Operational effects of the UNOT gate on classical and quantum correlations

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Abstract

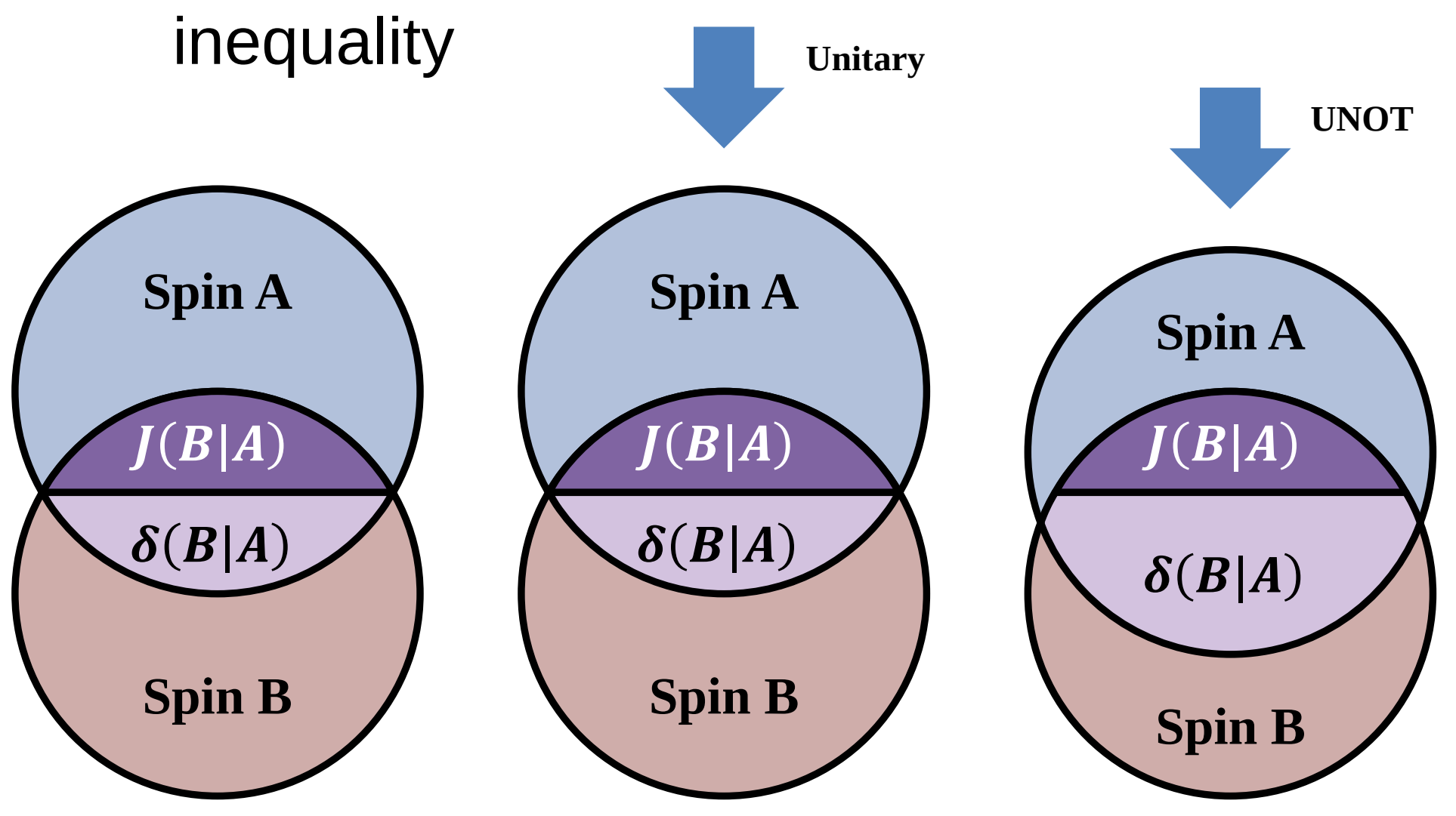
The NOT gate that flips a classical bit is ubiquitous in classical information processing. However its quantum analogue, the universal NOT (UNOT) gate that flips a quantum spin in any alignment into its antipodal counterpart is strictly forbidden. Here we explore the connection between this discrepancy and how UNOT gates affect classical and quantum correlations. We show that while a UNOT gate always preserves classical correlations between two spins, it can non-locally increase or decrease their shared discord in ways that allow violation of the data processing inequality. We experimentally illustrate this using a multi-level trapped ^{171}Yb^{+} ion that allows simulation of anti-unitary operations.

