Simulating the Effects of Quantum Error Correction Schemes

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1 Introduction

It is important to protect quantum information against decoherence and operational errors, and quantum error correcting codes (QECC) are the keys to solving this problem. Of course, just the existence of codes is not efficient. It is necessary to perform operations fault-tolerantly on encoded states [2, 6, 5] because error correction process (i.e., encoding, decoding, syndrome measurement and recovery) itself induces an error. By using simulation, this paper investigates the effects of three QECC (the five qubit code, the seven qubit code and the nine-qubit code) and their fault-tolerant operations when the error correction process itself induces an error.

2 Quantum Computer Simulation System (QCSS)

The features of our quantum computer simulation system (QCSS) are fully described in the Reference [4]. We briefly explain the error model implemented in the QCSS.

QCSS assumes that the quantum depolarizing channel as the decoherence error model. QCSS represents inaccuracies by adding small deviations to the angles of rotations and phase shifts. Each error angle is drawn from Gaussian distribution with the standard deviation.

QCSS does not deal with mixed states. Therefore, in order to compute the fidelity, the experiments are repeated $10^5 \sim 10^7$ times and the average values are used.

3 Error-correction codes and circuits

We deal with three QECC, that is, the nine qubit code, the seven qubit code and the five qubit code. They correct one arbitrary error.

The error correction circuits (i.e., encoding, decoding, syndrome measurement and recovery circuits) for the nine qubit code are shown in Figure 1. Of course, there exists the syndrome measurement and recovery circuit based on the stabilizer formalism [1]. However, we use this circuit*, since it is simpler and hence induces less errors.

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*Note that this recovery circuit cannot be used without decoding circuit.

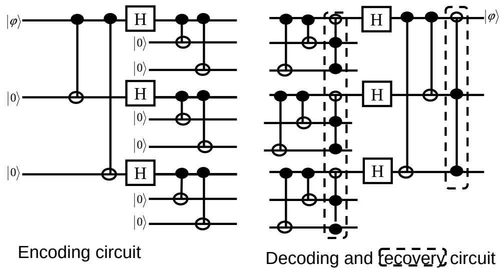

Figure 1: The error correction circuit for the nine qubit code

As for the seven qubit code, we use the error correction circuits described in the Reference [7]. As for the five qubit code, we use the error correction circuits described in the Reference [2].

It will be helpful to compare the features of these three QECC. Table 1 summarizes the comparison among three QECC.

Table 1: Comparison among the nine qubit code, the seven qubit code and the five qubit code

Depth of error-correcting circuits.

	[[9, 1, 3]]	[[7, 1, 3]]	[[5, 1, 3]]
Encoder (Decoder)	5	4	10
Syn. measurement and recovery	2	28	22

The number of (minimum) required qubits.

	[[9, 1, 3]]	[[7, 1, 3]]	[[5, 1, 3]]
qubits	9	8	6

Transversal operations implementation [2].

[[9, 1, 3]]	[[7, 1, 3]]	[[5, 1, 3]]
Hard	Easy	Hard

As for error-correcting circuit complexity, the nine qubit code is simplest. Furthermore, it does not require an ancilla qubit. About transversal operations, in which the operations act independently on each qubit in the block, the seven qubit code is best.

4 Simulation Results

Quantum computation using encoded qubits, is a sequence of computation directly (or indirectly) on the encoded qubits along with periodic error-correction processes. If these processes are not performed sufficiently, then multiple errors may occur between these processes, which results in an uncorrectable state. However, if these processes are performed too much, then the circuit size becomes too large, which increases the total amount of errors. Our simulation aims to investigate how often to perform these operations.

We adopt Hadamard transformation as main computation. We have performed the simulation using QCSS. The start state of the quantum register is $|0_L\rangle$ (logical 0 bit). The x-axis in the Figure 2 and 3 show the even iteration number of Hadamard transformation. The y-axis in the Figure 2 and 3 show the fidelity.

4.1 Effects of Ancilla operations

We have investigated the effects of fault-tolerant ancilla operations by using the seven qubit code. We set the number of main computation 200. We have performed the simulation under the following two conditions.

First, only one qubit is used to compute each bit of the syndrome. Second, four qubits are used to compute each bit of the syndrome. Of course, we prepare the ancilla in a Shor state [6] that reveals the errors without revealing the data, and four XOR gates are performed transversally in computing the syndrome. The depth of error-correcting circuits becomes 20.

