

# A semi-device-independent framework based on natural physical assumptions and its application to random number generation

T. Van Himbeeck, E. Woodhead, N. Cerf, R. García-Patrón, S. Pironio

arXiv:1612.06828

AQIS 2017 – 4-8 September 2017

# There exist vulnerabilities in quantum cryptography, successfully exploited by quantum hackers

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## Quantum hacking: Experimental demonstration of time-shift attack against practical quantum-key-distribution systems

[Y Zhao](#), [CHF Fung](#), [B Qi](#), [C Chen](#), [HK Lo](#) - Physical Review A, 2008 - APS

Abstract Quantum-key-distribution (QKD) systems can send quantum signals over more than 100 km standard optical fiber and are widely believed to be secure. Here, we show experimentally a technologically feasible attack—namely, the time-shift attack—against a

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## Quantum hacking of a continuous-variable quantum-key-distribution system using a wavelength attack

[JZ Huang](#), [C Weedbrook](#), [ZQ Yin](#), [S Wang](#), [HW Li](#)... - Physical Review A, 2013 - APS

Abstract The security proofs of continuous-variable quantum key distribution are based on the assumptions that the eavesdropper can neither act on the local oscillator nor control Bob's beam splitter. These assumptions may be invalid in practice due to potential

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## Hacking commercial quantum cryptography systems by tailored bright illumination

[L Lydersen](#), [C Wiechers](#), [C Wittmann](#), [D Elser](#)... - Nature ..., 2010 - nature.com

... For example, RSA public key cryptography has been subject to extensive scrutiny, which has led to the discovery of effective attacks based on implementation loopholes 25 . In our view, **quantum hacking** is an indication of the mature state of QKD rather than its insecurity. ...

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## Quantum hacking on quantum key distribution using homodyne detection

[JZ Huang](#), [S Kunz-Jacques](#), [P Jouquet](#), [C Weedbrook](#)... - Physical Review A, 2014 - APS

Abstract Imperfect devices in commercial quantum key distribution systems open security loopholes that an eavesdropper may exploit. An example of one such imperfection is the wavelength-dependent coupling ratio of the fiber beam splitter. Utilizing this loophole, the

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## Quantum eavesdropping without interception: an attack exploiting the dead time of single-photon detectors

[H Weier](#), [H Krauss](#), [M Rau](#), [M Fürst](#)... - New Journal of ..., 2011 - iopscience.iop.org

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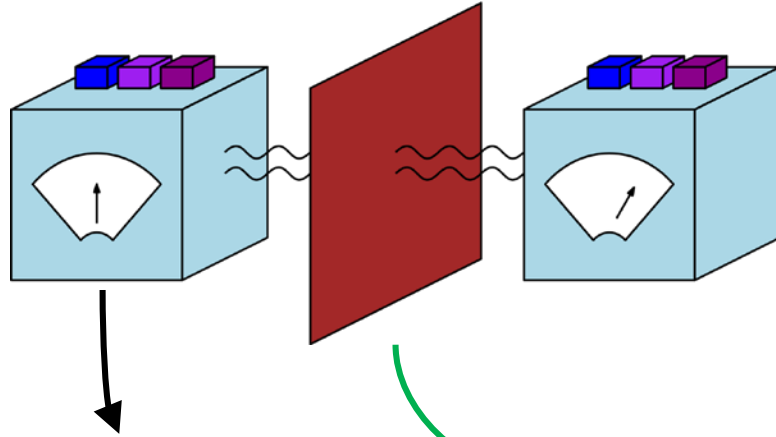
## [HTML] Optimised quantum hacking of superconducting nanowire single-photon detectors

[MG Tanner](#), [V Makarov](#), [RH Hadfield](#) - Optics express, 2014 - osapublishing.org

We explore bright-light control of superconducting nanowire single-photon detectors (SNSPDs) in the shunted configuration (a practical measure to avoid latching). In an

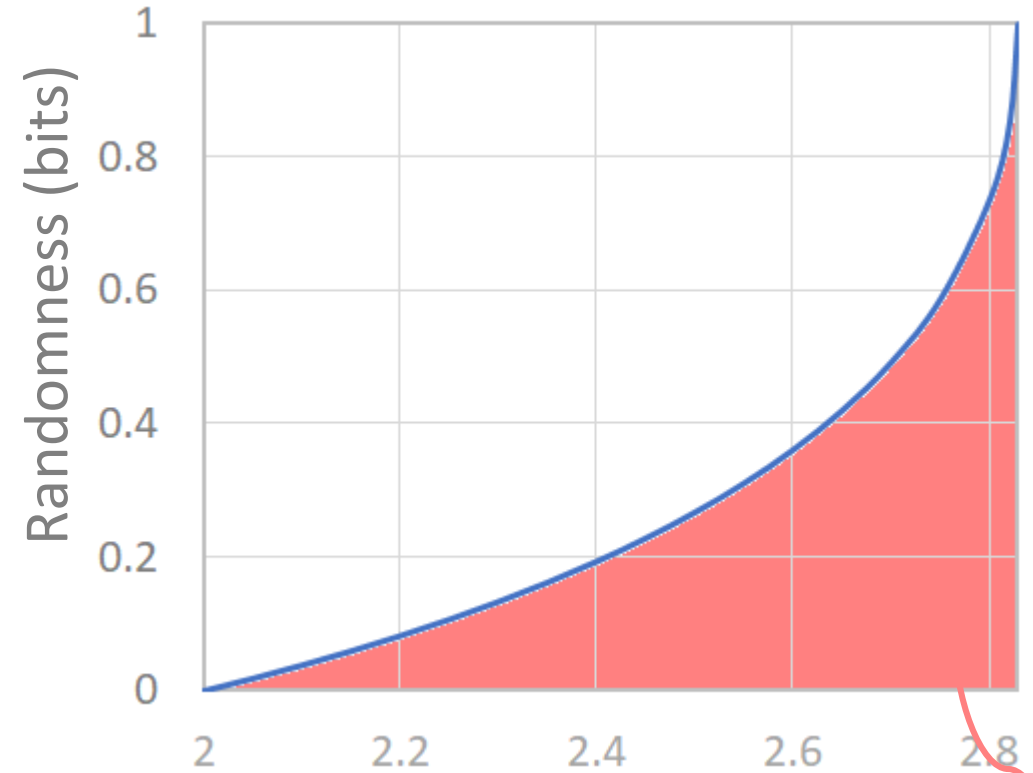
These attacks exploit a mismatch between the theoretical model used to prove security and the actual implementation

# Device-independent quantum cryptography



Devices viewed as black boxes

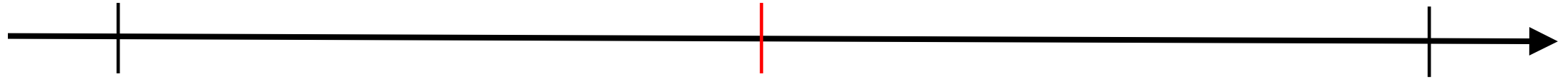
A single natural physical assumption



Bell inequality violation

Region not allowed by quantum theory  
if devices do not communicate

This approach can be used to certify the security of RNG and QKD protocols, or even the performance of quantum computers.



Usual, “device-dependent”  
quantum cryptography

Based on a detailed  
characterization of the  
devices

Semi-device-independent  
quantum cryptography

Based on a few assumptions.  
Devices are partly untrusted.

E.g.:

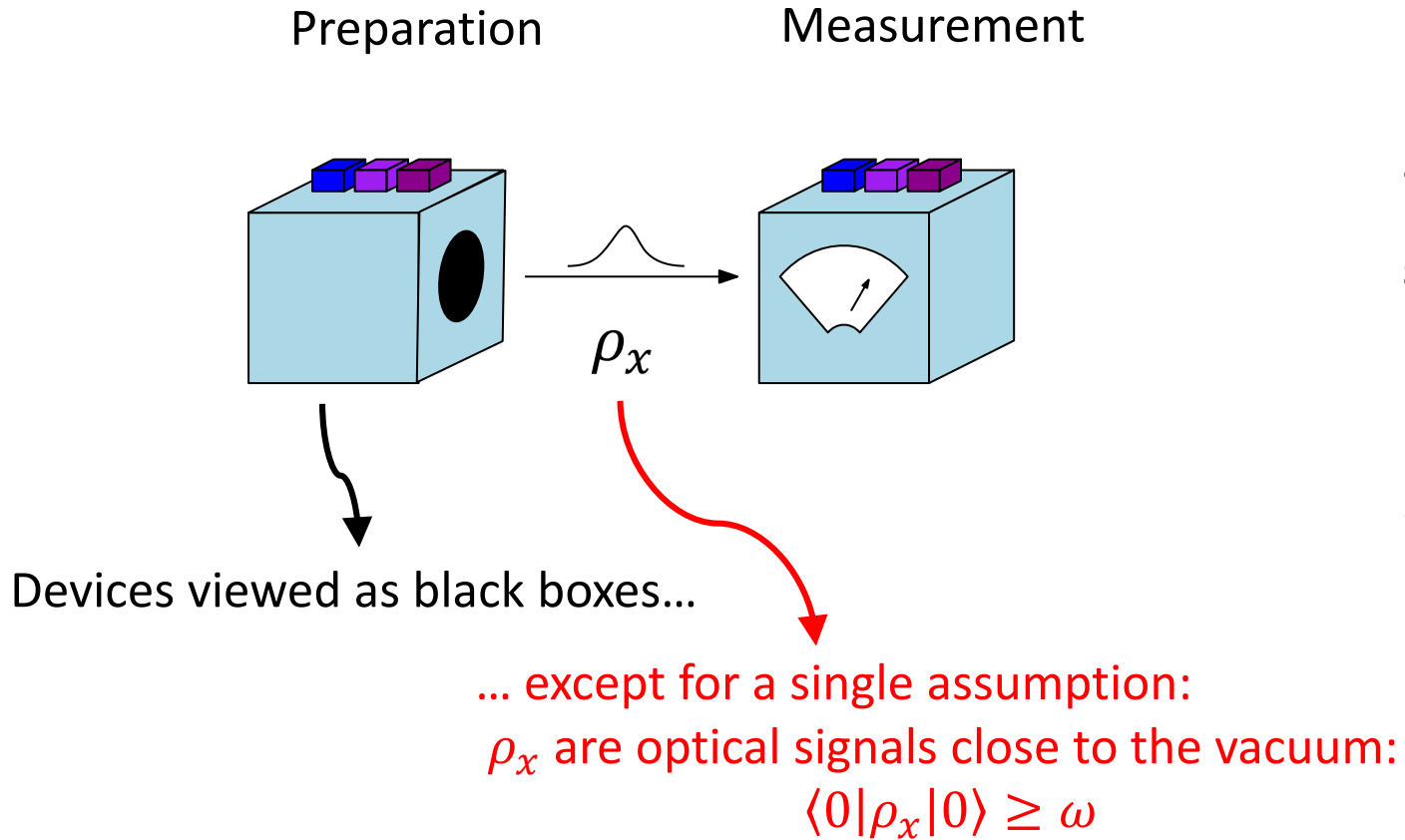
- Measurement-device-independence
- One-sided quantum cryptography
- Source-independent QRNG
- Qubit assumption
- Source & measurement independence
- ...

