Work Extraction and Fully Entangled Fraction

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Introduction

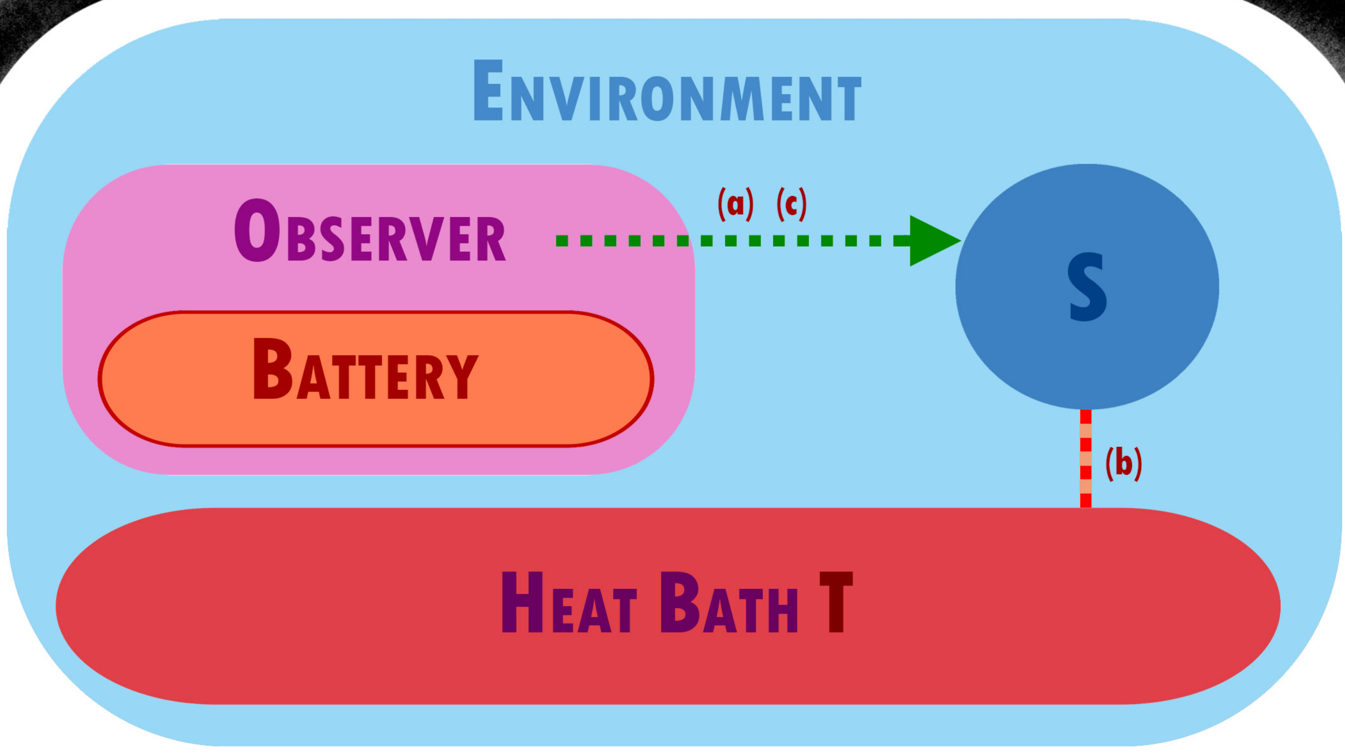
Quantum thermodynamics and quantum nonlocality share similar capacity to tell quantum and classical regimes apart. Even though the awareness of the relations between work and entanglement [1], coherence [2,3] has already been studied in the literature, quantitative connections to nonlocal properties such as quantum nonlocality [4], quantum steerability [5-7], and the usefulness of quantum teleportation [8-11] still remain as open questions.

In this work, we try to bridge quantum thermodynamics and quantum nonlocality together, by relating work gain under different processes to *fully entangled fraction* (FEF) of a given quantum state ρ which is defined by:

$$\mathcal{F}(\rho) := \max_{|\Psi\rangle} \langle\Psi|\rho|\Psi\rangle,$$

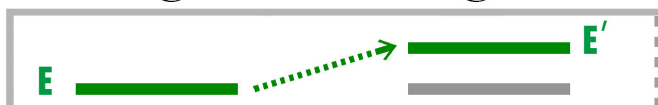
where the maximization is taken over all maximally entangled states $|\Psi\rangle \in \mathbb{C}^d \otimes \mathbb{C}^d$. From now on we only focus on $d = 2^l$.

Framework



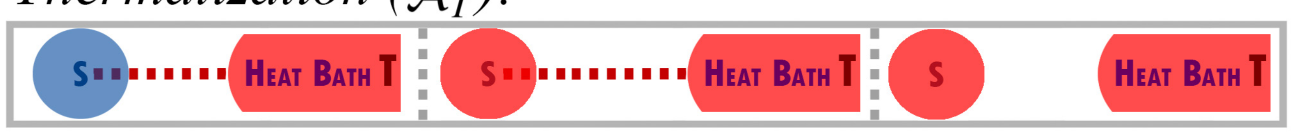
We introduce the *allowed actions* for the observer to apply:

(a) *Raising or lowering the energy level* ($\mathcal{A}_{\mathrm{R/L}}$).



