

On Irreversibility, Dilation in Conditional Isometric Process

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Abstract

Generation of arbitrary quantum state involving entangled state is a main subject in engineering theory. It requires more general mathematical theory rather than conventional physical theory. For example, dynamic procedure of entangled state is described by completely positive map from quantum filtering theory based on the most general physical model. In fact, any entanglement can be obtained as the unitary entanglement in the result of quantum filtering dilation by tracing out nonobservable degree of freedom of a quantum environment. It suggests that any generation of quantum state in future works must take into account "environment" or "control of interaction" which exceeds isolated systems.

We are concerned with treating generation scheme of quantum states, including truly quantum couplings, as quantum channel (quantum dynamical map). Mathematical methods to describe such channels have been discussed by many mathematicians. The most conventional one is application of normal completely positive maps to "states" on a von Neumann algebra, or compound state theory between input and output, which were devoted mainly to describe features of irreversibility and quantum coupling between input and output. The irreversibility as physical phenomena comes from a generalized H -theorem by Lindblad based on Umegaki quantum relative entropy. Although general compound states can describe general quantum channels, it is not dynamical map.

For discrete pure state input, conditional isometric theory, which can describe non unitary maps, is very convenient in the information theoretical point of view for extensive physical processes as mathematical channel model, and to draw the cut-off line of conventional quantum mechanics. Furthermore, it can describe channel which allow to "control the environment".

In this talk, we will introduce a brief theory of conditional isometric process and show an example of its application to quantum branching process in which physical meaning of no preserving property of norm in

CP map formalism is discussed. It will suggest that quantum Bayes like theorem appears automatically as conditional isometric sense. Based upon the above, we will present some problems for generation of entangled state of non-orthogonal state.