Fully “device-independent”  
quantum cryptography

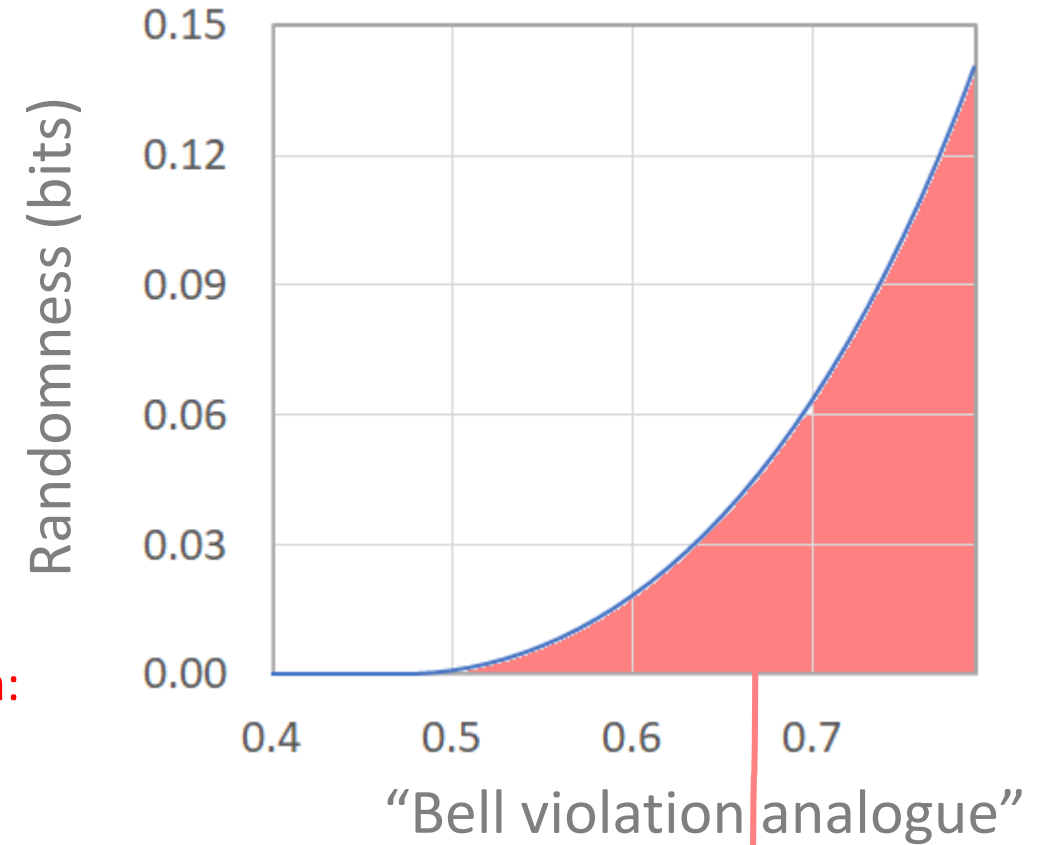
Based on minimal assumptions.  
Devices can be untrusted.

Advantage: higher rate, easier to implement  
than fully device-independent protocols

# Semi-device-independent protocols based on an energy constraint

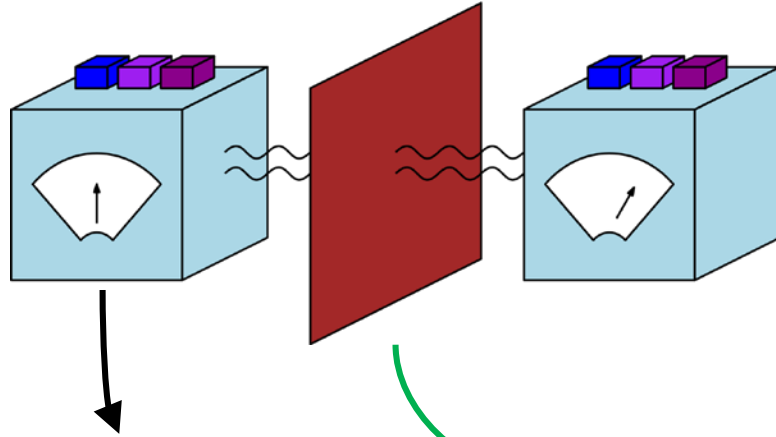


This assumption is sufficient to guarantee that devices behave in a genuinely quantum way.  
In particular, it allows for secure RNG protocols.  
Hopefully, it can also be used for QKD



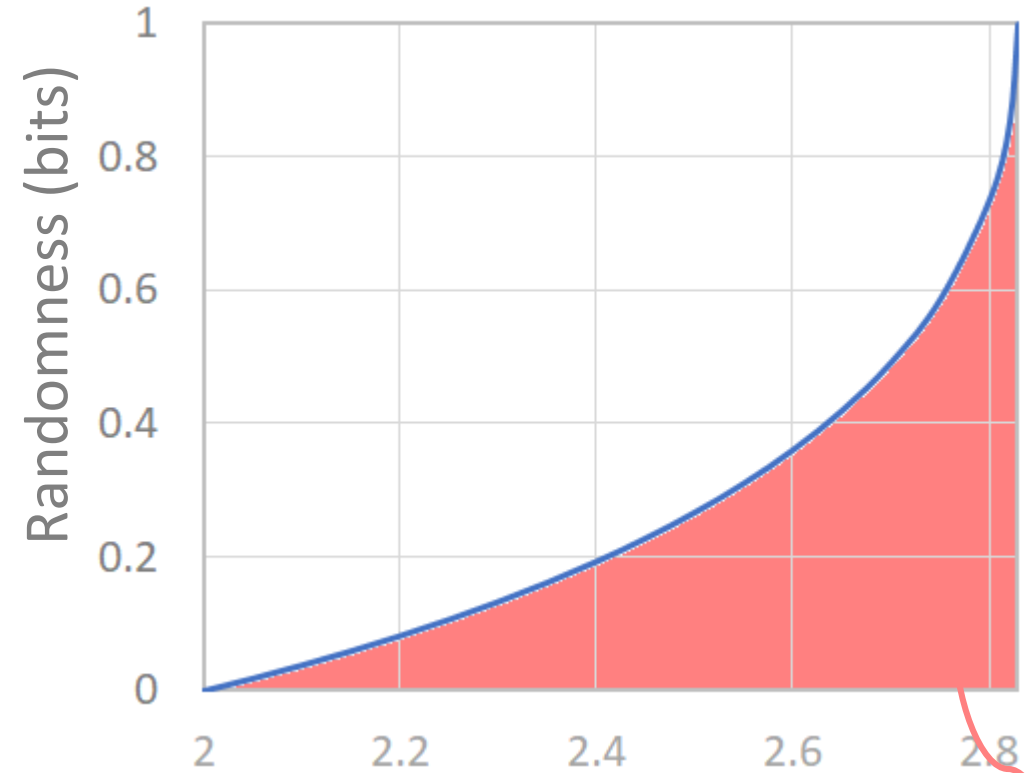
Region not allowed by quantum theory  
if prepared states satisfy the energy constraint

# Device-independent quantum cryptography



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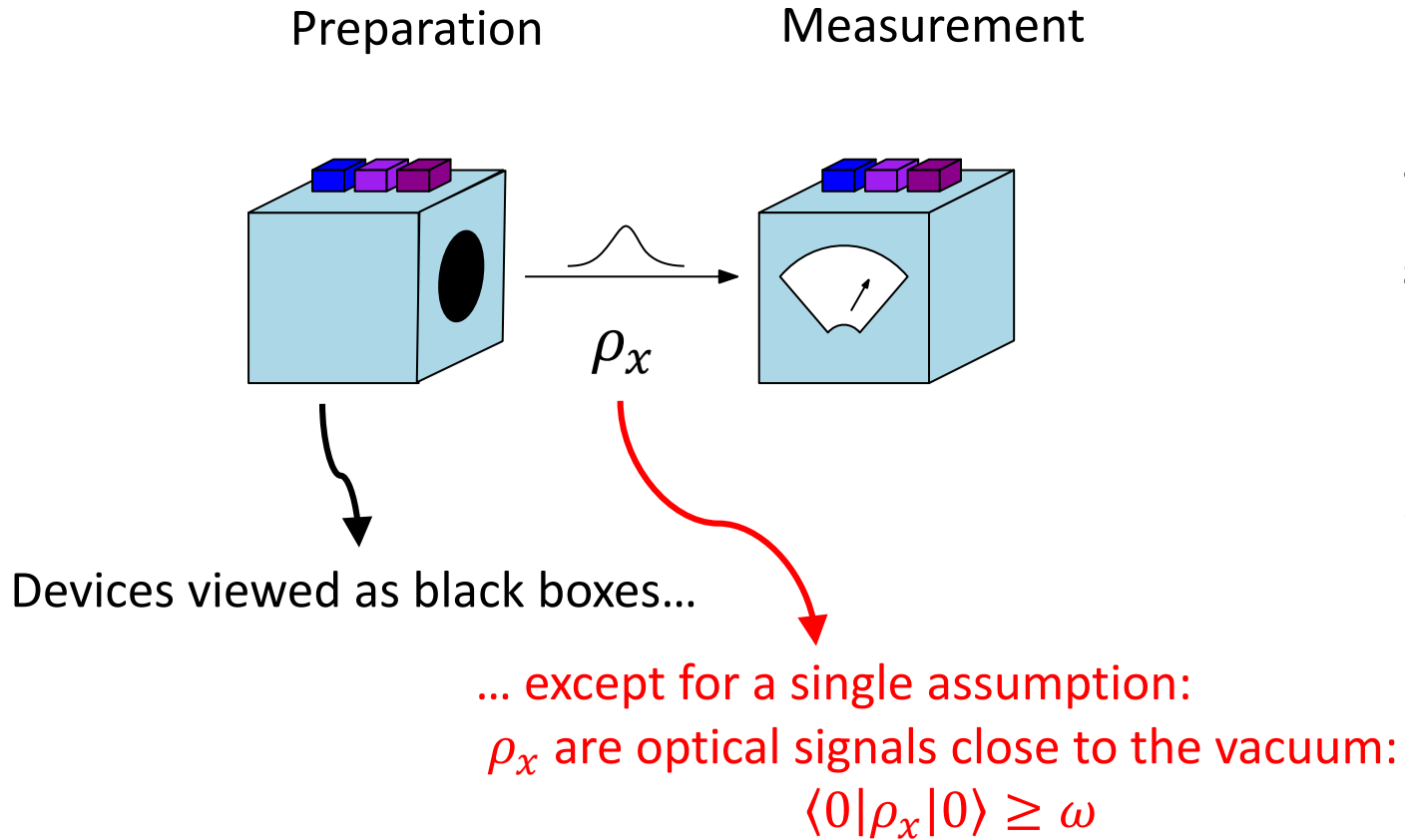