$\mathrm{tr}[\rho(H' - H)]$

(b) *Thermalization* ($\mathcal{A}_T$).



(c) *Unitary operations* ($\mathcal{A}_U$).

$$\rho \mapsto \int_{U(d^2)} U\rho U^\dagger P_U\, dU$$

(d) δ *approximation* ($\mathcal{A}_\delta$). $\rho \overset{\delta}{\approx} \sigma$ $\quad \frac{1}{2}\|\rho - \sigma\|_1 \leqslant \delta$

For a given state $\rho \in \mathcal{L}(\mathbb{C}^d \otimes \mathbb{C}^d)$, we defined an *allowed process* for ρ to be a finite sequence of actions denoted by

$$\mathcal{P}_\rho = \{A_n\}_{n=1}^N \subset \mathcal{A}_{\mathrm{R/L}} \cup \mathcal{A}_T \cup \mathcal{A}_U \cup \mathcal{A}_\delta$$

such that the initial and final Hamiltonians are identical and fully degenerate.

Definitions

$\mathcal{P}_\rho^{\mathrm{Er|A}}$ is a *local erasure process* on Alice's side if
(1) $\mathcal{P}_\rho^{\mathrm{Er|A}} = \mathcal{P}_{|\phi\rangle\langle\phi|}$, where $|\phi\rangle\langle\phi|$ is a purification of ρ.
(2) it maps $|\phi\rangle\langle\phi| \mapsto |\mathbf{0}\rangle\langle\mathbf{0}| \otimes \mathrm{tr}_{\mathrm{A}}|\phi\rangle\langle\phi|$.

The *minimal deterministic average work cost* in the iid limit of erasure process is defined by [12]

$$-W_{\mathrm{Er}}(\rho) := \inf\left\{w|\exists\{\mathcal{P}_{\rho^{\otimes k}}^{\mathrm{Er|A}}\}_{k=1}^\infty \text{ s.t. } \lim_{k\to\infty} P\left[-W\left(\mathcal{P}_{\rho^{\otimes k}}^{\mathrm{Er|A}}\right) \leqslant kw\right] = 1\right\}$$

Theorem 1

$$\mathcal{F}(\rho) > \frac{1}{d} \quad \Longrightarrow \quad W_{\mathrm{Er}}(\rho) \geqslant k_B T \ln \mathcal{F}(\rho) d$$

Definitions

$\mathcal{P}_\rho^{\mathrm{W}}$ is a *work extraction process on* ρ if it is an allowed process having $\frac{\mathbb{I}}{d^2}$ as the final state.

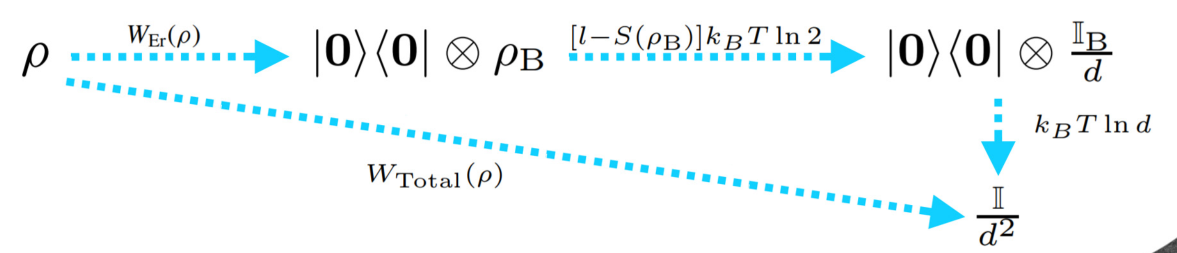
The *largest deterministic extractable work* in the iid limit for ρ is defined by

$$W_{\mathrm{Total}}(\rho) := \sup\left\{w|\exists\{\mathcal{P}_{\rho^{\otimes k}}^{\mathrm{W}}\}_{k=1}^\infty \text{ s.t. } \lim_{k\to\infty} P\left[W\left(\mathcal{P}_{\rho^{\otimes k}}^{\mathrm{W}}\right) \geqslant kw\right] = 1\right\}$$

Theorem 2

Given $0 < \epsilon \leqslant \frac{1}{2}$ and $\delta_\epsilon := -3\ln\epsilon$, if (i) $\mathcal{F}(\rho) > \frac{1}{d}$, (ii) there exists a process given in Ref. [13] with $P_{\mathrm{success}} \geqslant 2\epsilon$ which can extract $W_{\mathrm{Total}}(\rho)$, and (iii) $\log_2\|\rho\|_\infty = \log_2 \mathcal{F}(\rho)d - S_{\min}(\rho)$, then up to $\delta_\epsilon k_B T \ln 2$, we have: $S_{\min}(\rho) := \min\{S(\rho_{\mathrm{A}}); S(\rho_{\mathrm{B}})\}$

$$W_{\mathrm{Total}}(\rho) \approx k_B T \ln d^2 - S_{\min}(\rho) k_B T \ln 2 + W_{\mathrm{Er}}(\rho)$$



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