A generalised Quantum-Slepian Wolf

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arXiv:1703.09961

September 8, 2017

Outline for section 1

1. The coding problem of Slepian and Wolf [1973]
2. Quantum case:
3. Techniques
4. Conclusion

The communication task

The communication task

$$p(x_1, y_1)p(x_2, y_2)\ldots p(x_n, y_n)$$

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$$x_1, x_2, \dots x_n$$

$$\uparrow$$

$$p(x_1, y_1)p(x_2, y_2)\dots p(x_n, y_n)$$

$$\downarrow$$

$$y_1, y_2, \dots y_n$$

The communication task

nR_A

$x_1, x_2, \ldots x_n$

$y_1, y_2, \ldots y_n$

nR_B

The rate region

- Slepian and Wolf [1973]: Task achievable if
  - $R_A \geq H(X|Y)$,
  - $R_B \geq H(Y|X)$,
  - $R_A + R_B \geq H(XY)$.
- Optimal.

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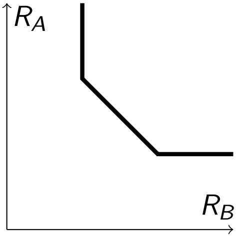

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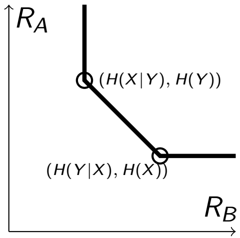
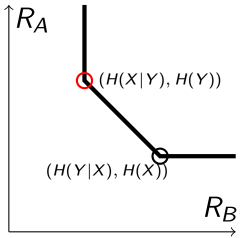
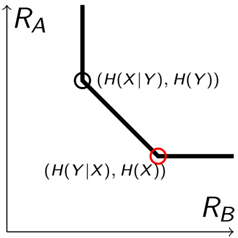


Approach 1: reduce to two-party task



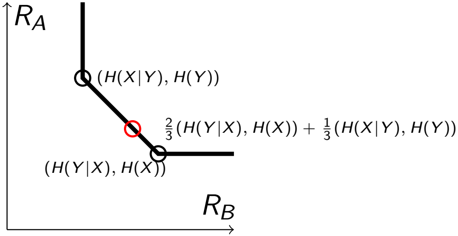
- To achieve red point:
  - Bob sends all of $y_1, y_2, \ldots y_n$ to Charlie with $nR_B = nH(Y)$.
  - Alice sends $x_1, x_2, \ldots x_n$ to Charlie with $nR_A = nH(X|Y)$.

Approach 1: reduce to two-party task



- To achieve red point:
  - Alice sends all of $x_1, x_2, \ldots x_n$ to Charlie with $nR_A = nH(X)$.
  - Bob sends $y_1, y_2, \ldots y_n$ to Charlie with $nR_B = nH(Y|X)$.

Approach 1: time sharing



- To achieve red point:
  - Alice and Bob group $(x_1, y_1), (x_2, y_2), \ldots (x_n, y_n)$ into $(x_1, y_1), (x_2, y_2), \ldots (x_{n/3}, y_{n/3})$ and $(x_{n/3+1}, y_{n/3+1}), (x_{n/3+2}, y_{n/3+2}), \ldots (x_n, y_n)$.
  - They follow first protocol on first group and second protocol on second group.

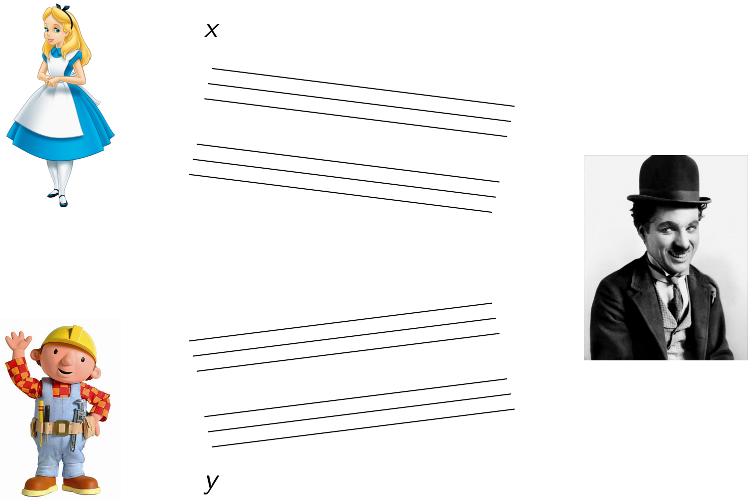
Approach 1: limitations

- Works only in the asymptotic, independent and identically distributed setting.
- Not suitable for following conceptually important settings:
  - One-shot setting: $n = 1$.
  - Asymptotic, non-i.i.d. setting: joint distribution $p(x_1, x_2, \ldots x_n, y_1, y_2, \ldots y_n)$.
  - Second order: more information about communication rate region.