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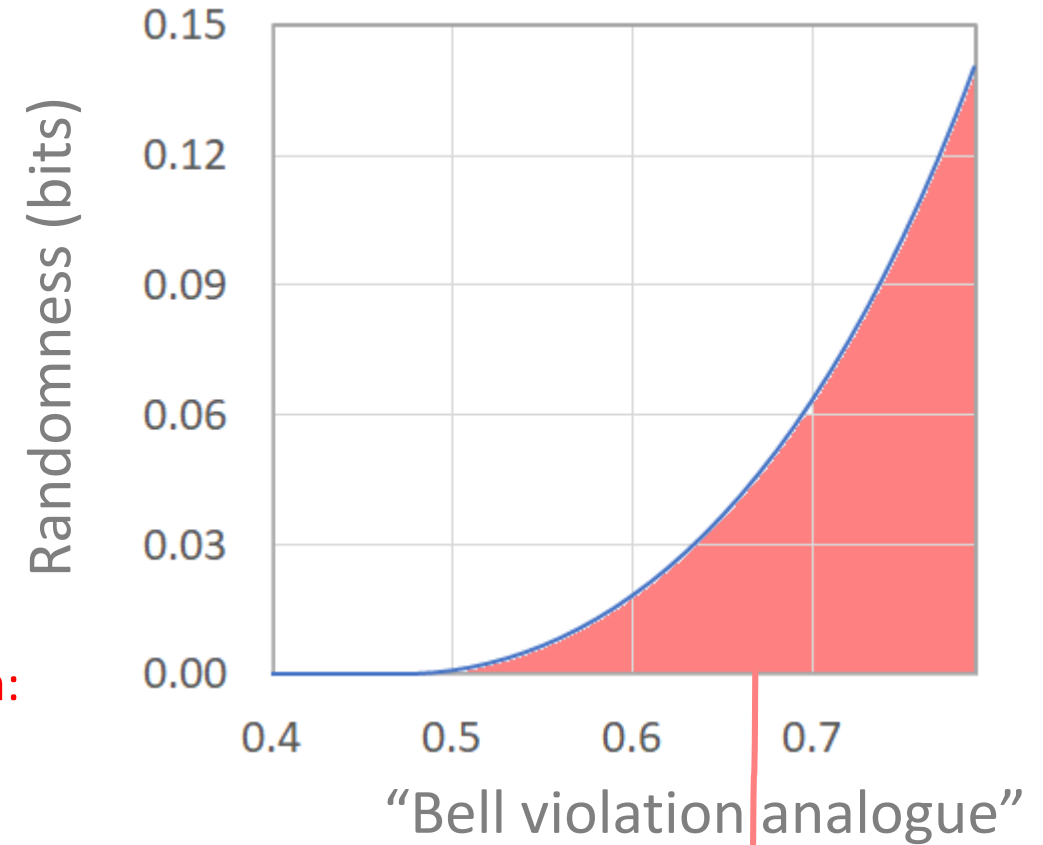
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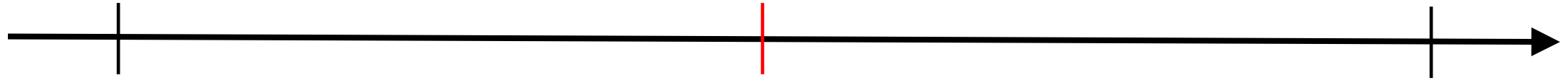
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Based on a detailed  
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Based on a few assumptions.  
Devices are partly untrusted.

E.g.:

- Measurement-device-independence
- One-sided quantum cryptography
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- Source & measurement independence
- ...
- Energy constraint

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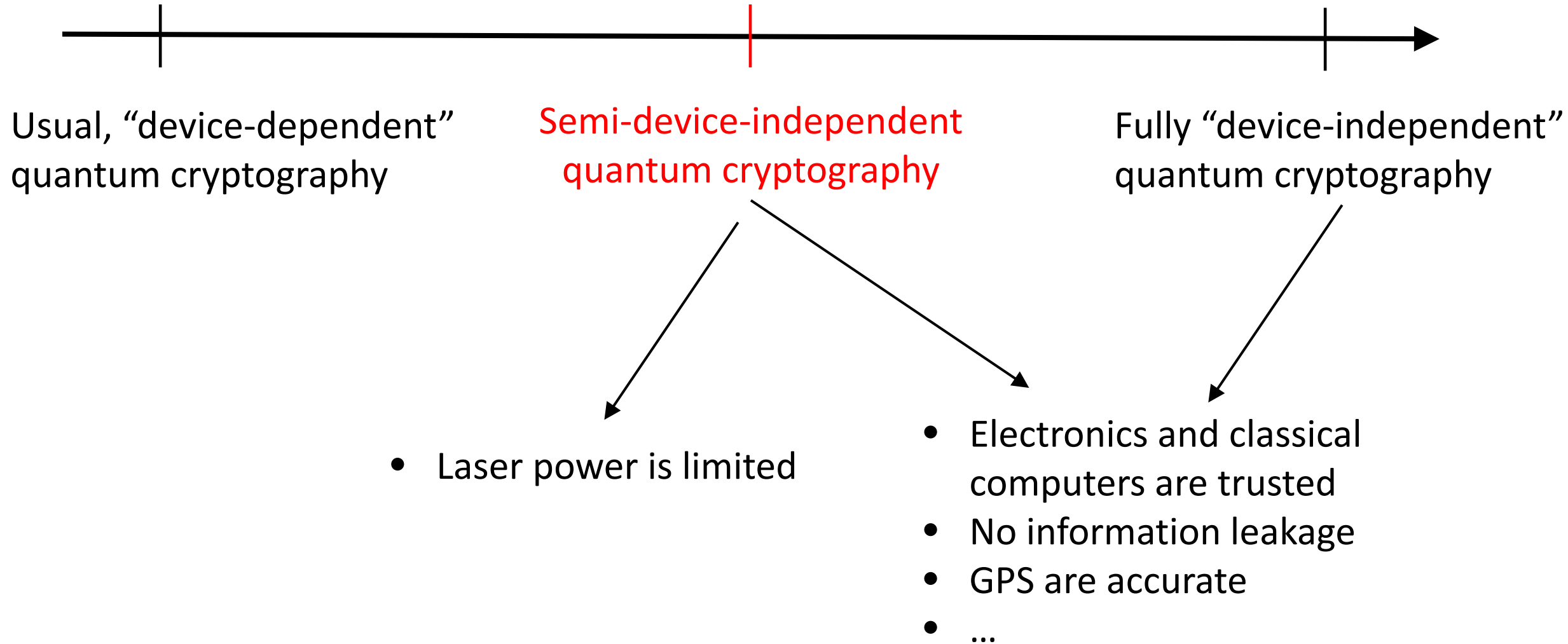
# Outline

- Why semi-device-independent quantum cryptography?
- Motivation for our energy constraint assumption
- Results

# Outline

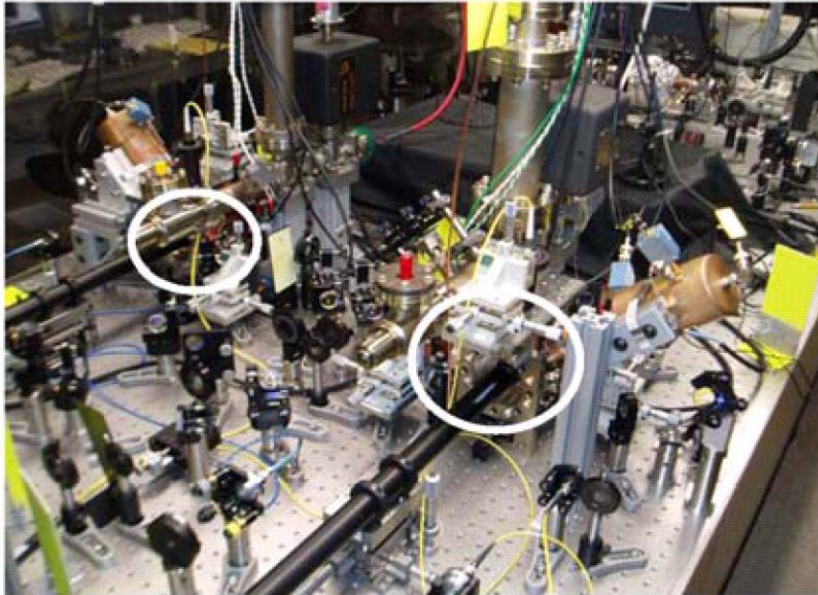
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# Even full DI requires non-trivial assumptions

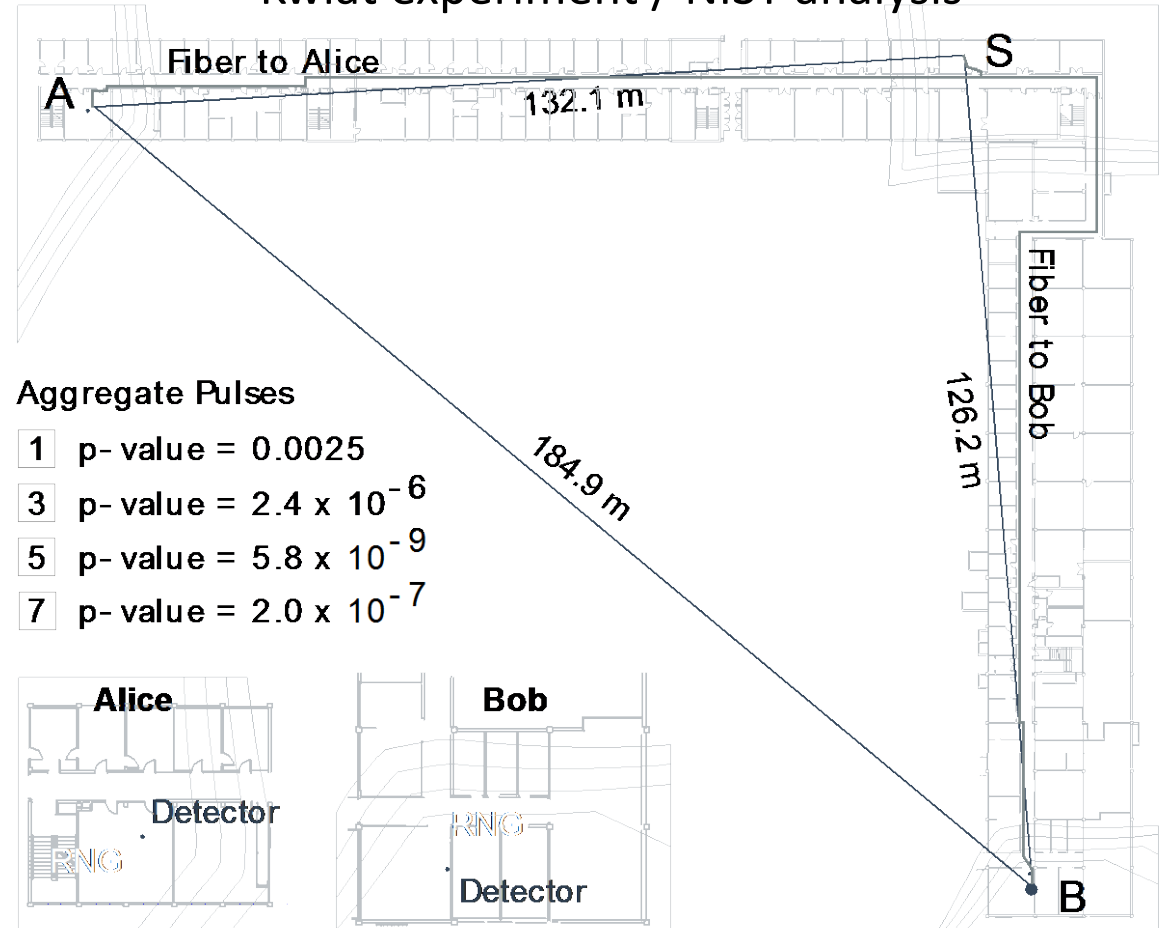


# DI RNG implementations

Monroe experiment



Kwiat experiment / NIST analysis



we extracted 256 bits, certified to be uniform to within 0.001.

in [18], which is titled “XOR 3” and consists of a total of 182,161,215 trials, acquired in 30 min of running the experiment, improving on the approximately one month duration of data

Experimental  
requirements

Device-independent



Device-dependent

Assumptions



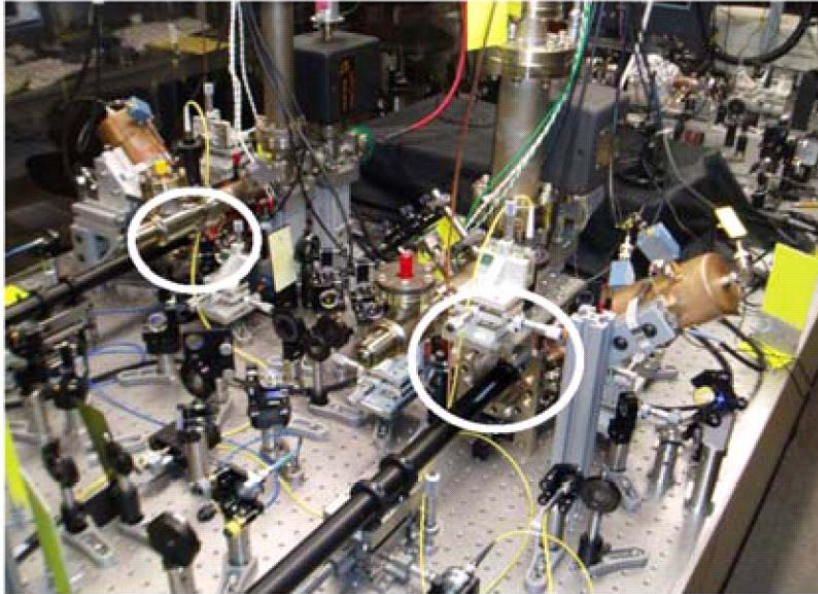
Don't waste time developing cars:  
in the future planes will be easy to  
build, common, and affordable.



