Deterministically Distillable Entanglement

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Quantum entanglement is a valuable resource in quantum information theory. In quantum teleportation [1], Alice and Bob communicate a quantum state via an entangled state,

$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB}), \tag{1}$$

where qubits A and B are possessed by Alice and Bob, respectively. It is essential that they share a Bell pair, $|\Phi^+\rangle_{AB}$, in advance. If Alice and Bob initially share partially entangled pairs, they need to prepare Bell pairs from the initial pairs only by operating locally on their respective systems and communicating classically. Extraction of maximally entangled pairs from partially entangled pairs by using local operations and classical communication (LOCC) is generally called entanglement concentration. Various studies on its limitations and efficiency have been carried out [2, 3, 4, 5, 6]. All the proposed entanglement concentration processes are probabilistic for a finite number of entangled pairs.

We discuss an entanglement concentration scheme within the framework of deterministic transformations [7, 8]. We focus on the deterministic distillation of Bell pairs from two-qubit partially entangled pairs:

$$|\psi_i\rangle = \sqrt{a_i}\,|00\rangle + \sqrt{1-a_i}\,|11\rangle, \quad i = 1, \ldots, n, \tag{2}$$

where $\frac{1}{2} \leq a_i \leq 1$. In other words, our scheme converts a collection of two-qubit partially entangled pairs having different amounts of entanglement into a bunch of Bell pairs with probability 1. Whereas the above probabilistic concentration processes run the risk of losing all the entanglement contained in the initial states with certain probability, our concentration scheme answers the question of how many Bell pairs can be distilled from the partially entangled pairs without gambling.

The maximum number of Bell pairs extracted in the deterministic entanglement concentration is

$$k_{max} = \left\lfloor \sum_{i=1}^{n} -\log_2 a_i \right\rfloor, \tag{3}$$

where $\lfloor x \rfloor$ represents the largest integer equal to or less than x. Furthermore, it is shown that only two-pair collective manipulations suffice for the optimality. This contrasts sharply with the fact that the probabilistic concentrations known so far require a collective manipulation of all entangled pairs for the optimality. Finally, this scheme reveals an entanglement measure for the deterministic concentration.

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