Approach 2: Slepian and Wolf's protocol

- Although designed for asymptotic and i.i.d. setting, key ideas work in:
  - One-shot setting: $n = 1$, [Warsi 2016, Anshu, Jain and Warsi 2017].
  - Asymptotic, non-i.i.d. setting: joint distribution $p(x_1, x_2, \ldots x_n, y_1, y_2, \ldots y_n)$ [Han 2003].
  - Second order: more information about communication rate region. [Tan and Kosut, 2012].

Approach 2: a one-shot protocol



Approach 2: a one-shot protocol

x

$\#2^{r_A}$

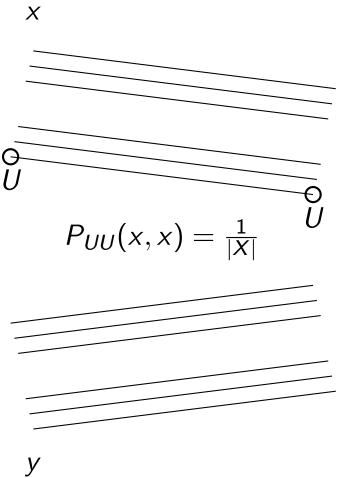
$\#$ blocks 2^{R_A}

$\#$ blocks 2^{R_B}

$\#2^{r_B}$

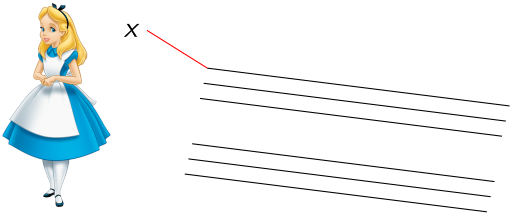
y

Approach 2: a one-shot protocol



$$P_{UU}(x, x) = \frac{1}{|X|}$$

Approach 2: Alice's strategy



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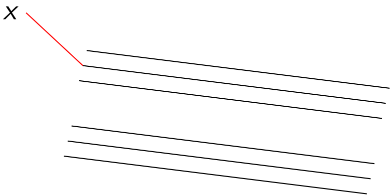

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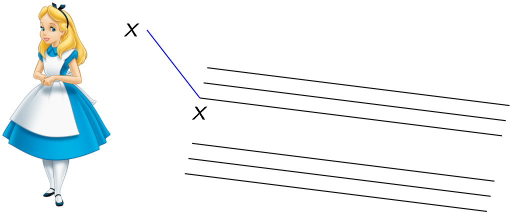

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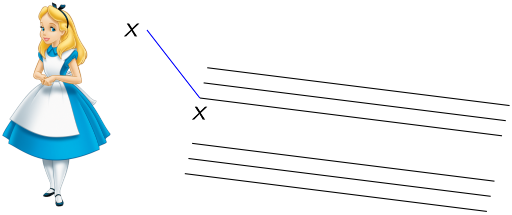
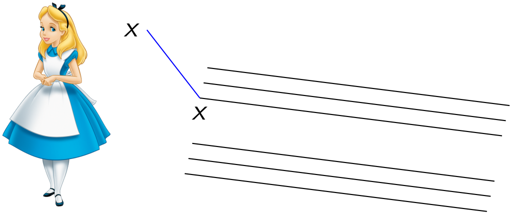


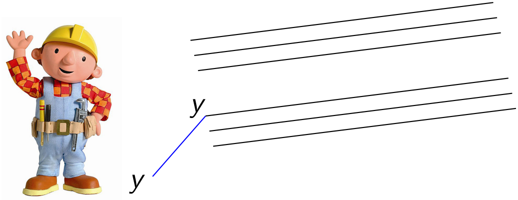
- Probability of finding no sample:
  - $(1 - \frac{1}{|X|})^{2^{R_A + r_A}}$.
  - $\approx \varepsilon$ if $2^{R_A + r_A} \geq |X|/\varepsilon$.