Introduction

- UNOT gate on one spin

$$\alpha|\uparrow\rangle+\beta|\downarrow\rangle \xrightarrow{\text{UNOT}} -\beta^{*}|\uparrow\rangle+\alpha^{*}|\downarrow\rangle$$

  - Anti-unitary thus unphysical
  - No radical contradiction
- Local UNOT on two spin
  - Be able to violate the data processing inequality



Classical and Quantum correlations

- Quantum mutual information

  Total correlations between A and B

$$I(A,B)=S(A)+S(B)-S(AB)$$

- Classical correlations

  Maximal information gain of B by measuring A

$$J(B|A)=S(B)-S(B|A)$$

- Quantum correlations

  Quantum discord

$$\delta(B|A)=I(A,B)-J(B|A)$$

Theoretical results

(i) Local UNOT gates always preserve $J(B|A)$ and $J(A|B)$.

(ii) If $\delta(B|A)=0$, then a local UNOT gate always conserves $I(A,B)$.

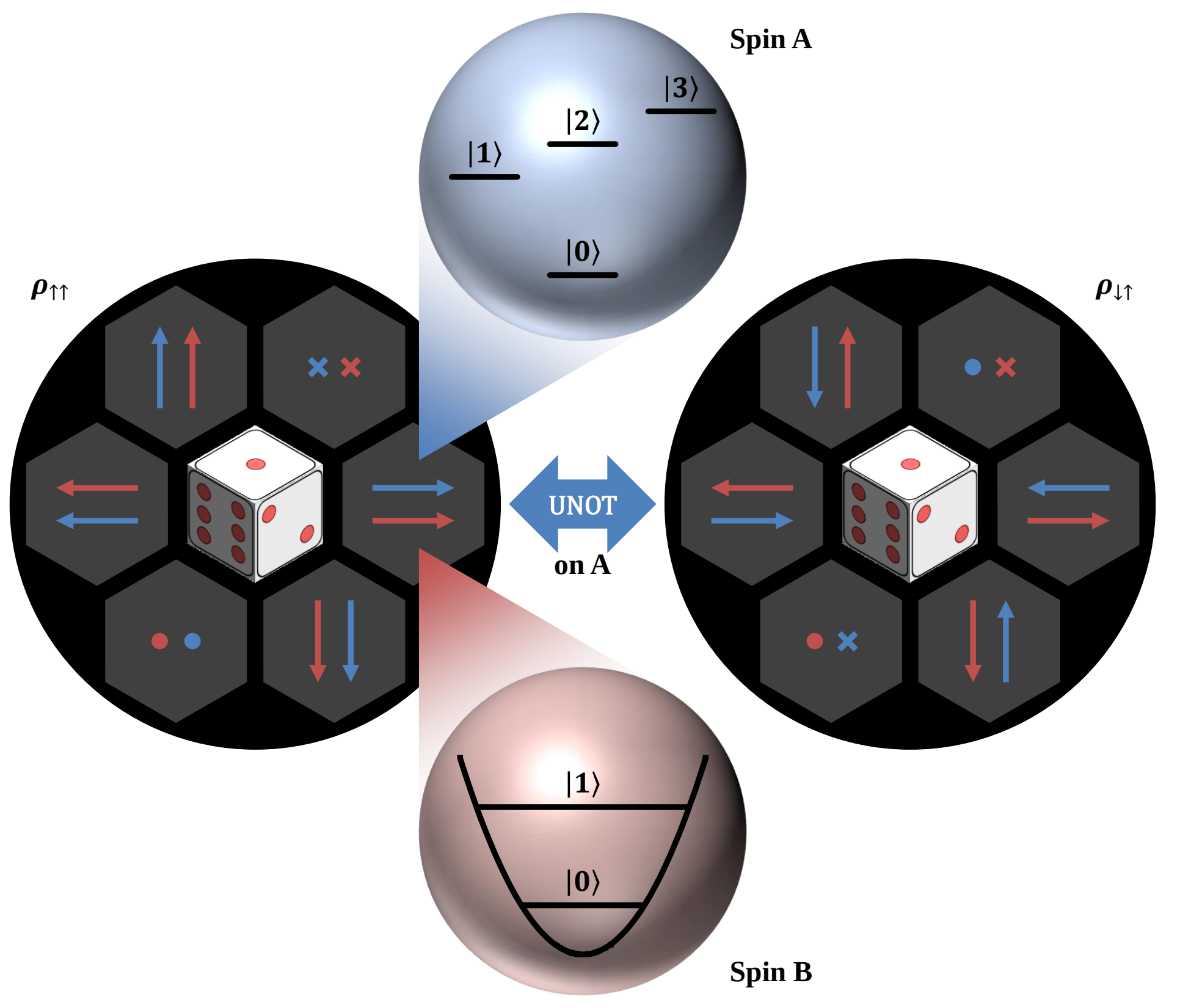
(iii) Local UNOT gates can nevertheless violate the data processing inequality, but only when $\delta(B|A)>0$.

