Quantum solitons as possible qubits

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Quantum solitons in optical fibres could both beequbits' for quantum information, quantum computing, logic gates, etc. Multi-quantum solitons are more easily realised experimentally than are the current realisations of qubits. The 1-soliton solution of the classical 1D attractive Nonlinear Schrödinger (NLS) equation is,

$$\psi^s(x,t) = 2|c|^{-\frac{1}{2}}\eta(\exp i\Phi(x,t))\mathrm{sech}[2\eta(x-x_0)+8\xi\eta t] \tag{1}$$

where $\Phi \equiv -4(\xi^2-\eta^2)t - 2\xi x + \theta_0$ and $c < 0$. Quantum features on this classical soliton are then introduced by imposing momentum-position uncertainly or number phase uncertainly, the latter in particular satisfying $\Delta n \Delta\varphi \geq \frac{1}{2}$. Because of the stability of classical NLS solitons, and because such classical ebits' must in fact be *quantum*ebits', and thereforeequbits', the quantum soliton solutions of the *quantum* attractive NLS equation seem worth investigating.

The quantum (attractive) NLS equation in 1D can be put in the form, with $c < 0$,

$$-i\hbar\hat{\psi}_t = \hbar^2/2m \cdot \hat{\psi}_{xx} - 2c\hat{\psi}^\dagger\hat{\psi}\hat{\psi} \tag{2}$$

where the fields $\hat{\psi}(x,t)$, adjoint $\hat{\psi}^\dagger(x,t)$, satisfy the boson commutators. We set $\hbar = 1$ and $m = 1/2$. Equation (2) derives from a Hamiltonian $\hat{H}$ and there are a number operator $\hat{N}$ with eigenvalues n (the photon numbers) and a momentum operator $\hat{P}$ with eigenvalues P. For soliton solutions we need $c < 0$ in (2)and it is $|c|$ which figures in the classical 1-soliton (1). Indeed in classical limit $\hat{\psi} \to \psi, \hat{\psi}^\dagger \to \psi^*$ so that $\{\psi,\psi^*\} = i\delta(x-x')$. One discovers [1] that the classical soliton (1) derives exactlly from a quantum mechanical matrix element for photon numbers $n \to \infty$,

$$\lim_{n\to\infty}\langle n, X', t|\hat{\psi}(x-vt)|n+1, X, t\rangle \tag{3}$$

where $\hat{\psi}(x-vt)$ with v a velocity, solves (2). The states $|n+1, X, T\rangle$ derive as wave packets from simultaneous eigenstates $|n+1, P\rangle$ of $\hat{H}, \hat{N}, \hat{P}$ etc. In general the eigenstates satisfy

$$\hat{N}|k_1,\dots k_n\rangle = n|k_1,\dots k_n\rangle, \quad \hat{P}|k_1,\dots k_n\rangle = \sum_{j=1}^{n} k_j|k_1,\dots k_n\rangle, \tag{4}$$

and the eigenvalue of $\hat{H}$ is $\sum_{j=1}^n k_j^2$; but for a quantum soliton these states are bound states forming an en-string' such that $k_j = p + \frac{1}{2}\{n-(2j-1)ic\}$ for $j = 1,2,\dots,n\cdot$ Thus, as n is fixed, they are infinitely squeezed in n and cannot be squeezed any more! On the other hand experiments on laser light traversing the two different loops of a Sagnac interferometer demonstrate pronounced squeezing of the photon number and the conclusion might be that quantum entanglement must be involved. It is suggested that two quantum solitons (two qubits) exhibit such entanglement and it is known [2] that two soliton solutions of the classical attractive sine-Gordon equation exhibit mutualephase shifts'. For the true quantum solitons being described here we need to work out matrix elements for two different n-string solutions and examine the quantum phase shifts.

[1] M. Wadachi and M. Sakagami, J. Phys. Soc. Japan **53**, 1933-1938(1984).
[2] R. K. Bullough and J. Timonen, In Statistical Mechanics and Field Theory, eds. V. V. Bazhanov and C. J. Burden(World Scientific, Singapore, 1995) pp. 336-414 and references.