Approach 2: Alice's strategy



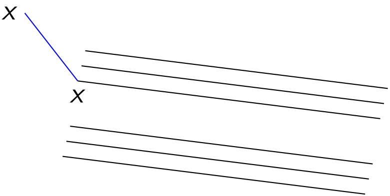
- Probability of finding no sample:
  - $(1 - \frac{1}{|X|})^{2^{R_A + r_A}}$.
  - $\approx \varepsilon$ if $2^{R_A + r_A} \geq |X|/\varepsilon$.
- So choose $R_A + r_A \geq \log |X| - \log \varepsilon$.

Approach 2: Bob's strategy



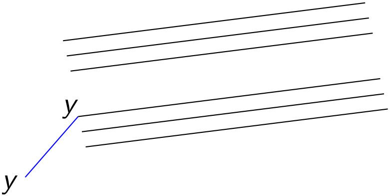
- Similar strategy for Bob.
- Choose $R_B + r_B \geq \log |Y| - \log \varepsilon$.

Approach 2: when both succeed



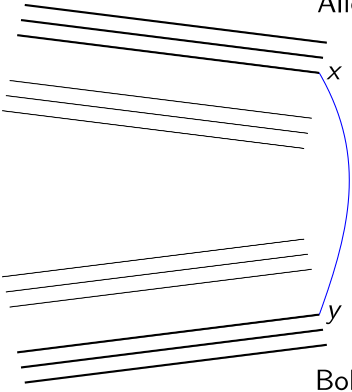
Alice tells: block # 5

Bob tells: block # 88



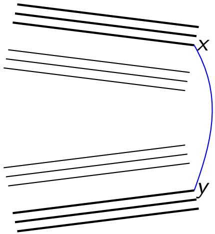
Approach 2: Charlie's strategy

Alice's message: block #5



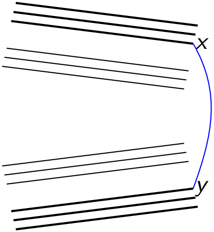
Bob's message: block #88

Approach 2: Charlie's strategy



- Charlie performs hypothesis testing.
- Succeeds with probability $1 - 3\varepsilon$ if
  - $r_A \leq \log|X| - H_0(X|Y)$,
  - $r_B \leq \log|Y| - H_0(Y|X)$,
  - $r_A + r_B \leq \log|X||Y| - H_0(XY)$.
- $H_0(.|.)$ is a one-shot version of $H(.|.)$.

Approach 2: Charlie's strategy



- Recall:
  - $R_A + r_A \geq \log|X| - \log \varepsilon$,
  - $R_B + r_B \geq \log|Y| - \log \varepsilon$,
- Implies the desired.

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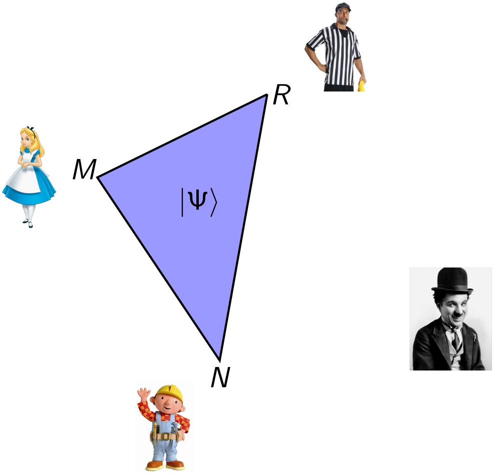
Quantum version of the problem



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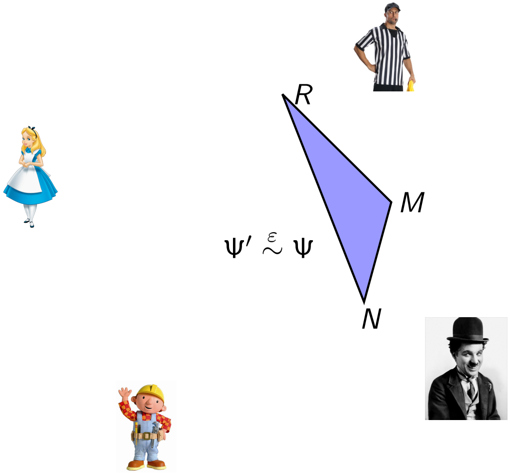

Prior work

- First considered in [Abeyesinghe, Devetak, Hayden, Winter, 2009] in asymptotic i.i.d. setting.
- Quantum state: $|\Psi\rangle_{RMN}^{\otimes n}$.

