Gate Reduction in Quantum Algorithms via Many-Qubit Interactions

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A universal quantum computer can be implemented by a sequence of single-qubit and two-qubit gates [1–6]. As several proposals take advantage of intrinsically pairwise interactions such as dipole-dipole processes, this paradigm for quantum computation seems natural, provided that one can couple and decouple qubits at specific intervals.

However, whereas a sequential implementation strategy is sufficient for universal computation, it is of interest to inquire whether the number of gates – and perhaps more importantly, the evolution time – required to implement a given operator in $\mathrm{SU}(2^m)$ can be reduced by the inclusion of many-qubit interactions.

The logical extreme of this idea is to consider a Hamiltonian $\mathcal{H}$ that consists of a weighting of all the generators for $\mathrm{SU}(2^m)$ [7]. For example, we might envision a system where $\mathcal{H}$ consists of a linear superposition of all possible tensor products of spin operators $\{\hat{I}, \hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z\}$; here, $\hat{I}$ corresponds to the identity operator. Any unitary operator could be realized by appropriately setting the coefficient for each product term at initialization, and allowing the system to evolve for a fixed time. In essence, we would be engineering the system to precisely solve the algorithm of interest in its time evolution.

However, it is evident that even were such a physical system to be engineered, the number of independent generators in the Hamiltonian is an exponentially large $4^m - 1$; in essence, we are substituting the difficult task of solving the problem underlying the algorithm with the difficult task of creating the operators themselves. The overhead required in determining and setting these parameters for an arbitrary unitary operator outweighs the advantage gained from reducing the evolution to a single gate.

An intermediate approach can be adopted that is a compromise between the sequential and concurrent implementations. If a physical system can be devised that demonstrates selected many-qubit interactions in addition to single and two-qubit gates, some operators that normally require several basic gates can be performed in one step. The entire algorithm still requires a sequence of gates, but the number is reduced compared to the case where only one and two-qubit gates are available.

The nature of the many-qubit interactions that one would like to include in a quantum computer depends on two factors: their utility in reducing the number of gates for a useful algorithm, and ultimately, the practicality of demonstrating and controlling such an interaction on a real physical system.

Here, we consider the utility of a particular m-qubit interaction in reducing the gates required to implement Grover's algorithm for database search [8]. An m-qubit controlled-phase rotation, which applies a phase shift of π if and only if all m qubits are logical one, is of use in the database search algorithm for the diffusion transform and also for implementation of the oracle storing the database. The operator can be implemented by a sequence of at most $48m - 162$ single-qubit gates and controlled-NOT gates [4]. We review the idea of using a single m-qubit interaction – analogous to a $(2m-1)$-order optical nonlinearity – to implement the controlled-phase rotation as a single gate.

We investigate the possibility of engineering this interaction by coupling selected modes of a microwave resonator via a rf SQUID loop. We demonstrate the how this interaction can be surmised from the nonlinear current-flux relation of the rf SQUID, and discuss the assumptions and geometrical constraints required to engineer this nonlinear interaction.

REFERENCES

[1] D. P. DiVincenzo, Phys. Rev. A **51**, 1015 (1995).
[2] A. Barenco, Proc. R. Soc. London Ser. A **449**, 679 (1995).
[3] S. Lloyd, Phys. Rev. Lett. **75**, 346 (1995).
[4] A. Barenco, C. H. Bennett, R. Cleve, D. P. DiVincenzo, N. Margolus, P. Shor, T. Sleator, J. Smolin, and H. Weinfurter, Phys. Rev. A **52**, 3457 (1995).
[5] D. Deutsch, A. Barenco, and A. Ekert, Proc. R. Soc. London Ser. A **449**, 669 (1995).
[6] H. F. Chau and F. Wilczek, Phys. Rev. Lett. **75**, 748 (1995).
[7] F. Yamaguchi, C. P. Master, and Y. Yamamoto (2000), e-print, quantum-ph/005128.
[8] L. K. Grover, Phys. Rev. Lett. **79**, 325 (1997).