Universal concentration, universal teleportation and universal dense coding

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In quantum systems, we can perform some information processes which do not appear in classical systems. For example, quantum teleportation, dense coding etc. For them it is necessary to share an entangled state between two systems. If the entangled state is the perfect Bell state, its analysis is very easy. Otherwise, it is not easy [1, 2].

We can produce perfect Bell states from some entangled states which are not perfect Bell states with local operations and classical communications (LOCC). Such an operation is called a concentration, and is discussed by Bennett et al. [3] When we share the n-tensor product state $|\phi\rangle\langle\phi|^{\otimes n}$ on the total tensor product system $\mathcal{H}_A^{\otimes n} \otimes \mathcal{H}_B^{\otimes n}$, where $|\phi\rangle\langle\phi|$ is a pure state on the total system $\mathcal{H}_A \otimes \mathcal{H}_B$. Bennett et al [3] proved that we can produce $nH(\rho_A)$-qubit perfect Bell states with the success probability going to 1, where $\rho_A := \mathrm{Tr}_B |\phi\rangle\langle\phi|$ and $H(\rho_A)$ is the entropy $-\mathrm{Tr}\,\rho_A \log \rho_A$.

In this paper we propose a universal concentration, in which, we can, independently of ρ_A, produce no less than $nH(\rho_A)$-qubit perfect Bell states and the success probability goes to 1. In the protocol, the concentration is performed without any classical communications.

In the quantum teleportation, we can send a quantum state with LOCC. In such a setting we maximize the number of teleported qubits only with LOCC. To share R-qubit perfect Bell state is equivalent with to send R-qubit of perfect Bell states only with LOCC. Therefore, we can perform $nH(\rho_A)$ qubits quantum teleportation, under the assumption that we share the n-tensor product state $|\phi\rangle\langle\phi|^{\otimes n}$ on the total system $\mathcal{H}_A^{\otimes n} \otimes \mathcal{H}_B^{\otimes n}$. Even if we do not know the density operator ρ_A, using the universal concentration we can perform $nH(\rho_A)$ qubits quantum teleportation with the success probability going to 1. In the protocol, it is enough to send the minimum classical communications of the size of $2nH(\rho_A)$ bits.

If entangled states are shared, we can send R_1 bits classical message by sending only $R_2(< R_1)$ qubits. This type information process is called (super) dense coding. The number $R_1 - R_2$ signifies the effect of entanglement. Thus, in this setting we can regard the maximum of $R_1 - R_2$ as the capacity. Our setting is different from the usual setting of the dense coding. We can prove that the maximum of $R_1 - R_2$ asymptotically equals to $nH(\rho)$. Even if we do not know the density ρ_A, using the universal concentration we can send $2nH(\rho)$ bits of classical information by sending only $nH(\rho)$ qubits.

References

[1] C. H. Bennet and S. J. Wiesner, "Communication via one- and two-particle operators on Einstein-Podolsky-Rosen states," Phys. Rev. Lett. vol 69, 2881, 1992.

[2] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres and W. K. Wootters, "Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels," Phys. Rev. Lett. vol 70, 1895, 1993.

[3] C. H. Bennett, H. J. Berstein, S. Popescu, and B. Shcumacher "Concentrating partial entanglement by local operations," Phys. Rev. A vol 53, 2046, (1996). quant-ph/9511030.

[4] A. Barenco, A. Ekert, "Dense coding based on quantum entanglement," J. Mod. Optics, vol. 42, 1253–1259, 1995.