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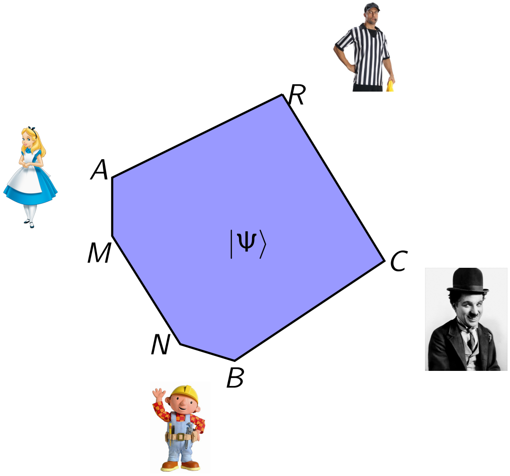
- Quantum state: $|\Psi\rangle_{RMN}^{\otimes n}$.
- Used time-sharing to reduce to Schumacher compression and quantum state merging.
- Rate region for the task: if Alice communicates nC_A qubits, Bob communicates nC_B qubits, then
  - $C_A \geq \frac{1}{2} I(R:M)$,
  - $C_B \geq \frac{1}{2} I(R:N)$,
  - $C_A + C_B \geq \frac{1}{2} I(R:M:N)$.

Prior work

- One shot version studied in [Dutil and Hayden, 2010]. Hence no time-sharing.
- Studied the entanglement cost in this task, rather than communication cost.
- Technique involved decoupling by random unitary.

A generalized Quantum Slepian-Wolf



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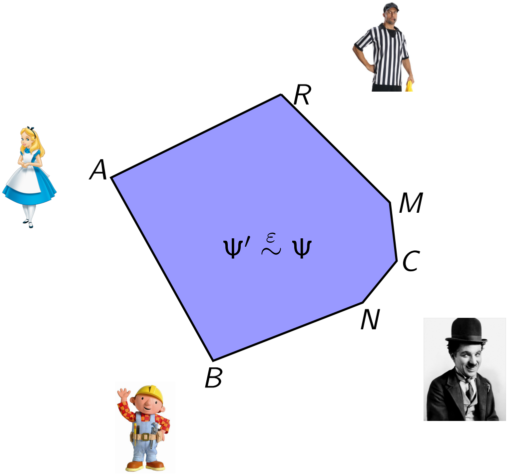

Our results

- One-shot rate region for communication required by Alice and Bob.
- Asymptotic i.i.d. analysis for:
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  - Registers N, B trivial. Recovers Quantum state redistribution [Devetak and Yard, 2009].

Outline for section 3

1 The coding problem of Slepian and Wolf [1973]

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Techniques

- Convex-split technique:
  - Introduced in [Anshu, Devabathini and Jain, 2014] as a coherent quantum version of rejection sampling.
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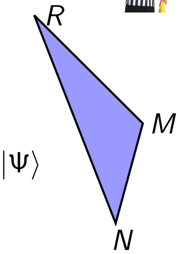
  - We use a bipartite version of the technique.
- Position-based decoding technique:
  - Introduced in [Anshu, Jain and Warsi, 2017] for entanglement assisted quantum channel coding. Used to obtain near-optimal one-shot code for point to point quantum channel.

