Detecting the inseparability and distillability of continuous variable states in Fock space

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The partial transposition(PT) operation is an effecient tool in detecting the inseparability of a mixed state. We give an explicit formula for the PT operation for the continuous variable states in Fock space. We then give the necessary and sufficient condition for the positivity of Gaussian operators. Based on this, a number of creterions on the inseparability and distillability for the multimode Gaussian states are naturally drawn. We finally give an explicit formula for the state in a subspace of a global Gaussian state. This formula, together with the known results for Gaussian states, gives the criterions for the inseparability and distillability in a subspace of the global Gaussian state.

Quantum entanglement or inseparability plays a fundamentally important role in the fascinating area of quantum computation and quantum information. To check the inseparability of a (mixed) state, a particularly elegant mathematical tool, partial transposition(PT) operation was given by Peres. This method was later on applied to various discrete states. While a lot of studies in quantum computation and quantum information(QCI) are based on the discrete states, recently a number of new proposals in the area were raised by using the continuous variable states. In certain respects, the continuous variable states can have advantages to the discrete states. Consequently, the entanglement detection for continuous variable states is an interesting and important topic. The distillability and and inseparability for Gaussian states were broadly studied recently. In these works, the starting point is the Wigner function, whose PT operation is simple and clear. Actually, the PT operation in Fock space is also simple. In this presentation, we directly use Peres′ PT operation to the density operator itself. We shall first give an explicit formula for the PT operation for the continuous variable states in Fock space. We then check the inseparability of various types of practically existing states. We formulate a new type of practical impure squeezed state in the form of mixture of different squeezed vacuum states and Fock number states. We show this state and the squeezed state in phase damping channel are always inseparable. We then apply the PT operation to the Gaussian operator. We shall give the general criterion on the positivity of multi-variable Gaussian states in Fock space. This can be used to give the necessary and sufficient condition for inseparability or distillability for $N \otimes 1$ system or arbitrary bipartite system, respectively. More generally, with the positivity condition for Gaussian state in Fock space we can easily investigate the entanglement prpoerty in a subspace of a global Gaussian state.

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