Phase matching in quantum searching: Theory and NMR experimental Studies

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We report our recent results on the phase matching condition in a generalized quantum search algorithm. Grover's quantum search algorithm is a very important development in quantum computation [1]. When the two phase inversions in the Grover algorithm are replaced by arbitrary phase rotations, we found that [4] arbitrary phase rotation of the marked state alone can not be used to construct a quantum search algorithm which is contrary to previous anticipation [2,**?**]. We then discovered that when both phase inversions are replaced by arbitrary phase rotations, they must satisfy a phase matching condition [4], and an approximate formula for amplitude of the marked state is also given. We have also established an SO(3) picture for quantum search which enables the study of quantum search problems in simple geometry terms [5]. In this picture, a quantum search process is a series of rotations in a 3-dimensional space. The phase rotation angles determines the rotational axis, and together with the unitary transformation, they determine the angle of rotation of the state vector around this axis. We have found that the error tolerance in Grover algorithm [7] is $O(N^{1/2})$, and this sets a less stringent requirement than the result of [8]. We have improved the Grover algorithm to zero failure rate for any given number of N where N is the number of items in the database [10]. In this improvement, the algorithm uses an phase rotation angle which is closes to π. Very recently, we have given a generalized phase matching condition which depends both on the initial state of the problem and the search engine itself [6], which clarifies a confusion as to what is the exact phase matching condition in the literature.

We have implemented the phase matching algorithm in a 2-qubit NMR system [9]. In this experiment, two versions of the generalized quantum search algorithm are performed. In the first one, the phase inversions are replaced by $\pi/2$ phase rotations. In the second experiment, the phase inversion for the marked state is taken as $\pi/2$, and the phase rotation angle of the prepared state $|0\rangle$ is taken as $2\pi/2$. The first version is a phase matched quantum search algorithm which can successfully find the marked state, while the 2nd one is phase mismatched search algorithm and fails to find the marked state. The experiment data agree with theoretical expectation well.

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