Time-reversed problem



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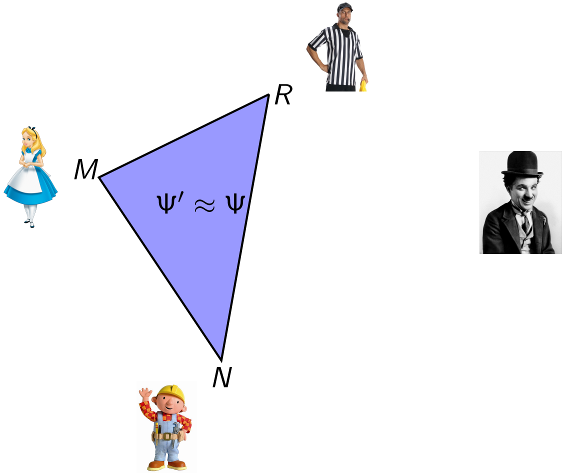
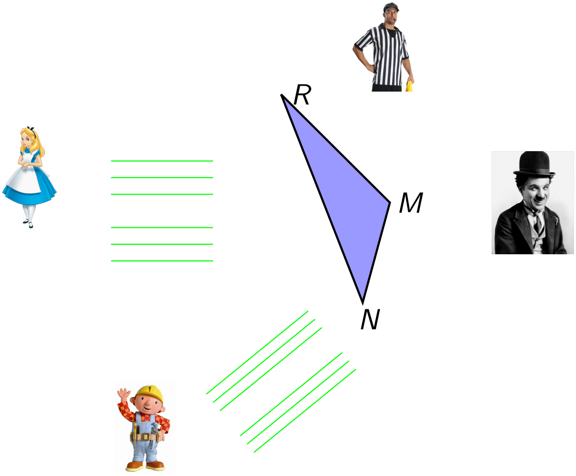


Shared entanglement



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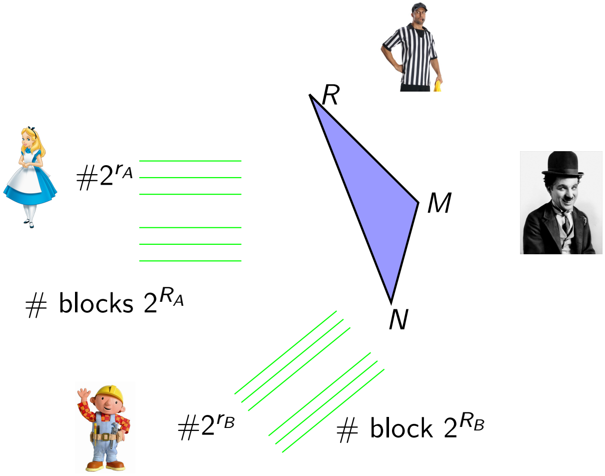
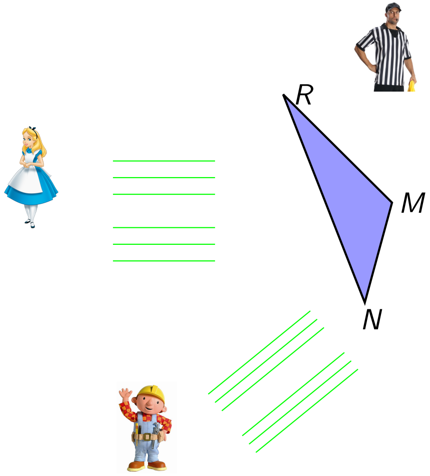
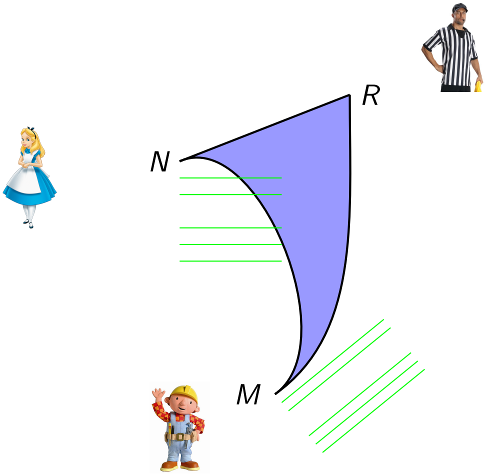


Actual state with Alice, Bob, Reference



Desired states with Alice, Bob, Reference



Sufficient condition for actual $\approx$ desired

- Bipartite convex-split lemma.
- Gives conditions for $R_A + r_A$, $R_B + r_B$ and $R_A + r_A + R_B + r_B$ so that Reference is randomly entangled with Alice and Bob.
- Charlie holds purification, hence he can steer the actual state to one of desired states.

Decoding by Alice and Bob

- Charlie only communicates partial information: block number.
- Alice and Bob use quantum hypothesis testing to obtain the correct location in the block [position-based decoding].

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- Obtained one-shot achievability results, and analysed its asymptotic i.i.d. property when Charlie has no side information (else if Bob has no registers: quantum state redistribution).
- Can we find nice asymptotic i.i.d. behaviour when Charlie has side information?
- Can we prove a tight converse for these tasks?