Generation and Detection of Entangled Electrons

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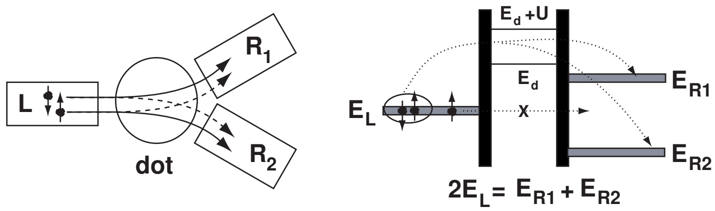

Fig. 1. Quantum dot entangler and energy band diagram in the empty dot regime.

This poster addresses how one might generate entangled Bell-singlet states with electrons [1]. In Figure 1, we consider an empty quantum dot. The input lead is at energy E_{L} and the output leads are at energies E_{R1} and E_{R2}. These energy levels have narrow width, and energy conservation suppresses single-electron tunneling. Therefore, two-electron co-tunneling through virtual states becomes the lowest-order contribution to the total current, I, from the left lead to the right leads. Furthermore, energy conservation dictates that one electron must go to R_1, and the other electron must go to R_2. Using perturbation theory in the small tunneling matrix element, V, of the Anderson Hamiltonian in the interaction picture, we calculate all possible virtual "paths" that two electrons can traverse the dot. The interference between these paths results in an *entangled Bell singlet state only* at the output. The triplet states destructively interfere. The Anderson Hamiltonian and the resulting singlet current, I^{singlet}, (proportional to transition rate, ω) is:

$$H_{\mathrm{Anderson}} = \sum_{k,\sigma,\eta\in\{\mathrm{L},\mathrm{R}_1,\mathrm{R}_2\}} E_\eta a^+_{\eta,\sigma,k} a_{\eta,\sigma,k} + \sum_{\sigma} E_d c^+_{d,\sigma} c_{d,\sigma} + U n_{d\uparrow} n_{d\downarrow} + \sum_{k,\sigma,\eta\in\{\mathrm{L},\mathrm{R}_1,\mathrm{R}_2\}} \left(V_{\eta,k} a^+_{\eta,\sigma,k} c_{d,\sigma} + h.c. \right)$$

$$I_{\mathrm{L}\to\mathrm{R}_1,\mathrm{R}_2} = I^{\mathrm{singlet}}_{\mathrm{L}\to\mathrm{R}_1,\mathrm{R}_2} \propto \omega_{\mathrm{L}\to\mathrm{R}_1,\mathrm{R}_2} = \left| \frac{2UV_{\mathrm{L}}^2 V_{\mathrm{R}_1} V_{\mathrm{R}_2}}{E_{\mathrm{L}} \left(2E_{\mathrm{L}} - U \right) E_{\mathrm{R}_1} E_{\mathrm{R}_2}} \right|^2$$

where the input state consists of two electrons in the left lead, $a^+(a)$ and $c^+(c)$ are the creation (annihilation) operators for the leads and dot respectively, σ is the spin, k is the momentum, U is the charging energy for two electrons in the dot, and V is the tunneling matrix element connecting the leads and the dot. For simplicity, we have taken E_{d} as an energy reference (E_{d} =0) and k=k' for the two electrons at the input. Note that since $E_{\mathrm{L}} < 0$, $I_{L\to R_1,R_2}$ increases to its saturated value in the limit of large U. Further, the entangled Bell singlet state generation does require a non-linear Coulomb interaction, as manifest in the charging energy, U. This type of entangled electron pair generation is analogous to a photon $\chi^{(3)}$-parametric amplification (a four-wave mixing process).

A Hanbury Brown and Twiss-type intensity interferometer [2,3] and electron collision analyzer [4,5] could be used to characterize entanglement in a proposed entangled electron bunching/anti-bunching experiment [6,7]. A quantum dot may provide a means to generate the entangled Bell-singlet state. Once an electron entangler is demonstrated, an interesting next step would be to violate Bell's inequality in an electron system [8].

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