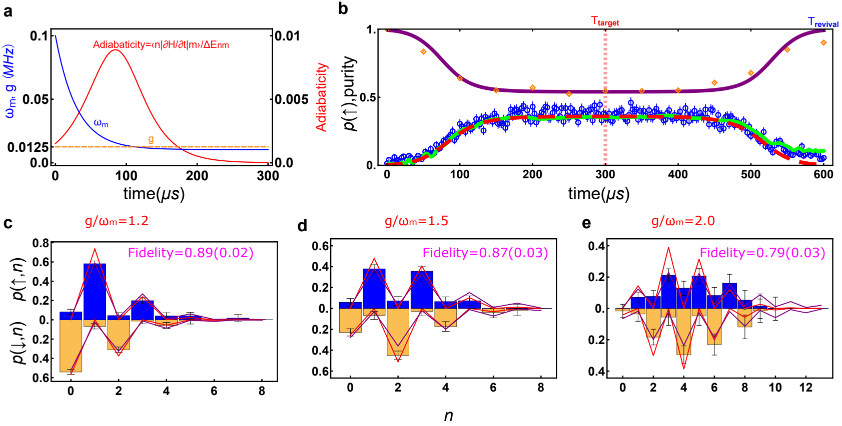
Spin and motion evolution at different coupling regime



Phonon bounce back and forth in the DSC regime for the degenerate $\omega_0 = 0$ and non-degenerate case $\omega_0 \neq 0$ case



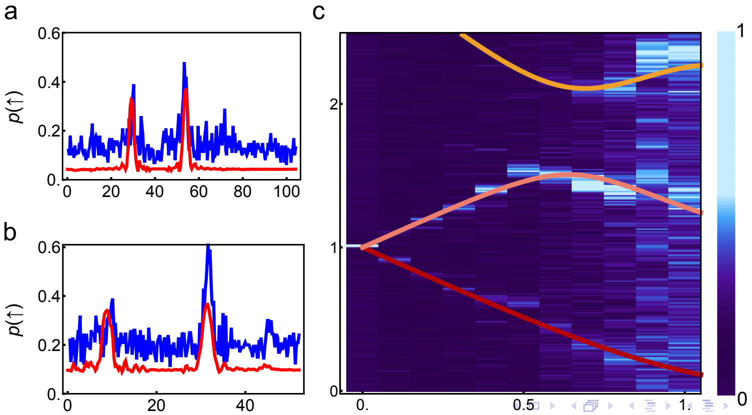
Adiabatically prepare the ground state in the DSC regime



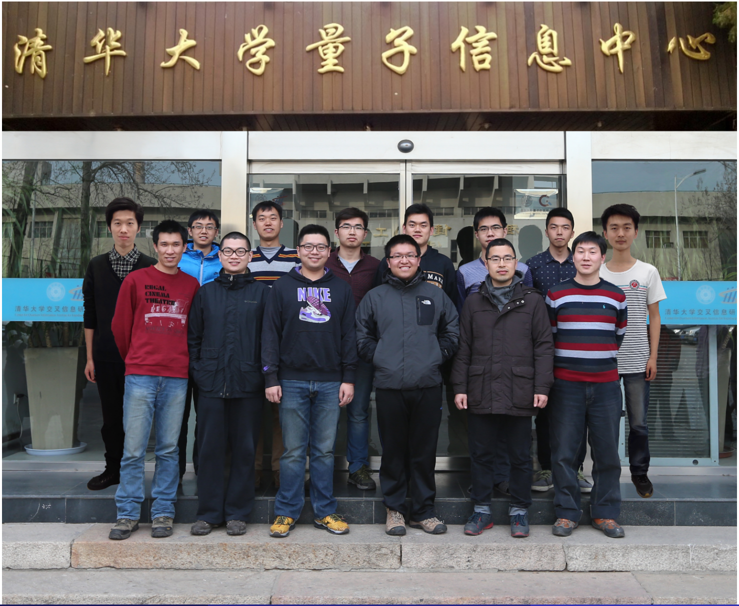
Add modulate field that break the parity

$$\hat{H}_{modulate} = \hat{H}_{QRM} + ggg * \hbar\Omega\hat{\sigma}_x \quad (12)$$

$$\hat{H}_{modulate} = \hat{H}_{QRM} + ggg * \hbar\Omega\eta(\hat{a} + \hat{a}^\dagger) \quad (13)$$



Group photos



Thanks for your attention!