

Information-Spectrum Approaches to Quantum Information Theory

Hiroshi Nagaoka *

One of the principal themes of information theory is to establish a link between two different kinds of quantities: one is an operational quantity which is defined through the asymptotic optimality on some concrete concept such as code length, compression rate, error probability, etc., while the other is an abstract information quantity such as the entropy, divergence, mutual information, etc. The same theme is now being pursued extensively in quantum information theory, and lots of mathematical results have been reported so far.

In the so-called information-spectrum methods in the classical information theory [1], the process of establishing a link between an operational quantity and an entropy-like information quantity is intentionally divided into two parts, making the third kind of quantities — *information spectrum* — intervene. In the methods, the asymptotic optimality of an operational quantity is characterized by a limiting expression on information spectra (asymptotic behavior of logarithmic likelihoods) in the most general setting, while the information-spectrum quantity is rewritten to an entropy-like quantity in a specific situation mostly as a direct consequence of a limiting theorem in the probability theory such as the law of large numbers, the Shannon-McMillan-Breiman theorem, etc. Such a framework brings not only generality but also transparency of mathematical arguments.

Extending the information-spectrum methods to the quantum case is an attractive subject, which is expected to provide both the asymptotic optimality of operational quantities and the limiting law governing quantum stochastic situations with transparent and comprehensive understanding. In this talk I would like to review recent attempts to develop the quantum information-spectrum methods made by Hayashi, Ogawa and myself, mainly based on [2, 3, 4], and discuss their significance in the quantum information theory.

References

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*Graduate School of Information Systems, University of Electro-Communications, 1-5-1, Chofugaoka, Chofu-shi, Tokyo 182-8585, Japan. Email: nagaoka@is.uec.ac.jp