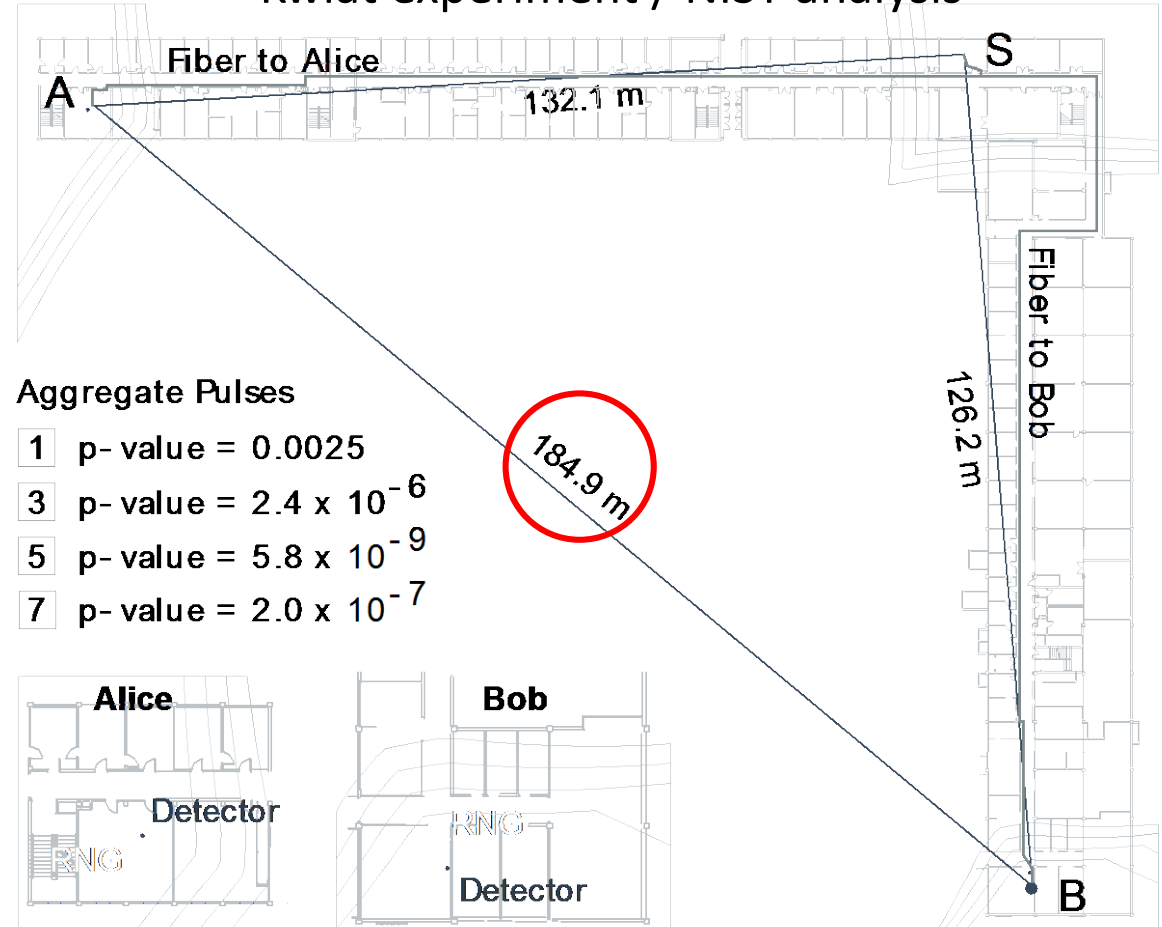


# DI RNG implementations

Monroe experiment



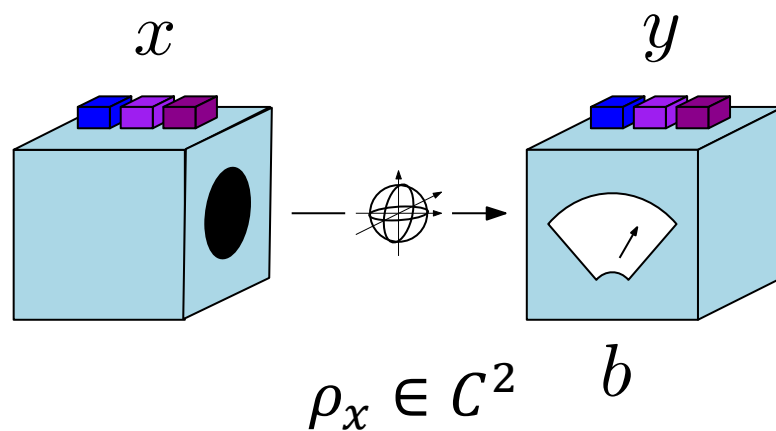
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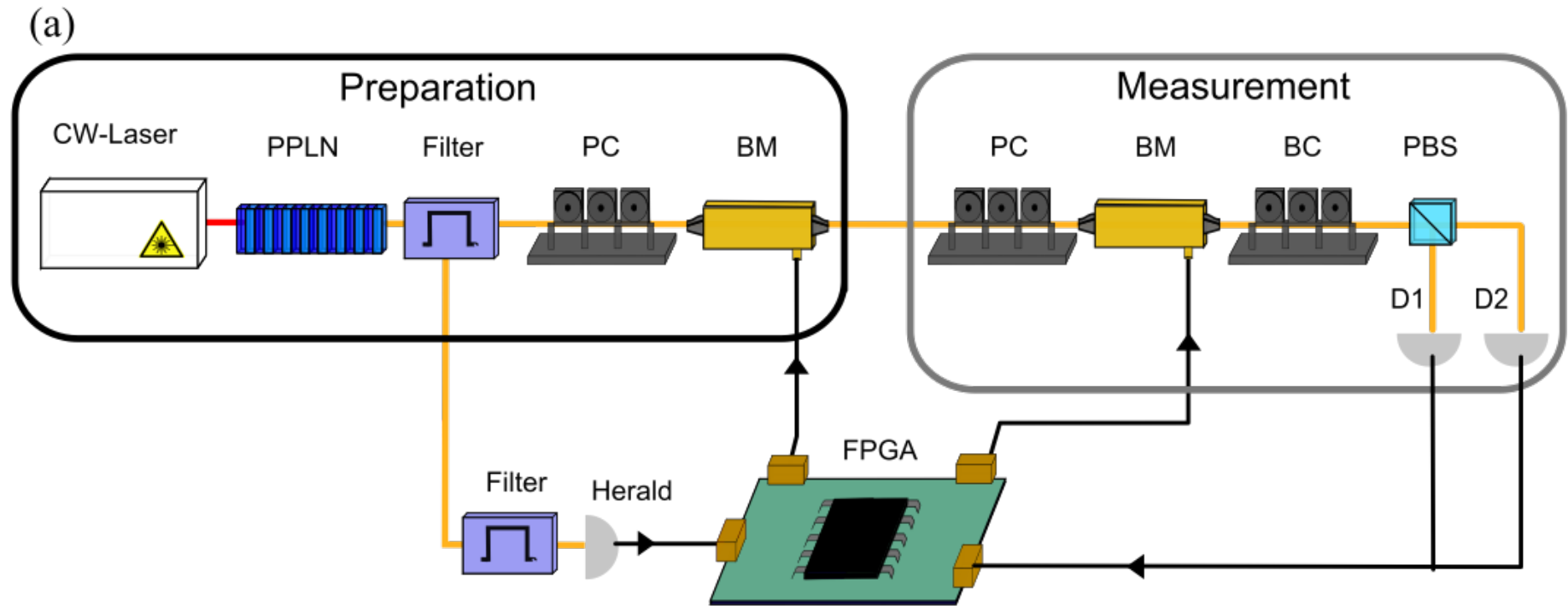
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# Semi-DI protocols based on a qubit assumption



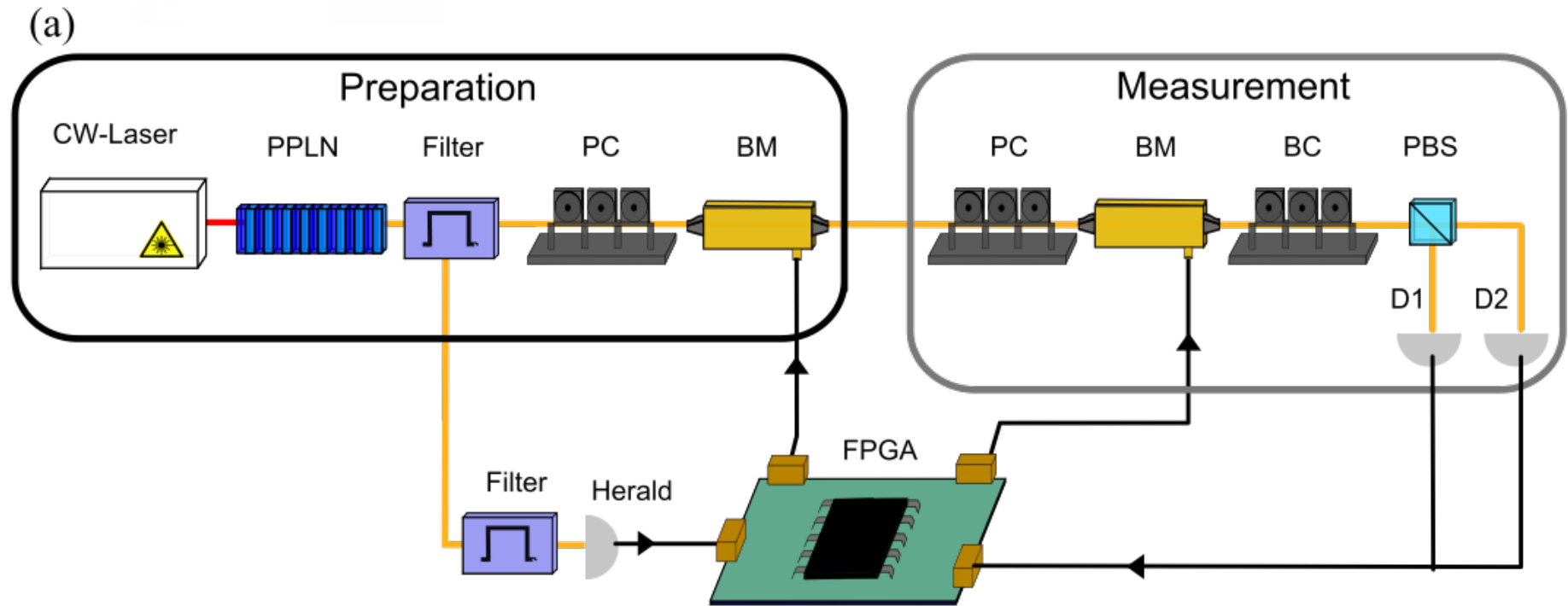


# Semi-DI protocols based on a qubit assumption





Mismatch between model used for security proof and implementation!