- (i) and (ii) are theoretically proven
- (iii) is demonstrated by experiments

Protocol

Change $\delta(B|A)$ by transforming the quantum state between aligned spins $\rho_{\uparrow\uparrow}$ and anti-aligned spins $\rho_{\downarrow\uparrow}$ using local UNOT gates

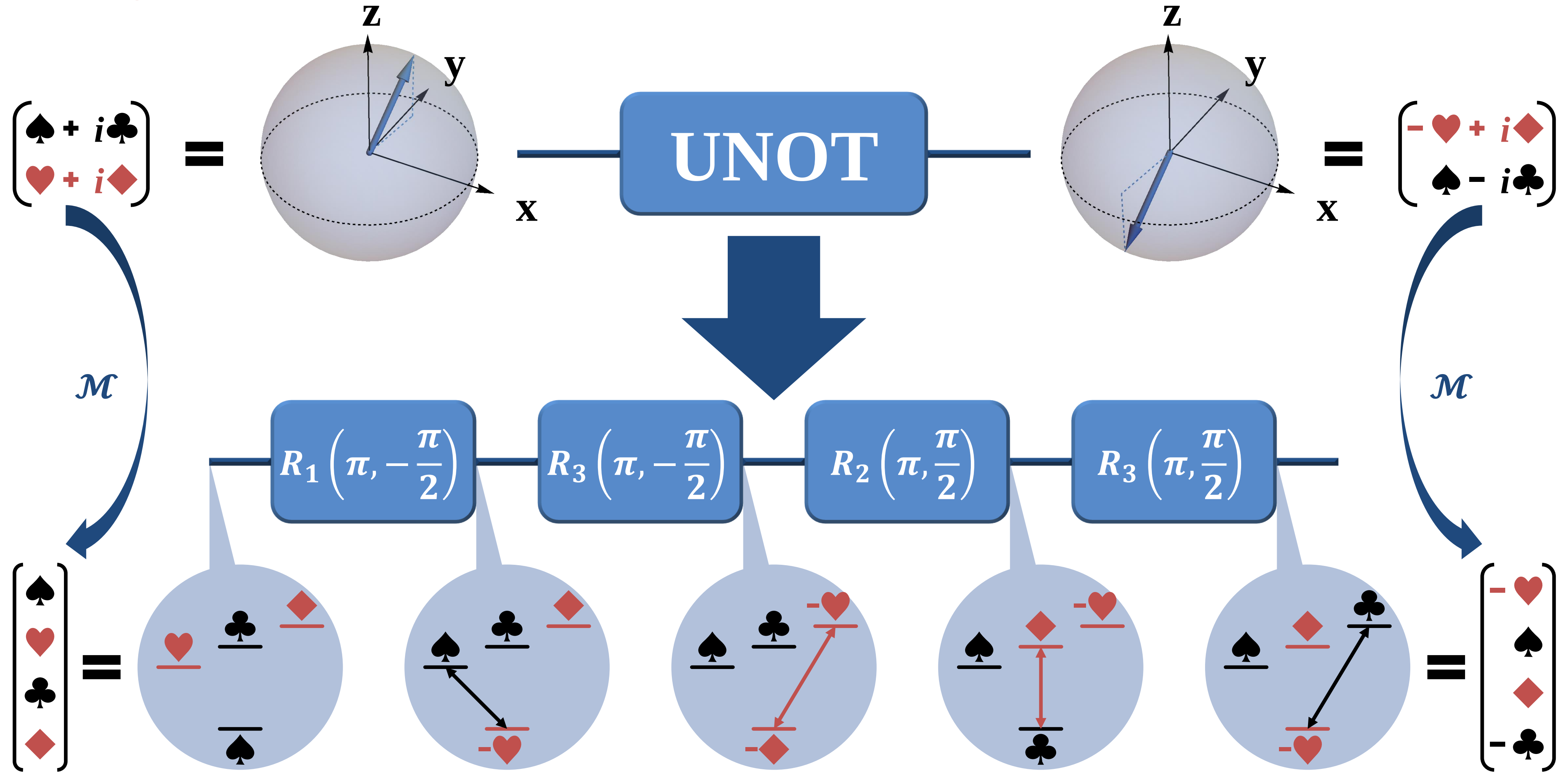
- $J(\rho_{\uparrow\uparrow})=0.082=J(\rho_{\downarrow\uparrow})$
- $\delta(\rho_{\uparrow\uparrow})=0.415>0.208=\delta(\rho_{\downarrow\uparrow})$



