Quantum channel coding : theory and experiment

Masahide Sasaki, Mikio Fujiwara, Masahiro Takeoka, and Jun Mizuno
Communications Research Laboratory,
Koganei, Tokyo 184-8795, Japan
e-mail:psasaki@crl.go.jp

Quantum channel coding deals with how reliably one can transmit classical alphabet over a quantum channel. This is one of the central issues of quantum commuication. Given a source of quantum states (letter states), $\{ |\psi_i\rangle\}$, block sequences are made as direct products of the letter states, $\{ |\psi_i\rangle\}^{\otimes n}$, and some of them are selected, assigned to represent each alphabet and transmitted as *codeword states*, $\{|\psi_{i_1}\rangle \otimes \cdots \otimes |\psi_{i_n}\rangle\}$. They are then decoded quantum mechanically. We introduce basic notion and present recent experimental progress toward photonic realization of quantum channel coding.

As a letter source, we consider the ternary symmetric states of a single photon polarization. Each letter is used with equal a priori probability. Since they are nonorthogonal states, they cannot be distinguished perfectly. This fact can, in turn, be used for, e.g. quantum key distribution [1]. For reliable transmission with this source, let us first consider the conventinal channel coding scheme depicted in Fig.1. The k bits of message $\{X_{\mathbf{i}}\}$ are encoded by the codewords of length n. Each optical pulse is detected separately, and converted to an electrical pulse. The pulse sequence is then decoded by the electric decoder.

The first question is what kind of measurement should be used. The conventional way is the von Neumann measurement which is mathematically a projection of the signal states onto the orthonormal basis, $|H\rangle$ and $|V\rangle$. Physically it is implemented by a polarizer and two photon counters. The extracted information is measured by the mutual information $I(X:Y)$, which is $I_1^{\mathrm{vN}} = 0.459$ bit. This means that if the transmission rate $R = (k/n)\log_2 3$ is kept below 0.459, then the decoding error P_{e} can be arbitrarily small by taking $n \to \infty$. (The factor $\log_2 3$ is due to the ternary letters used.) On the other hand, the optimal measurement is realized by the polarization interferometer (Fig.2) which is a physical implemetation of the generalized measurement consisting of the three nonorthogonal basis vectors $\{|\omega_0\rangle, |\omega_1\rangle, |\omega_2\rangle\}$ [2]. The attainable information, called the accessible information, is then $I_1^{\mathrm{Acc}} = 0.585$ bit. This means that the information transmitted can be increased from $0.459n/\log_2 3$ bit to $0.585n/\log_2 3$ bit under the same error level.

This optimal decoder is implemented in our laboratory, and the predicted performance is demonstrated by using an attenuated CW laser light and single photon counters. Our polarization interferometer exibits the discrimination performance of the signals, corresponding to 96% of the theoretical maximum I_1^{Acc} at the extrapolated point of the perfect efficiency of the single photon counter. Fig. 3 shows the experimental mutual information as a function of the relative offset angle between the signal set and the measurenent vectors [3].

Thus by installing the generalized measurement into the conventional channel coding, one can enhance the practical communication capacity. The next step is to introduce quantum computation to the decoding process. If a two-bit quantum computation becomes possible, one can then install this into the conventional decoding scheme as shown in Fig. 4. A well-known example of length 2 coding consists of the set of codewords $\{|\psi_0\rangle\otimes|\psi_0\rangle, |\psi_1\rangle\otimes|\psi_1\rangle, |\psi_2\rangle\otimes|\psi_2\rangle\}$ and the two bit collective decoding based on the square root measurement construction [4]. In the scheme of Fig. 4, each pair of two letter states is first processed by an appropriate quantum circuit, then converted to the classical signals (electric signals), and finally processed by the classical decoder.

The attainable transmission rate in this scheme is $I_2^{\mathrm{SRM}}/2 = 0.685$ bit. Now the single shot channel capacity C_1 for the ternary signals is conjectured as $C_1 = 0.645$ bit which is realized by using only two letters with equal probabilities 0.5 and by detecting with the von Neumann measurement [5]. This suggests that the transmittable information can be increased in a superadditive manner as the length n increases.

This is indeed an information theoretic quantum coding gain which cannot be seen in conventional memoryless channel coding. We will present the experimental demonstration of this superadditive quantum coding gain in the pulse position plus polarization coding of the ternary letter set.

[1] S.J.D. Phoenix, S.M. Barnett, and A. Chefles, J. Mod. Opt. **47**, 507 (2000).
[2] M. Sasaki, S.M. Barnett, R. Jozsa, M. Osaki, and O. Hirota, Phys. Rev. A **59**, 3325 (1999).
[3] J. Mizuno, M. Fujiwara, M. Akiba, T. Kawanishi, S. M. Barnett, and M. Sasaki, Phys. Rev. A,**65**, 012315 (2002).
[4] A. Peres and W. K. Wootters, Phys. Rev. Lett. **66**,1119 (1991)
[5] M. Osaki, et al., *Quantum Commun., Comput., and Meas. 2* (Eds. P. Kumar, M. D'Ariano, and O. Hirota, Prenum publishers, New York, 2000) 17.