Qubit assumption is an idealization.  
→ Shows that it is important to choose well the assumptions.

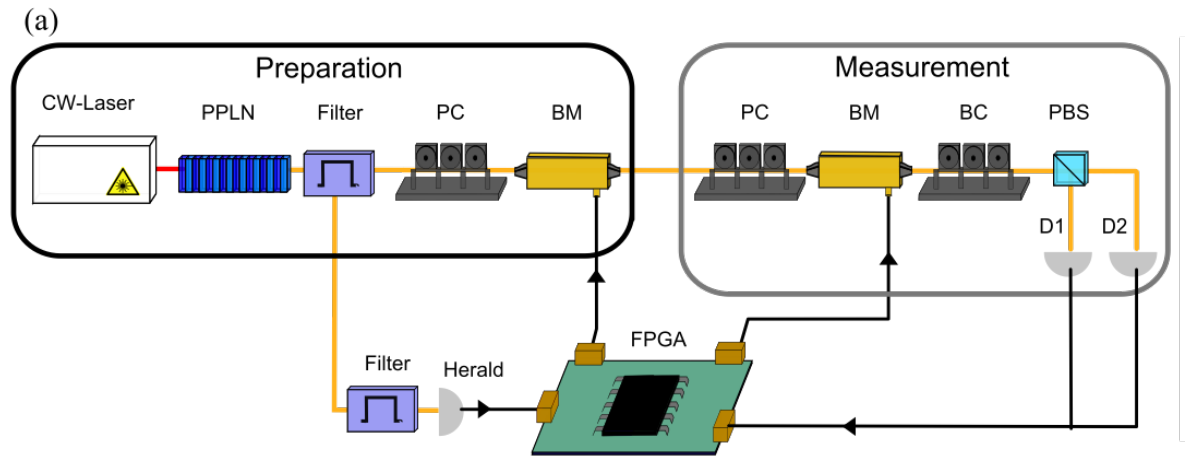
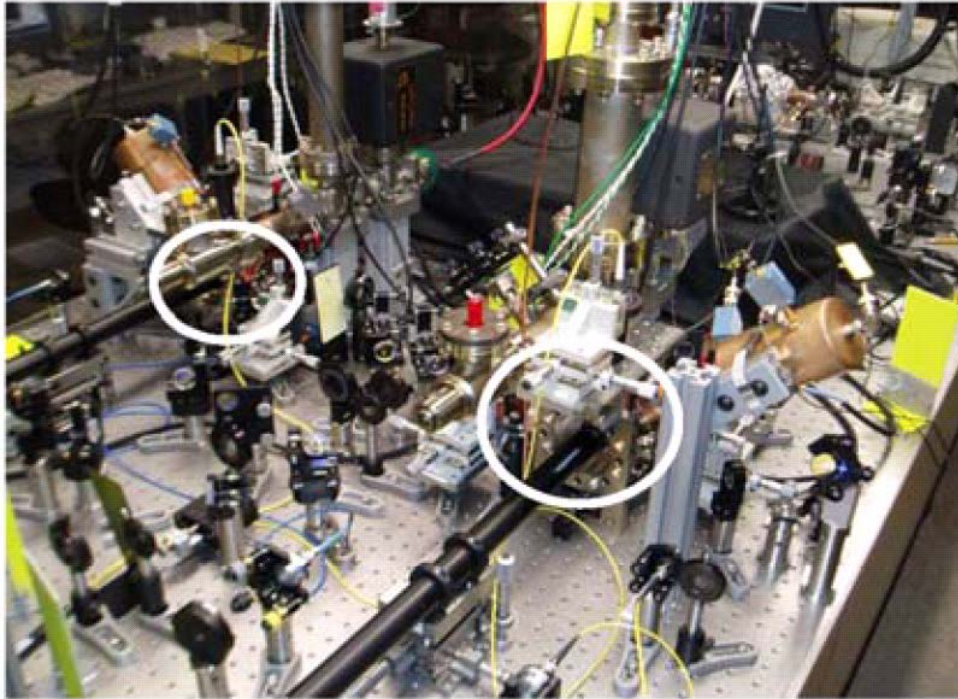




$$\alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle + \dots$$



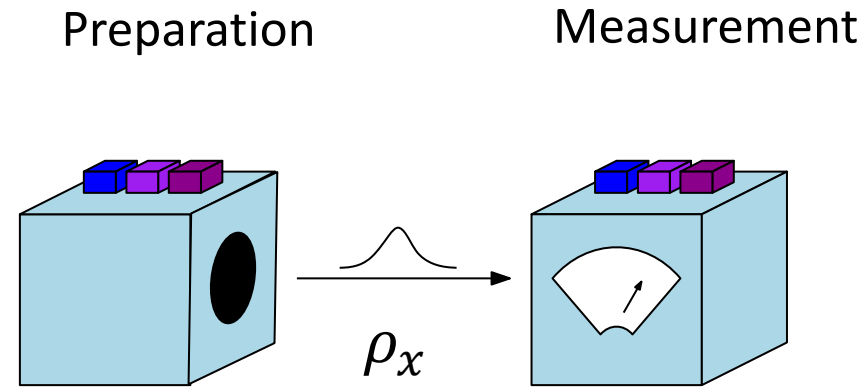
Click with prob  $1 - \alpha^2$   
No-click with prob  $\alpha^2$



# Outline

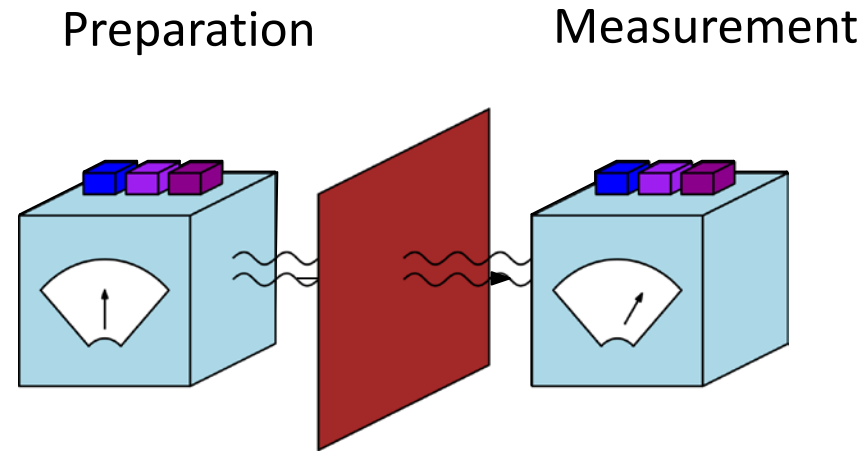
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# Semi-device-independent protocols based on an energy constraint



Assumption:  $\langle 0 | \rho_x | 0 \rangle \geq \omega_x$

# Semi-device-independent protocols based on an energy constraint

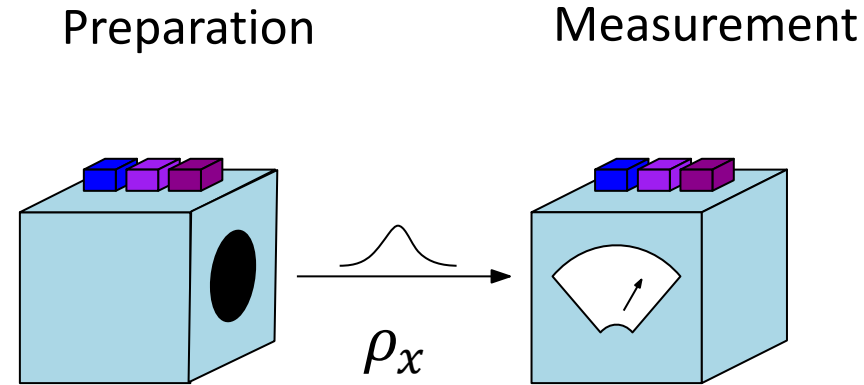


Assumption: no-communication

$$\Leftrightarrow \langle 0 | \rho_x | 0 \rangle = 1$$



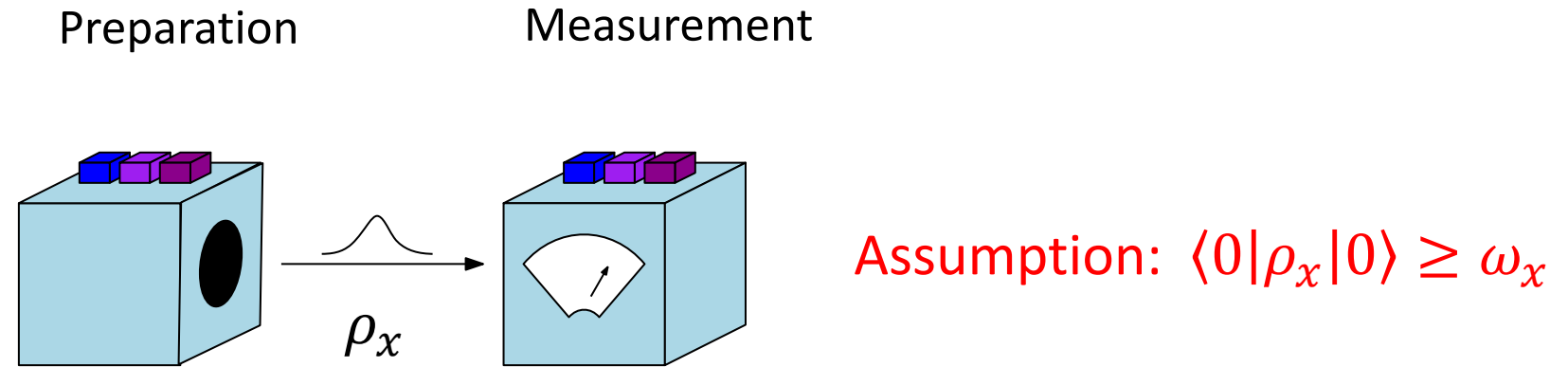
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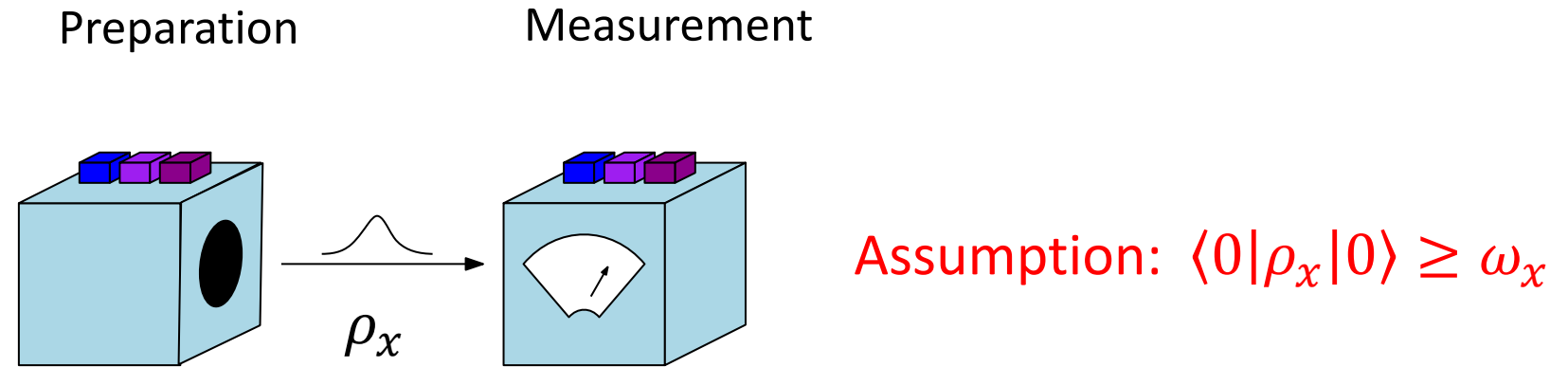
- Natural relaxation of the no-communication assumption of full DI protocols

