On the distribution of the entropy of a subsystem

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Abstract

If a composite quantum system of Hilbert space dimension mn consisting of two subsystems A and B of dimensions m and n respectively is in a random pure state, the average entropy of the subsystem of dimension $m \leq n$ is given by $\langle S_A \rangle = \sum_{k=n+1}^{mn} \frac{1}{k} - \frac{m-1}{2n}$, conjectured by D. Page [Phys. Rev. Lett. 71, 1291 (1993)] and proved by the authors in 1994 [Phys. Rev. Lett. 72, 1148 (1994)]. We have now computed numerically the probability distribution $\rho_{m,n}(S_A)$ of S_A, and its standard deviation for $m = 2$ and 3, and for n up to 20.

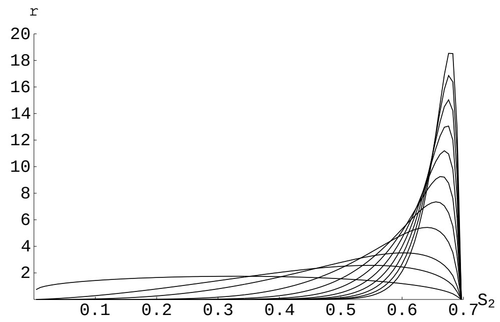

Figure 1: Distribution $\rho_{2,n}(S_2)$ for $n = 2, 3, 4, 6, 8, \cdots, 20$.

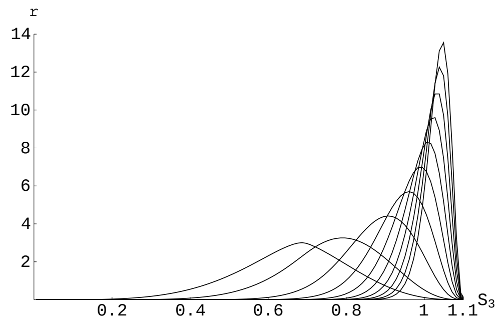

Figure 2: Distribution $\rho_{3,n}(S_3)$ for $n = 3, 4, 6, 8, \cdots, 20$.

The figures show that for n not much larger than m, the distributions are rather spread out though skew towards the maximum entropy. As n increases, the distributions become increasingly skew towards the maximum entropy and develop sharp peak.

Two remarks: 1) By a theorem due to Araki and Lieb, $S_A = S_B$ if $S_{AB} = 0$. Therefore, the distribution of S_A and that of S_B are the same. 2) If information content I of a system is defined as the deficit of the entropy of the system from the maximum (i.e. $\ln m$), then for subsystem A, $I_A = \ln m - S_A$, and the distribution of I_A for each value of m and n is obtained from the $\rho_{m,n}(S_A)$ by a reflection about the point $(\ln m)/2$.

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