Quantum Circuit Design for NMR Quantum Computers

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The quantum algorithm is represented by unitary operations. However, the quantum computer cannot directly execute the general unitary operation. The operations that the quantum computer can execute are restricted by the physical system we use as the quantum computer. Therefore we need to decompose the unitary operation which realize the quantum algorithm into the unitary operations that the quantum computer can execute. Our purpose is to do that for NMR quantum computers. Some decompositions are known [1, 2]. However, they are not necessarily suitable for NMR quantum computer, because many unnecessary gates appear. Therefore we need a simplification of quantum gates derived by these decompositions, or we adopt new way of the decomposition.

First, we decompose the unitary operation for the quantum algorithm into conditional gates by using the CS decomposition [2]. Second, we decompose conditional gates that derived by first step into 1-qubit unitary gates and 2-qubit controlled unitary gates. Finally, we replace these gates with the operations we can use in NMR quantum computer.

The conditional gate is as follows:

$$U|x_1,\ldots,x_{n+m}\rangle = \begin{cases} |x_1,\ldots,x_n\rangle \otimes U_1|x_{n+1},\ldots,x_m\rangle & \text{if} \quad |x_1,\ldots,x_n\rangle = |0,0,\ldots,0,0\rangle \\ |x_1,\ldots,x_n\rangle \otimes U_2|x_{n+1},\ldots,x_m\rangle & \text{if} \quad |x_1,\ldots,x_n\rangle = |0,0,\ldots,0,1\rangle \\ |x_1,\ldots,x_n\rangle \otimes U_3|x_{n+1},\ldots,x_m\rangle & \text{if} \quad |x_1,\ldots,x_n\rangle = |0,0,\ldots,1,1\rangle \\ \qquad\vdots & \\ |x_1,\ldots,x_n\rangle \otimes U_{2^n-1}|x_{n+1},\ldots,x_m\rangle & \text{if} \quad |x_1,\ldots,x_n\rangle = |1,\ldots,1,0\rangle \\ |x_1,\ldots,x_n\rangle \otimes U_{2^n}|x_{n+1},\ldots,x_m\rangle & \text{if} \quad |x_1,\ldots,x_n\rangle = |1,\ldots,1\rangle \ , \end{cases}$$

where $n = 1, 2, \ldots, m = 1, 2, \ldots$ and $U_1, \ldots, U_{2^n}$ is $m \times m$ unitary operator.

We also devise the decomposition of the 2-qubit controlled unitary gate. Generally the 2-qubit controlled unitary gate is decomposed by using the controlled-NOT gate. However, this is not suitable for NMR quantum computer. We could decrease the number of decomposed gates by using the controlled rotation gate (elementary gate in NMR quantum computer).

We could decompose any n-qubit unitary gate ($n \leq 4$) into the unitary gates that we can execute in NMR quantum computer and n-qubit gate ($n > 4$) into conditioned gates.

References

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