# Semi-device-independent protocols based on an energy constraint



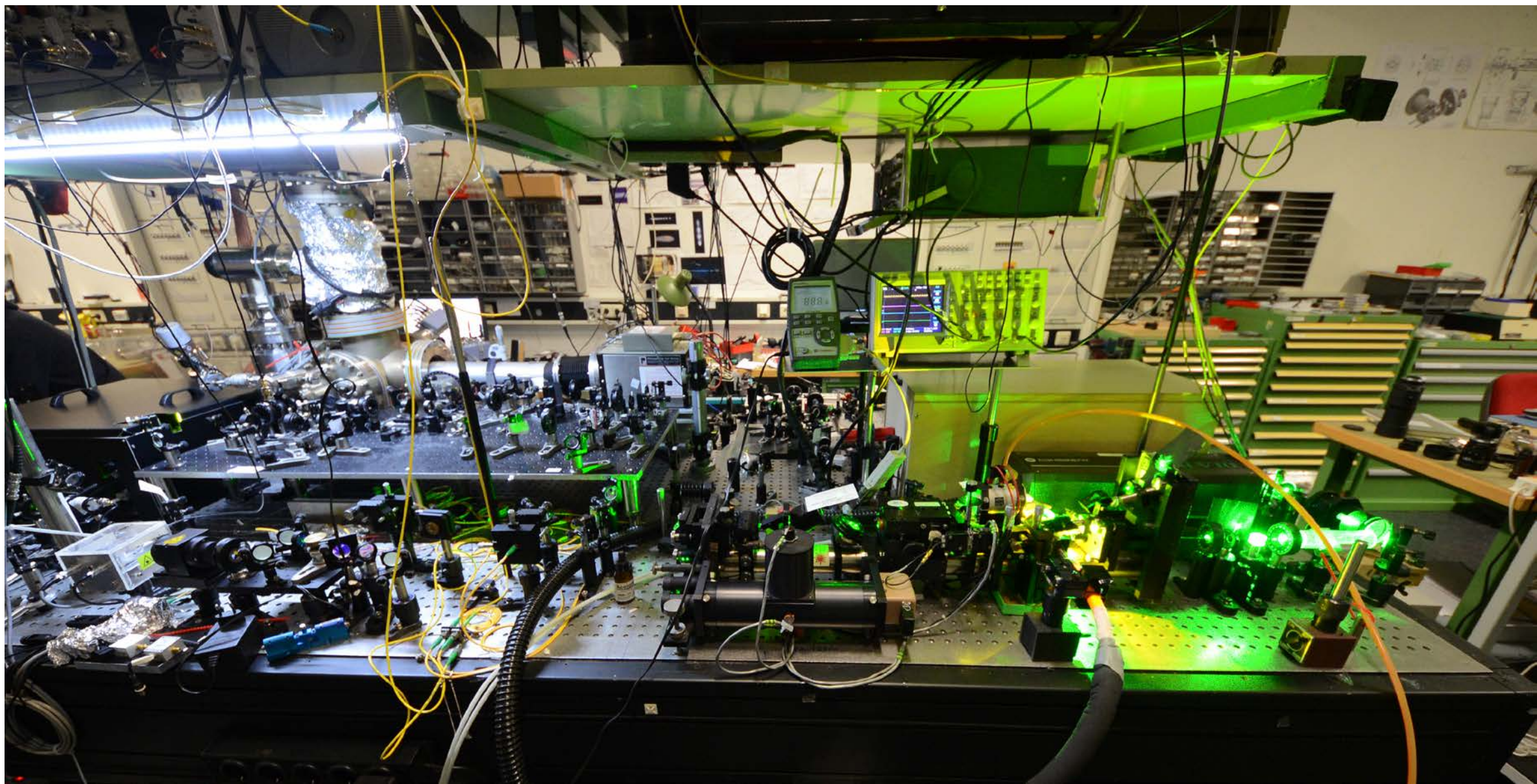
- Natural relaxation of the no-communication assumption of full DI protocols
- The appropriate space to describe quantum optics experiments is the Fock space of several quantum optical modes. In this context, it is natural to bound the average number of photons.
- This is an assumption anyway made in many quantum optics experiments in which attenuated laser sources are used.

# Semi-device-independent protocols based on an energy constraint

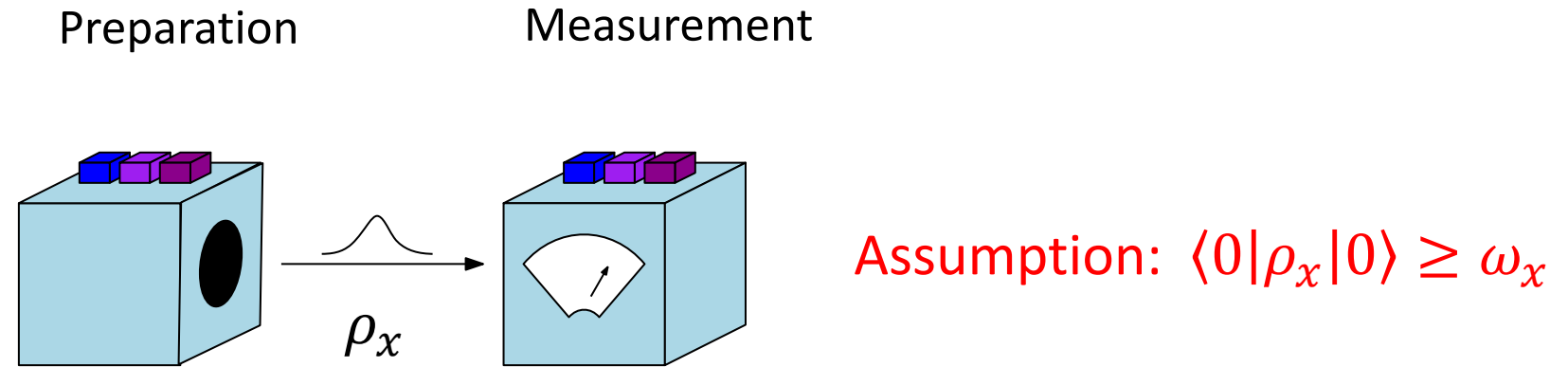


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- It is directly related to simple characteristics of the device components (laser power, attenuator) and robust to device imperfections.





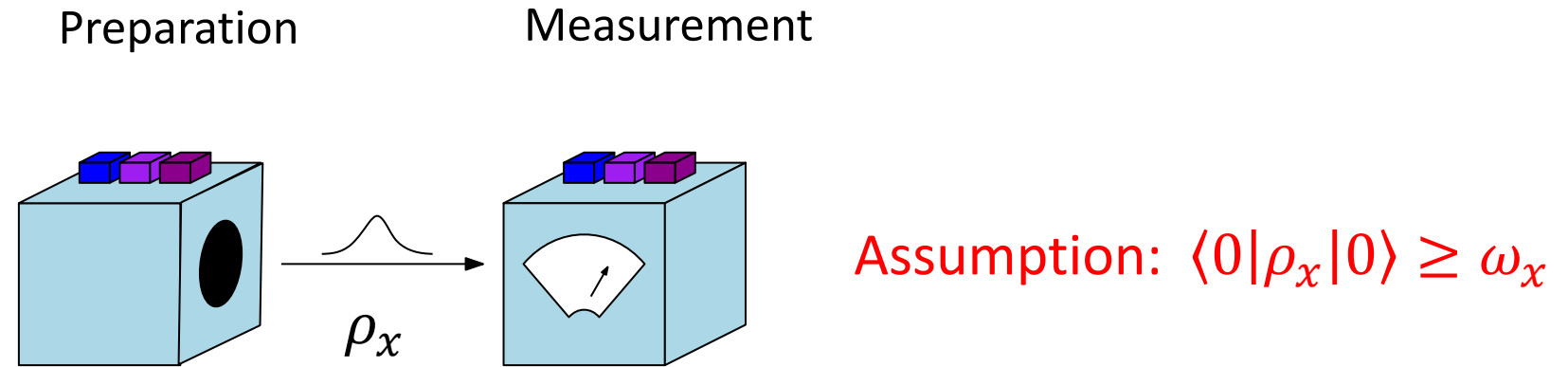
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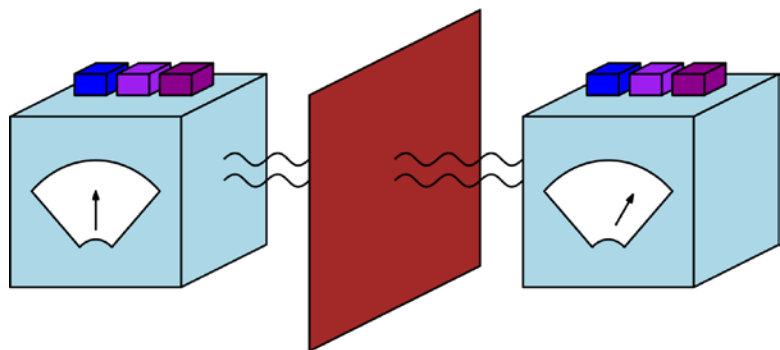


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- This is an assumption anyway made in many quantum optics experiments in which attenuated laser sources are used.
- It is directly related to simple characteristics of the device components (laser power, attenuator) and robust to device imperfections.
- It could be directly monitored (calibrated power meter) or enforced (optical fuse).

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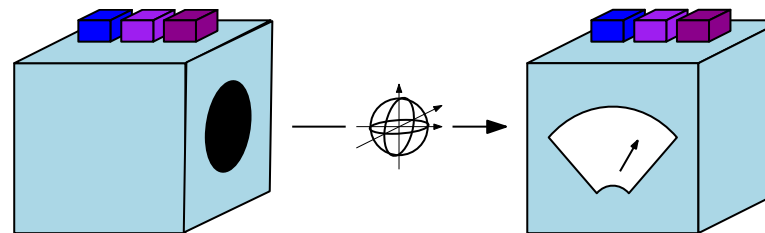
No-communication assumption



Violation of Bell inequalities

$$Q > C$$

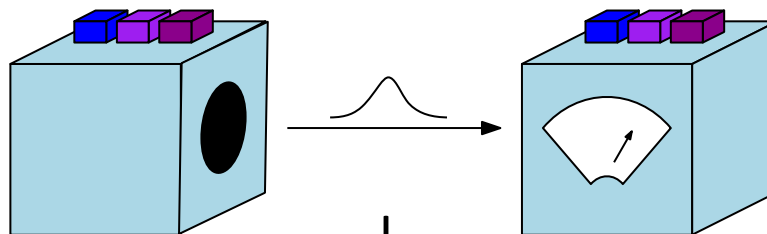
Qubit assumption



Violation of “dimension witnesses”

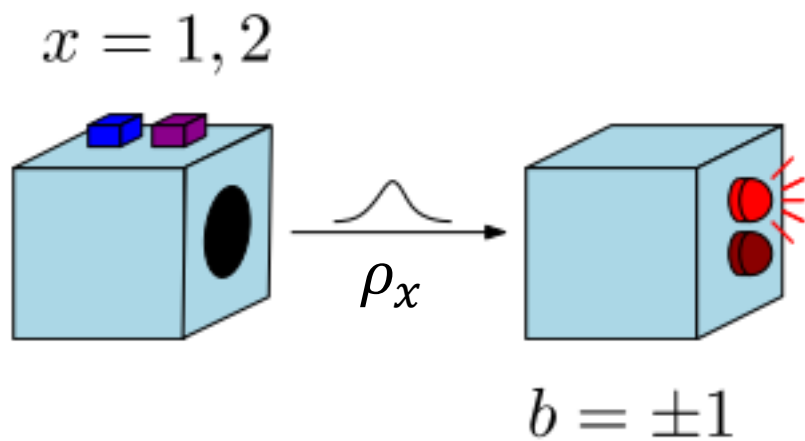
$$Q > C$$

Energy constraint assumption



$$Q > C ??$$





### Input-output statistics

$$P(b|x) = \text{Tr}[\rho_x M_b] \quad \text{or} \quad P(b|x) = \sum_{\lambda} p_{\lambda} \text{Tr}[\rho_x^{\lambda} M_b^{\lambda}]$$

