Optimal Superdense Coding

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Abstract

I investigate superdense coding with a general mixed entangled state on the Hilbert space $C^d \otimes C^d$ shared between a sender and receiver. In this general superdense coding, the sender (Alice) performs one of the local unitary transformations $U_i \in U(d)$ on her d-dimensional quantum system to put the initially shared entangled state ρ in $\rho_i = (U_i \otimes \mathbf{I}_d)\rho(U_i^\dagger \otimes \mathbf{I}_d)$ with *a priori* probability p_i $(i = 0, 1, \cdots, i_{max})$, and then she sends off her quantum system to the receiver (Bob). Upon receiving this quantum system, Bob performs a suitable measurement on ρ_i to extract the signal. The problem is to find the optimal signal ensemble $\{\rho_i; p_i\}_{i=0}^{i_{max}}$ that maximizes χ, the capacity of superdense coding. I show that the d^2 signal states ($i_{max} = d^2 - 1$) generated by mutually orthogonal unitary transformations with equal probabilities yield the maximum χ. The optimal capacity χ^* is given as the difference between the von Neumann entropy of the average signal state $\overline{\rho}^* = d^{-2}\sum_{i=0}^{d^2-1}\rho_i$ and that of the initially shared entangled state ρ;

$$\chi^* = S(\overline{\rho}^*) - S(\rho).$$

The average state for the optimal signal ensemble is shown to be

$$\overline{\rho}^* = \frac{1}{d}\mathbf{I}_d \otimes \rho^B$$

where $\rho^B = tr_A(\rho)$. It is also proved that the optimal capacity of dense coding χ^* satisfies

$$E_R(\rho) \leq \chi^* \leq E_R(\rho) + \log_2 d$$

where $E_R(\rho)$ is the relative entropy of entanglement of the shared entangled state.

Keywords: superdense coding, mixed state entanglement, Holevo quantity, relative entropy of entanglement