Experimental Realization

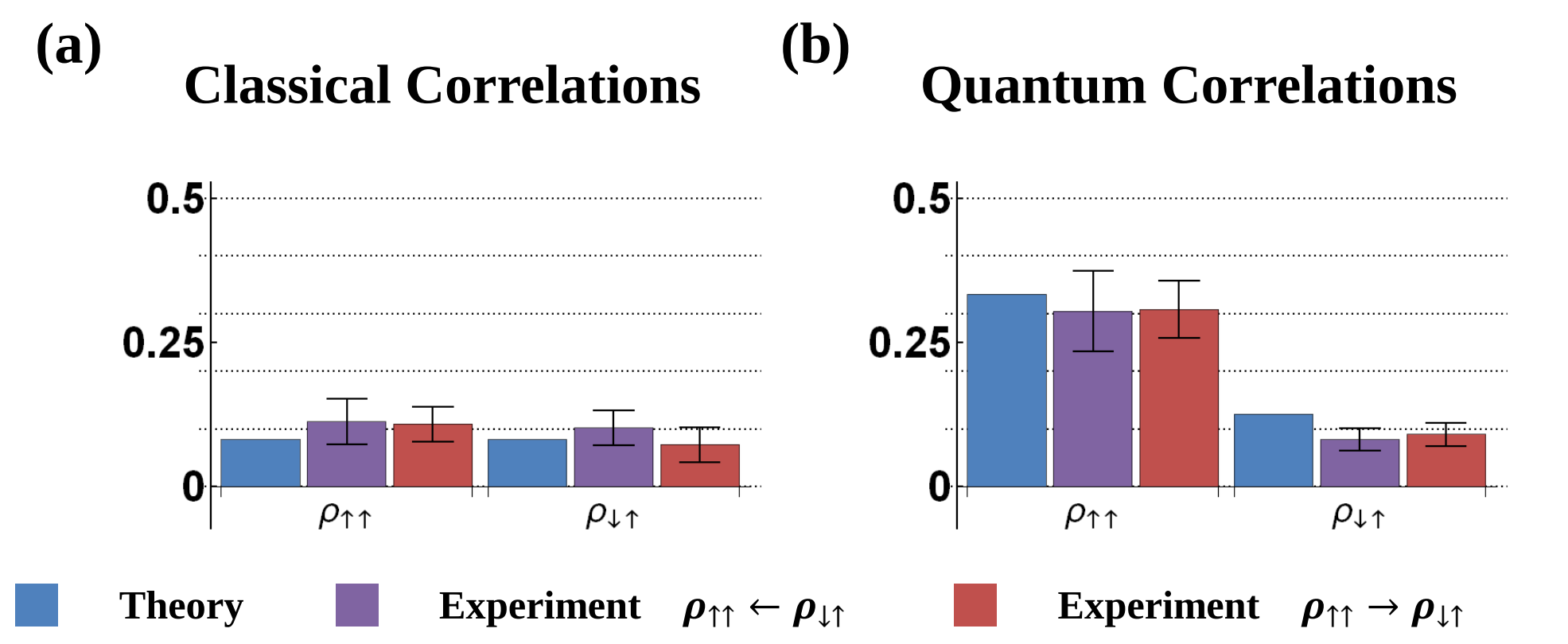
- Encoding scheme
  - Spin A is encoded within the internal 4-level of a trapped ^{171}Yb^{+}.
    - $|0\rangle_{\mathrm{A}}=|F=0,m_{\mathrm{F}}=0\rangle$
    - $|n=1,2,3\rangle_{\mathrm{A}}=|F=1,m_{\mathrm{F}}=n-2\rangle$
  - Spin B is encoded within the ground and first excited external motional states of ^{171}Yb^{+}.
    - Denoted $|0\rangle_{\mathrm{B}}$ and $|1\rangle_{\mathrm{B}}$
- Correlations measurement
  - The classical and quantum correlations are characterized by tomography
    - Develop an efficient tomography scheme that directly reconstructs the 4 × 4 density operator of the spin pair
    - Develop synthesizing operations to manipulate the internal and external degrees of freedom

Simulating UNOT



- The UNOT gate Θ_{UNOT} can be simulated by unitary opration $\bar{\Theta}_{\mathrm{UNOT}}=|1\rangle\langle 0|-|0\rangle\langle 1|-|3\rangle\langle 2|+|2\rangle\langle 3|$
- $\bar{\Theta}_{\mathrm{UNOT}}$ is realized with 4 microwave pulses, where $R_n(\pi,\phi)=-i\left(e^{-i\phi}|n\rangle_{\mathrm{A}}\langle 0|_{\mathrm{A}}+\mathrm{h.c.}\right)$, $n=1,2,3$

Results