Equivalent to knowledge of the bias of  $b$  given  $x$ :

$$E_x = P(b = 1|x) - P(b = -1|x)$$

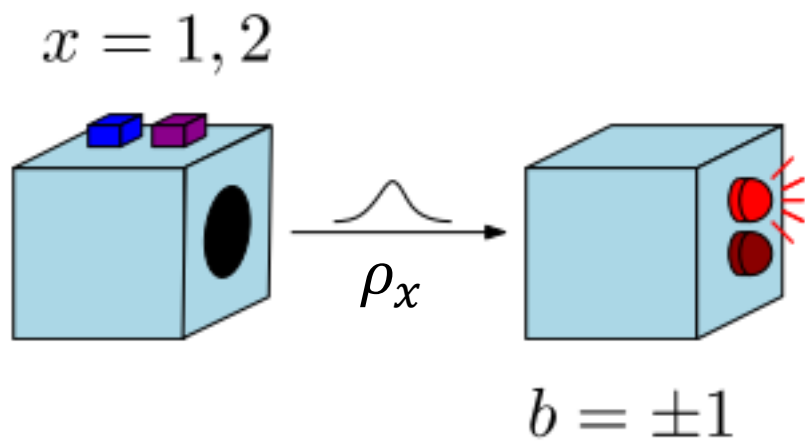
$$E_x = \text{Tr}[\rho_x M] \quad \text{or} \quad E_x = \sum_{\lambda} p_{\lambda} \text{Tr}[\rho_x^{\lambda} M^{\lambda}]$$

Output of devices is non-trivial if  $b$  is correlated to  $x$   
 Amount of correlations can be measured by quantity

$$E_- = (E_1 - E_2)/2$$

Probability to guess correctly  $x$  given  $b$  is  $\frac{1}{2} + \frac{|E_-|}{2}$

- $b$  does not depend on  $x$ :  $E_- = 0$
- $b$  fully correlated to  $x$ :  $|E_-| = 1$



### Assumption

Energy constraint:

$$\langle 0 | \sum_{\lambda} p_{\lambda} \rho_x^{\lambda} | 0 \rangle \geq w_x$$

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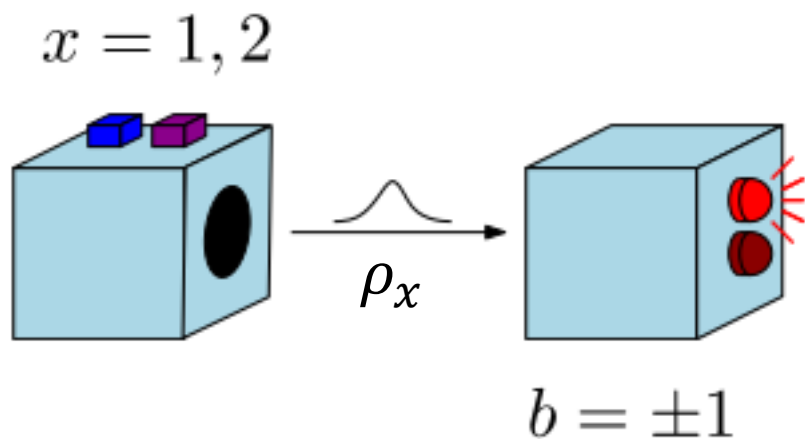
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or

$$\sum_{\lambda} p_{\lambda} \text{Tr}[H \rho_x^{\lambda}] \leq 1 - w_x$$

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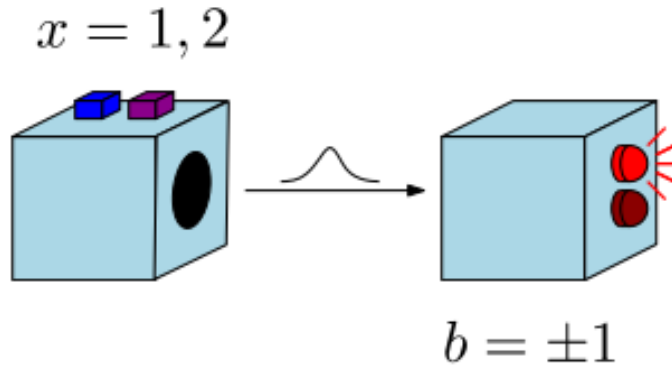
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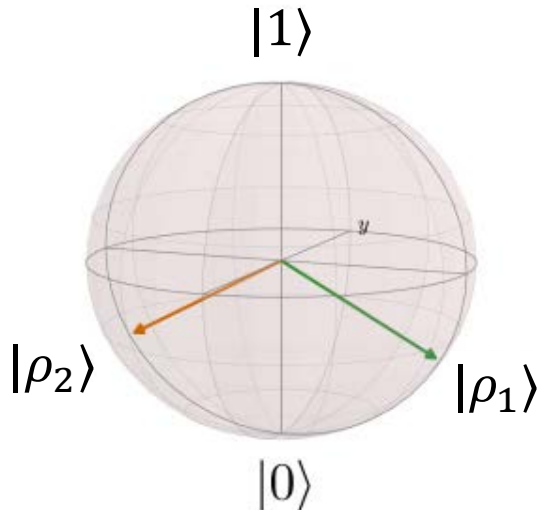
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Maximal value for  $E_-$  given  $w_1 = w_2 = w$ ?



- $w = 1 \rightarrow E_- = (E_1 - E_2)/2 = 0$
- $w = 1/2 \rightarrow |\rho_{1,2}\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$   
 $\rightarrow E_- = (E_1 - E_2)/2 = 1$
- $\frac{1}{2} \leq w \leq 1$  arbitrary  
 $|\rho_1\rangle = \sqrt{w}|0\rangle + \sqrt{1-w}|\phi_1\rangle$   
 $|\rho_2\rangle = \sqrt{w}|0\rangle + \sqrt{1-w}|\phi_2\rangle$



Scalar product minimal if  $|\phi_1\rangle = -|\phi_2\rangle = |1\rangle$   
 $\Rightarrow |\rho_{1,2}\rangle = \sqrt{w}|0\rangle \pm \sqrt{1-w}|1\rangle$

Best distinguishing measurement:  $M = \sigma_x$

$\rightarrow$  We find the inequality  $E_- = \frac{E_1 - E_2}{2} \leq 2\sqrt{w(1-w)}$

According to quantum strategies:  $E_- = \frac{E_1 - E_2}{2} \leq 2\sqrt{w(1-w)} = Q_{\max}$

Maximal value for “classical” strategies?

How to define “classical” strategies?

One possibility:

“classical” strategies = “deterministic” strategies (or convex combinations thereof)

$$E_x = \sum_{\lambda} p_{\lambda} E_x^{\lambda} \text{ with } E_x^{\lambda} = \pm 1$$

Let's be more conservative and compare  $Q_{\max}$  to strategies where only  $E_1$  is deterministic

$$E_1 = \sum_{\lambda} p_{\lambda} E_1^{\lambda} \text{ with } E_1^{\lambda} = \pm 1, \text{ no constraint on } E_2$$

→ If  $Q_{\max} > D_{\max}$  → the output of  $x = 1$  is random (even to adversary with arbitrary knowledge of the devices)

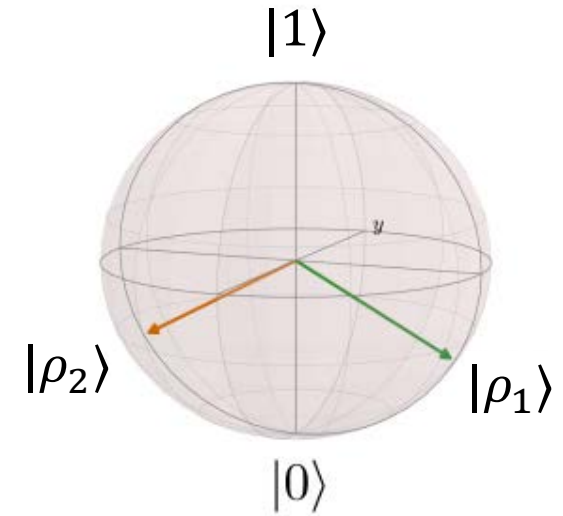
$$E_1 = \langle \rho_1 | M | \rho_1 \rangle = 1 \Rightarrow M = 2|\rho_1\rangle\langle\rho_1| - I$$

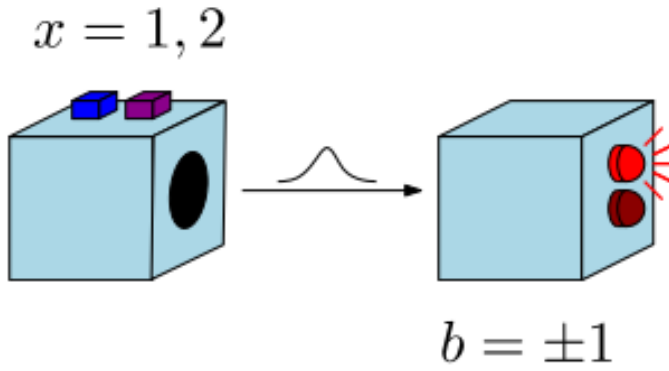
$$\Rightarrow E_2 = \langle \rho_2 | M | \rho_2 \rangle = 2|\langle \rho_2 | \rho_1 \rangle|^2 - 1$$

$$\Rightarrow E_- = \frac{E_1 - E_2}{2} = 2 - 2|\langle \rho_2 | \rho_1 \rangle|^2$$

