Quantum computation in computational geometry

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We discuss applications of quantum computation to geometric data processing such as convex hulls, minimum enclosing balls, linear programming, and intersection problems. Technically, we apply well-known Grover's algorithm and its variants [1] to solve them in sublinear time.

We consider geometric data processing problems, where the input is a set of n geometric objects in a d-dimensional space. In the standard RAM model, we construct a data structure to handle the data. Voronoi diagrams, convex hulls, and simplex range search data structures are examples of data structures for solving a variety of geometric problems; Unfortunately, they are not very effective if the number of dimensions is large (say, larger than three).

The aim of this paper is to show that the quantum model can be considerably advantageous to the RAM model for geometric data processing. As random sampling methods are useful in probabilistic methods to geometric data processing, we show that quantum computation can be applied to it even in higher dimensions.

We first utilize a view of Grover's algorithm as a biased random sampling method [1], which we call *quantum sampling*, and show that several query problems in a set of n points can be done in $O(\sqrt{n}\log n)$ time with high probability for any d without any preprocessing. For example, the extremal query, the nearest neighbor query, and the farthest neighbor query are such queries. In the dual form, if the input is a set of n hyperplanes (or halfspaces), we can do the separation query and ray shooting query in $O(\sqrt{n})$ time.

Next, as a showcase problem of our approach, we consider the problem of finding the lowest point in the intersection of n upper-halfspaces in the d-dimensional Euclidean space. The problem can be solved in $\Theta(n)$ time if d is a constant in RAM model. Regarding the problem as a parametric minimax problem [2], and by combining the quantum minimum finding algorithm and multidimensional searching technique, we can solve it in $O(\sqrt{n}\log^{2d-1} n)$ time.

Then, we show some other geometric optimization problems such as the minimum enclosing ball problem and linear programming problems can be solved efficiently in quantum model if d is a constant. We can regard the minimum enclosing ball problem as a minimization of the parametric farthest-neighbor-query problem [2]; in other words, it is computation of the lowest point in the upper envelope of surfaces defined from the distance functions from points. Then it can be solved in $O(\sqrt{n}\log^{2d+1} n)$ time.

Also, some output-sensitive convex-hull algorithms and the ellipsoid algorithm for solving a general dimensional convex programming problem can be accelerated by using the quantum model.

References

[1] L. Grover, Rapid sampling through quantum computing, *Proc. 32nd STOC* (2000), 618–626.

[2] T. Tokuyama, Minimax parametric optimization problems and multi-dimensional parametric searching, *Proc. 33rd STOC* (2001) 75–83.