Geometric Strategy for Optimal Quantum Search

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Abstract:

Our motivation is based on two points: (i) while it is quite straightforward to verify Grover's quantum search algorithm and Zalka's proof of its optimality, it would be yet to be understood from the geometric aspects why quantum search algorithm works efficiently. (ii) Though it is often said that entanglement is useful to enhance the quantum information processing, that remains obscure on theories as well as on the Nuclear Magnetic Resonance (NMR) experiments after, in particular, Lloyd's proposition of "quantum search without entanglement". Thus, in this talk, we explore quantum search through (algebraic and differential) geometric viewpoints of a complex projective space $\mathbb{C}\mathcal{P}$, a space of rays, and discuss how entanglement gives quantum computation its power.

First, we show that the optimal quantum search can be geometrically identified with the shortest path along the geodesic joining a target state, an element of the computational basis, and such an initial state as overlaps equally, up to phases, with all the elements of the computational basis. Second, we calculate the entanglement through the algorithm for any number of qubits n as the minimum Fubini-Study distance to the submanifold formed by separable states in Segre embedding, and find the entanglement is used almost maximally for large n. The computational time seems to be optimized by the dynamics as the geodesic, running across entangled states away from the submanifold of separable states, rather than the amount of entanglement itself.

Keywords:

quantum computation, Grover's quantum search algorithm, entanglement, complex projective Hilbert space $\mathbb{C}\mathcal{P}$, geodesic, Segre embedding.

Reference:

A. Miyake and M. Wadati, "Geometric strategy for the optimal quantum search," Phys. Rev. A (to be published).