# Lower bound for the quantum capacity of a discrete memoryless quantum channel

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The quantum capacity of a quantum channel is the amount of quantum states that can be reliably transmitted through the channel. It is one of fundamental unsolved problems in the quantum information theory. Except the quantum erasure channel, we know only lower and upper bounds for the quantum capacity of a quantum channel, and in addition, a tight lower bound is not known for a general memoryless quantum channel. In this talk we shall demonstrate a lower bound for the capacity of a general memoryless quantum channel. A quantum channel is said to be memoryless if the state change of one transmitted quantum system (of the fixed degree of freedom) is statistically independent of the state change of another.

The problem of quantum capacity has attracted great attention, and rapid progress has been made. To be precise, the quantum capacity of a binary memoryless channel $\Gamma$ is the maximum number $Q(\Gamma)$ such that for any rate $R < Q(\Gamma)$ and any $\epsilon > 0$ there exists an $[[n, k]]$ quantum code $Q$ with $k/n \geq R$ such that the fidelity between the recovered state and the original state $|\varphi\rangle \in Q$ is at least $1 - \epsilon$ for any $|\varphi\rangle$.

It is the random coding that the most common technique in classical information theory to show that a specific rate is achieved by a code in a specific class of codes. For example, Elias showed that the capacity of the binary symmetric channel is achieved by binary linear codes using random coding. A proof by random coding is usually proceeded as follows: one first calculates the average of error probability of all codes in a specific class of codes of the same rate and the same code length, then shows that the average converges to 0 as the code length increases, and finally concludes that there exists at least one sequence of codes of the fixed rate with which the error probability converges to 0.

The technique of random coding is also used in the quantum information theory. Gottesman showed using random coding that the lower bound for the quantum capacity of the depolarizing channel can be achieved by stabilizer codes. However, his proof does not seem to extend easily to the case of general memoryless channel. The aim of the present talk is to derive a lower bound for the quantum capacity of a general memoryless channel by using random coding of stabilizer codes. In our argument of random coding, we shall use the fidelity as a replacement of error probability in the classical random coding, and use the idea behind the proof of the quantum Gilbert-Varshamov bound for the stabilizer codes. As a byproduct, we also improve the quantum Gilbert-Varshamov bound for stabilizer codes. Our improved bound is slightly better than the quantum Gilbert-Varshamov bound for general codes. The full version of this abstract can be found as `arXiv:quant-ph/0105151`.