Minimal value of  $|\langle \rho_2 | \rho_1 \rangle|^2$  given  $w$

$$\Rightarrow E_- \leq 4w(1 - w) = D_{\max}$$





### Assumption

Energy constraint:

$$\langle 0 | \sum_{\lambda} p_{\lambda} \rho_x^{\lambda} | 0 \rangle \geq w_x$$

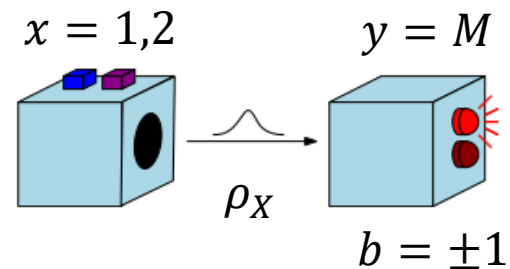
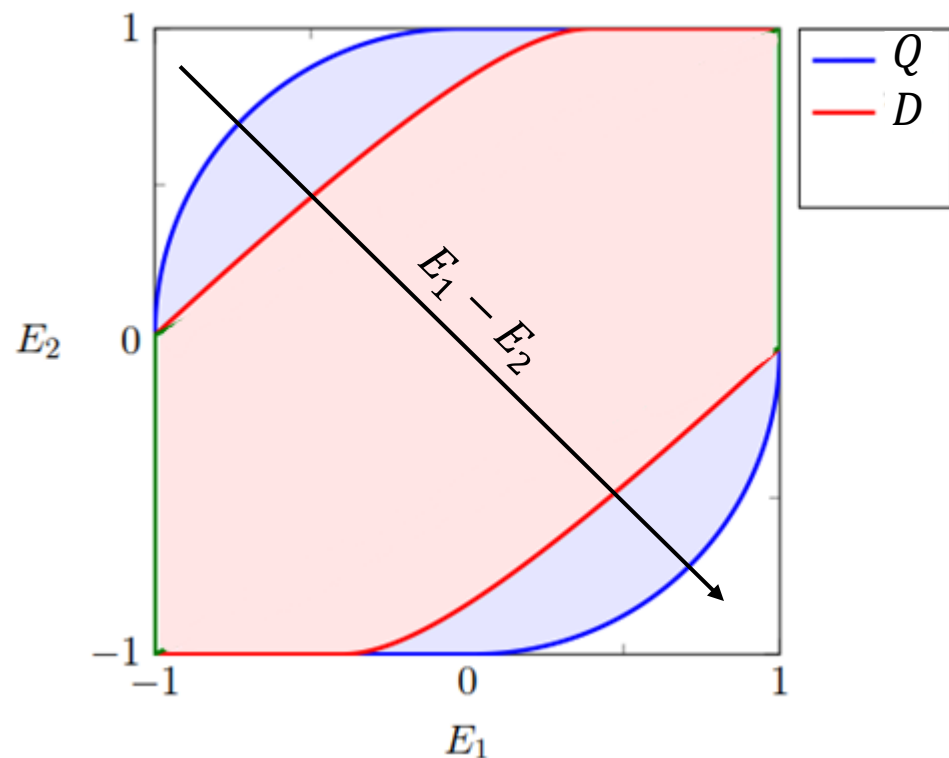
- $E_- = \frac{E_1 - E_2}{2}$
- If  $E_1$  is deterministic, we have the “Bell inequality”  

$$E_- \leq 4w(1 - w) = D_{\max}$$
- According to a general quantum strategy  

$$E_- \leq 2\sqrt{w(1 - w)} = Q_{\max} = \sqrt{D_{\max}} > D_{\max}$$

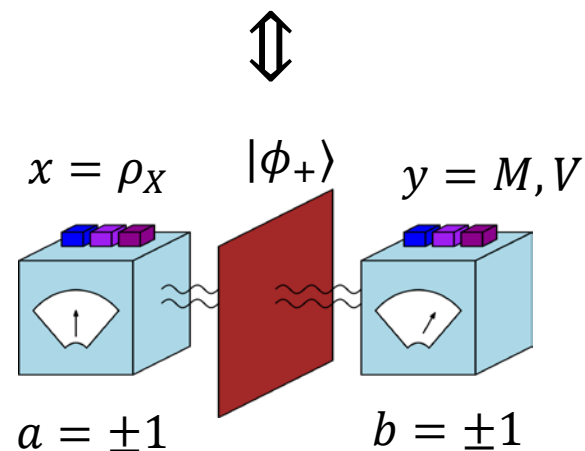
More generally, it is possible to characterize completely the set of allowed values  $(E_1, E_2)$  for given energy bounds  $(w_1, w_2)$

One nice way to do it:



$$E_x = \text{Tr}[\rho_x M]$$

$$w_x = \text{Tr}[\rho_x V]$$



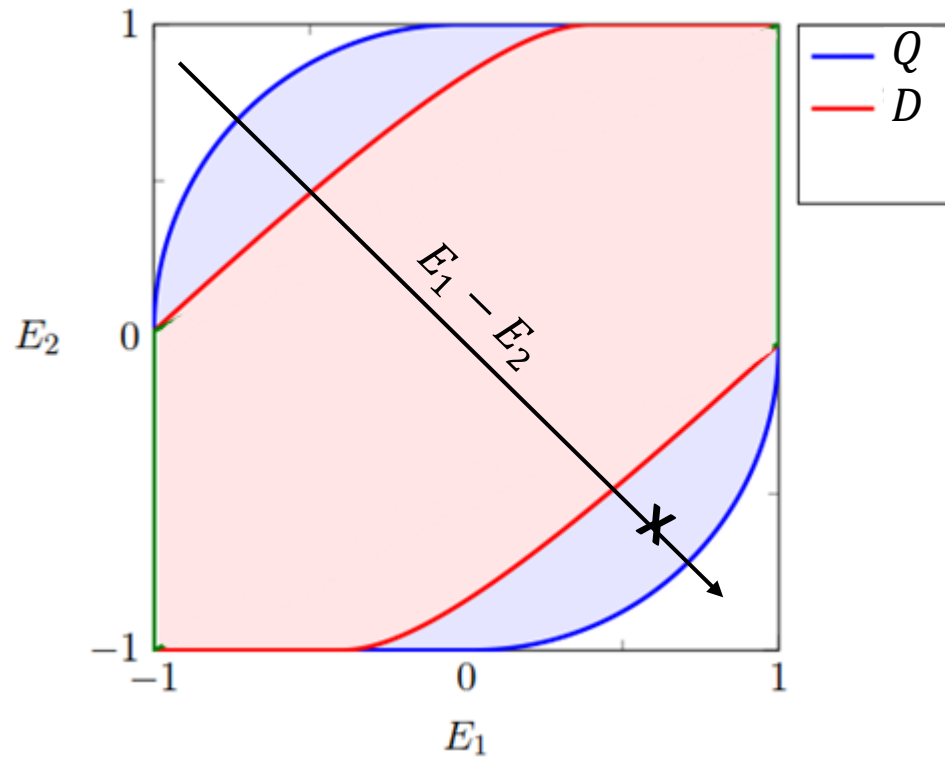
$$\langle A_x M \rangle = E_x$$

$$\langle A_x V \rangle = w_x$$

Given this characterization, one can also put rigorous bounds on the output entropy given  $(E_1, E_2)$   
 → straightforward to build a RNG protocol where amount of randomness produced is evaluated assuming only the energy bound, but no other assumption on the devices.

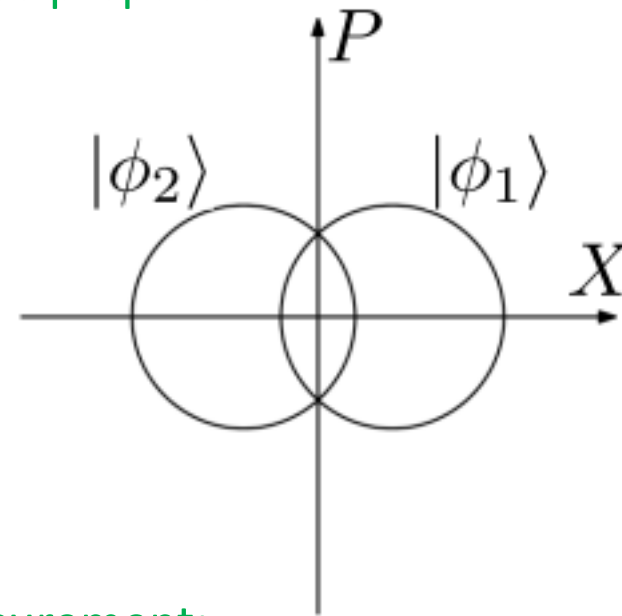


How to produce “non-deterministic” correlations in the lab?



A practical implementation with gaussian states and homodyne measurements:

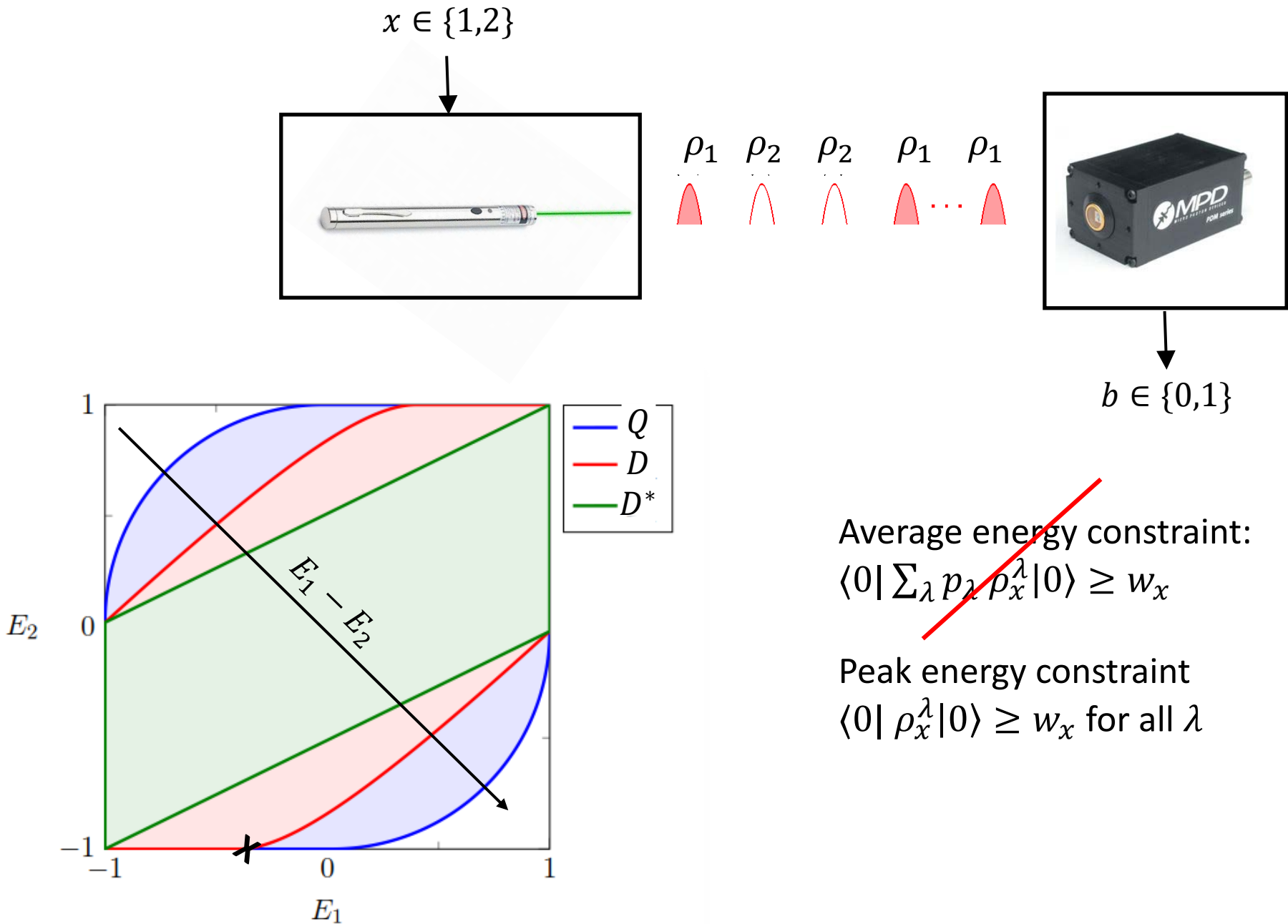
Source prepares:



Measurement:

Homodyne measurement of  $X$  quadrature  
with  $b = \text{sign}(X)$

A simpler implementation with a slightly stronger assumption



# Summary

- We propose to use a bound on the energy of optical signals as a unique assumption on which to prove the security of prepare-and-measure quantum cryptography protocol (with no other assumptions on the devices)
- We have shown that there is a gap between what can be achieved with very simple quantum implementations and deterministic strategies. This is equivalent to the violation of Bell inequalities in full DI protocols.
- These results immediately imply the existence of RNG protocols where the amount of randomness produced can be certified without making any assumptions about the devices except the energy assumption.

# Open question

- Is the energy assumption sufficient to prove the security of a QKD protocol?
- We implicitly assumed that the preparation and measurement device did not share prior entanglement. Can this be relaxed?
- One extra motivation for the energy assumption is that it is in principle compatible with CV protocols for which no DI or semi-DI implementations have been introduced.  
Can we analyze the security of a genuinely CV protocol in a DI setting using the energy assumption?