On Universal Quantum Cloning Machines

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Abstract

The no-cloning theorem describes one of the most fundamental nonclassical properties of quantum systems. It states that an unknow quantum state cannot be cloned exactly. And the no-cloning theorem for pure states is extended to other cases. However, the no-cloning theorem does not forbid imperfect cloning. It is interesting to know how well we can clone the quantum states. Bužek and Hillery proposed a 1 to 2 UQCM which produces two indentical copies whose quality is independent of the input qubit (two-level system). This UQCM was later proved to be optimal if the measure of quality is the fidelity between the input and the ouput. Gisin and Massar considered a general case in which M identical copies are generated from N ($M \geq N$) identical qubits, and proved that the fidelity of the copies is optimal. The complete positive (CP) map of the optimal N to M cloning transformation and the optimal fidelity for arbitrary dimensional system were derived and studied extensively by Werner and Keyl and Werner.

Generally, the known quantum cloning transformations are just for pure input states. Bužek and Hillery studied the 1 to 2 cloning of impure states for states in d-dimensional. In this paper, we shall propose a generalized N to M ($M \geq N$) universal quantum cloning machine which allows the input to be an arbitrary mixed state restricted to symmetric subspace of N qubits. And it reduces to the well-known N to M universal quantum cloning machine (UQCM) if the input are N identical pure states. The generalized cloner is optimal if we use fidelity between the input and output reduced density operators at a single qubit as the measure of quality. The output of a UQCM is in symmetric subspace, we can now concatenate a generalized UQCM and take the output of the UQCM as input, and produces more copies. The result for qubits is extended to arbitrary-dimensional states.

The generalized cloning transformation for d-level quantum system takes the following form

$$\begin{aligned}
&U(d:N,M)|j_0\Psi, j_1\Psi_1^{\perp}, \cdots, j_{d-1}\Psi_{d-1}^{\perp}\rangle \otimes R \\
&= \sum_{\vec{k}}^{M-N} \alpha_{\vec{j}\vec{k}}(M,L)|(j_0+k_0)\Psi, (j_1+k_1)\Psi_1^{\perp}, \cdots, (j_{d-1}+k_{d-1})\Psi_{d-1}^{\perp}\rangle \otimes R_{\vec{k}},
\end{aligned} \tag{1}$$

$$\alpha_{\vec{j},\vec{k}}(N,M) = \sqrt{\frac{(M-N)!(N+d-1)!}{(M+d-1)!}} \sqrt{\prod_{i=0}^{d-1} \frac{(j_i+k_i)!}{j_i!k_i!}}. \tag{2}$$

where $\sum_{i=0}^{d-1} j_i = N, \sum_{i=0}^{d-1} k_i = M-N$ are assumed. The ancilla states $R_{\vec{k}}$ can be realized in symmetric subspace $R_{\vec{k}}(\Psi) = |k_0\Psi, k_1\Psi_1^{\perp}, \cdots, k_{d-1}\Psi_{d-1}^{\perp}\rangle$.