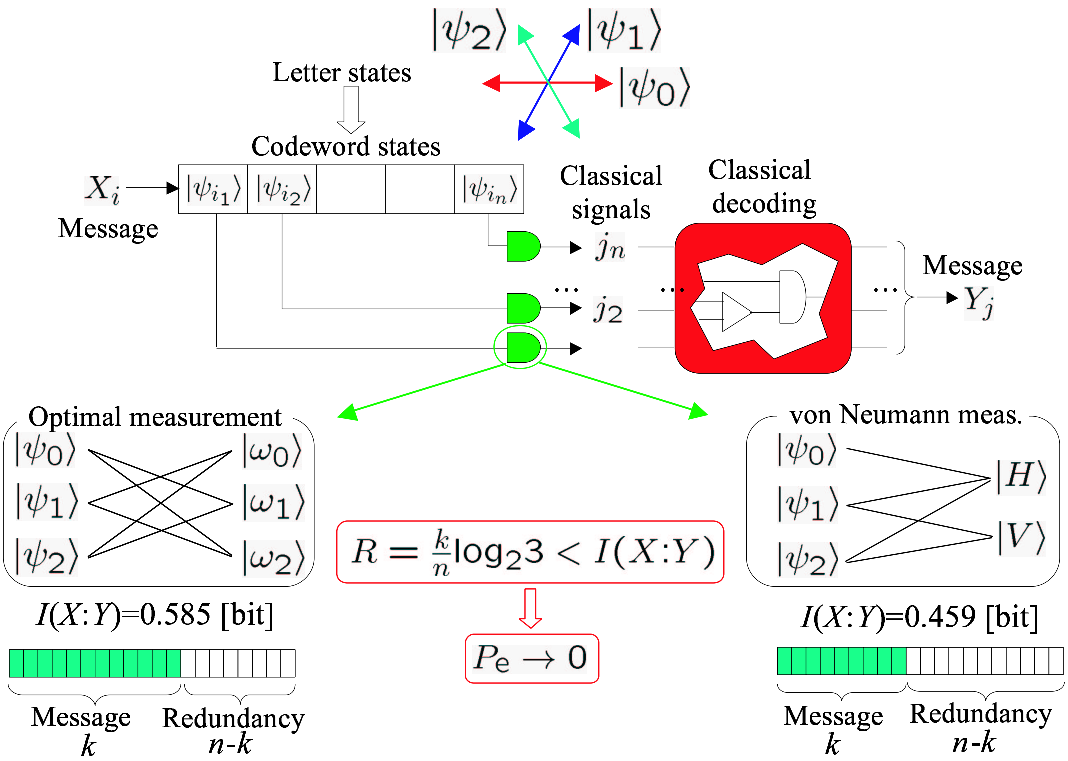

Fig. 1. Scheme of conventional channel coding.

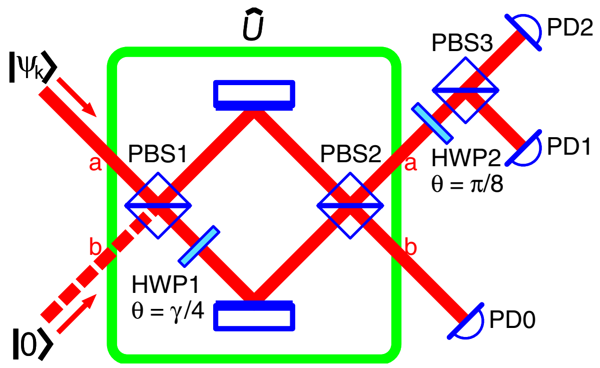

Fig. 2. Decoder structure for accessible information.

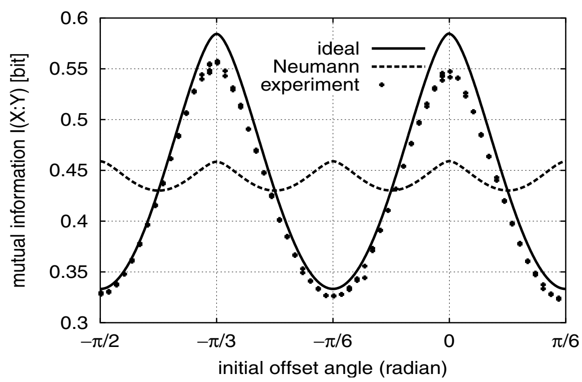

Fig. 3. Experimental mutual information.

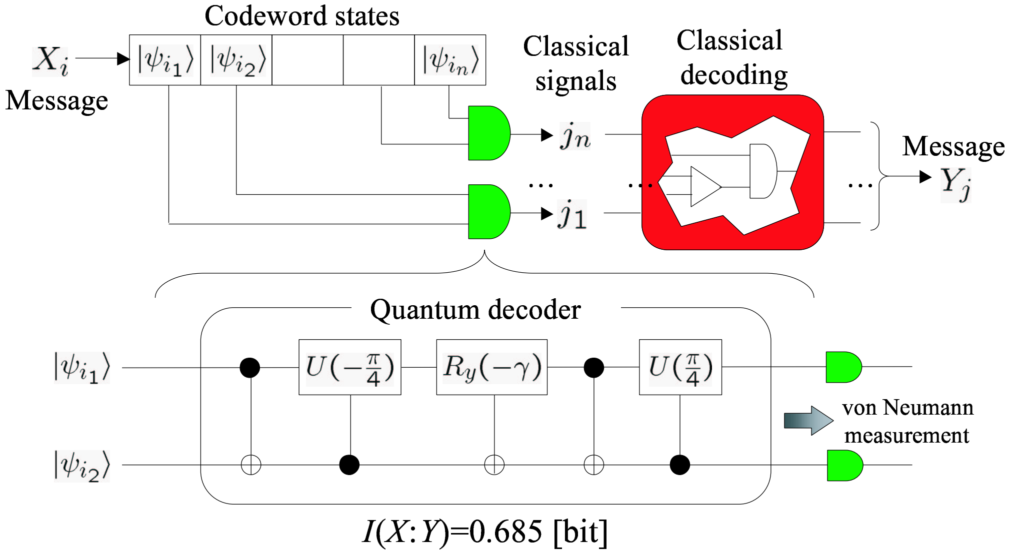

Fig. 4. Quantum-classical hybrid channel coding scheme.