Analysis of entanglement by simultaneous finite resolution measurements of non-commuting variables

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The violation of Bell's inequalities by entangled two level systems is perhaps the most direct demonstration of non-classical statistical properites in quantum systems. In order to explore the relationship between this non-classical feature and the local effects of quantum coherence, a simultaneous finite resolution measurement of the non-commuting variable pairs is. proposed. By preserving some of the quantum coherence between eigenstates of the measured variable, such finite resolution measurements allow an identification of quantum coherence as joint probabilities between the original measurement variable $\hat{A}$ and a non-commuting variable $\hat{B}$.

Specifically, an analysis of the measurement statistics shows that the off-diagonal density matrix elements $\langle\ A_1 \ \mid \hat{\rho}_{\text{in}} \mid \ A_2 \ \rangle$ contribute positive and negative probabilities to the joint probabilities of $A = (A_1 + A_2)/2$ and the eigenvalues of $\hat{B}$. In the case of spin operators with eigenvalues of ± 1, quantum coherence thus corresponds to spin values of zero. By determining the positive and negative joint probabilities for an entangled state using finite resolution measurements, the violation of Bell's inequalities can be identified as a consequence of the non-classical negative joint probabilities originating from spin values of zero.

Finite resolution measurements suggest that quantum coherence should be interpreted in terms of negative joint probabilities. This interpretation can then be applied to explain the violation of Bell's inequalities in terms of local non-classical statistics. Details of the violation of Bell's inequalities may be obtained directly from the finite resolution measurement statistics, which represent a simultaneous measurement of all spin components.

References:

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