The theoretically predicted effect of UNOT gate (blue bars), together with experimentally measured effects when acting on $\rho_{\downarrow\uparrow}$ (purple bars) and $\rho_{\uparrow\uparrow}$ (red bars) are displayed for

(a) Classical correlations $J(B|A)$

Experimental results agree with theoretical prediction within experimental error.

(b) Quantum correlations $\delta(B|A)$

Theory predicts that the difference in $\delta(B|A)$ between $\rho_{\uparrow\uparrow}$ and $\rho_{\downarrow\uparrow}$ is 0.207. This agrees with experiment, where we see respective increase and decrease of 0.22 ± 0.07 and 0.22 ± 0.05 when converting to and from $\rho_{\uparrow\uparrow}$.

Acknowledgments

This project was supported by the National Key Research and Development Program of China under Grants No. 2016YFA0301900 (No. 2016YFA0301901), and the National Science Foundation of China Grants No. 11374178, and No. 11574002, the John Templeton Foundation Grant 53914 "Occam's Quantum Mechanical Razor: Can Quantum theory admit the Simplest Understanding of Reality?", the National Research Foundation of Singapore (NRF Award No. NRF--NRFF2016--02), the Competitive Research Programme (CRP Award No. NRF-CRP14-2014-02), the Ministry of Education in Singapore, and the Oxford Martin School.

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