Initialization-free Generalized Deutsch-Jozsa Algorithm

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We generalize the Deutsch-Jozsa algorithm by exploiting summations of the roots of unity. The generalized algorithm distinguishes a wider class of functions promised to be either constant or many-to-one and onto an evenly spaced range. As previously, the generalized quantum algorithm solves this problem using a single functional evaluation.

A convenient definition of the more general class of functions which are many-to-one and onto evenly spaced ranges employs $\mathbb{Z}_N$, $\mathbb{Z}_M$ and $\mathbb{Z}_K$ for $K \leq M, N$. In more detail, the domain of f, $\mathbb{Z}_N$ is mapped to K evenly spaced elements in the range $\mathbb{Z}_M$ with constant separation ($\mu = M/K$), so each element in the range can be indexed by every element of $j \in \mathbb{Z}_K$ via $\mu j + t \in \mathbb{Z}_M$. Here, t is a possible initial shift less than μ in the $\mathbb{Z}_K$ assignments. Further, each element is mapped exactly $\nu = N/K$ times. When these conditions are satisfied, we refer to the function f as evenly distributed. Our generalization of the Deutsch-Jozsa problem for evenly distributed functions is to determine whether f is nonconstant or not evenly distributed. When K is known we need $\nu+1$ evaluations of f classically in worst case in order to solve this problem. Unless K is known, $\frac{1}{2}N+1$ evaluations are required in worst case before determining the answer with certainty. Thus this problem has the same computational complexity as that of the Deutsch-Jozsa problem. We establish that summations of the M^{th} roots of unity allow distinguishing constant functions from functions which are many-to-one and onto evenly spaced ranges.

We also consider the problem to distinguish constant and evenly balanced functions. A function $f : \mathbb{Z}_N \to \mathbb{Z}_M$ is said to be evenly balanced if half output values of f have parity 0 and half have parity 1. Then the generalized Deutsch-Jozsa problem for evenly balanced functions is to determine whether f is nonconstant or not evenly balanced. Any classical algorithm for this problem would require $\frac{1}{2}N + 1$ evaluations of f in worst case before determining the answer with certainty. Thus this problem has the same computational complexity as that of the Deutsch-Jozsa problem. We present a quantum algorithm for this problem that does not require any initialization of an auxiliary register involved in the process of functional evaluation and after solving the problem recovers the initial state of an auxiliary register.

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