Table 2: Final fidelity with decoherence errors for the one qubit ancilla case and the four qubit ancilla case

rate	Ancilla bit	Frequency of recovery process				
		200G	100G	50G	10G	1G
10^{-5}	1bit	0.9989	0.9985	0.9983	0.9948	0.9584
	4bit	0.9976	0.9969	0.9959	0.9891	0.9631
10^{-4}	1bit	0.9842	0.9835	0.9807	0.9491	0.7018
	4bit	0.9621	0.9766	0.9557	0.9442	0.7031
10^{-3}	1bit	0.7245	0.7411	0.7411	0.5915	0.4384
	4bit	0.6693	0.6696	0.6580	0.5136	0.3997

Table 2 shows the final fidelity with decoherence errors ($rate = 10^{-5} \sim 10^{-3}$) for the one qubit ancilla system and the four qubit ancilla system. Theoretically, the four qubit ancilla system is better than the one qubit ancilla system in terms of fault-tolerant operations. However, from Table 2, we can see that the four qubit ancilla system is not always better than the one qubit ancilla system. This is because the error probability of four qubit ancilla system is higher than that of the one qubit ancilla system.

4.2 Effects of the frequency of the error correction operations

We have experimented to investigate how the frequency of the error correction operation affects the fidelity with decoherence errors ($rate = 10^{-6} \sim 10^{-2}$) for three QECC. We briefly state the results due to limitations of space. Computation with error correction is much worse than computation without it for any frequency of the error correction operations as for the five qubit code and the nine qubit code.

The seven qubit code is effective for a decoherence rate of $10^{-6} \sim 10^{-4}$ if the error correction operation is performed at every 50 $\sim$ 200 main gate, as shown in Figure 2. We set the number of main computation 4000. The main computation is performed transversally. For example, "QECC/50" in the figure means that the error-correction operation is performed every 50 main gates. This figure also shows that error correction step performed at every 1 gate is excessive. We have also investigated the combined effect of operational and decoherence errors for three QECC. Briefly stated, the combined effect of two errors (decoherence errors and operational errors) is the product of each factor.

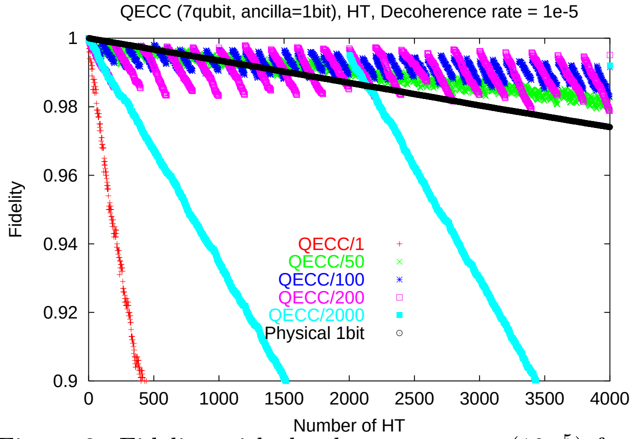

Figure 2: Fidelity with decoherence errors (10^{-5}) for the seven qubit code.

4.3 Comparison among three codes

We have made a comparison among the effects of three QECC in the asymptotic case. Figure 3 shows the fidelity with decoherence errors (rate = 10^{-3}). We set the number of main computation 2000. An error correction operation is performed at every 1 main gate. The result shows that the nine qubit code always gives the best result.

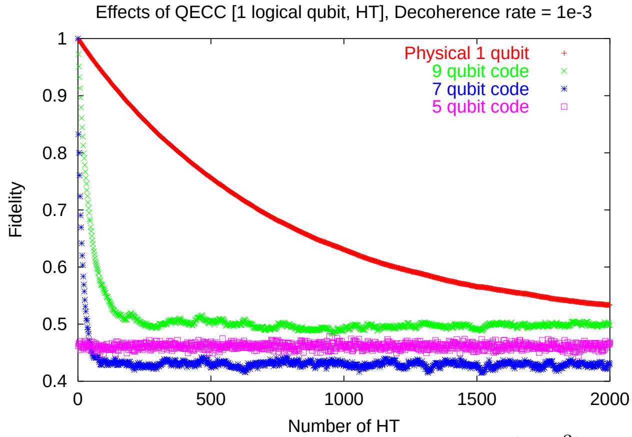

Figure 3: Fidelity with decoherence errors (10^{-3}) for three QECC

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References

[1] D. Gottesman. Phd Thesis, 1997.

[2] D. Gottesman. Physical Review A, 57(1), 1998.

[3] L. K. Grover. In Proc. of the ACM STOC, 1996.

[4] J. Niwa, K. Matsumoto and H Imai. quant-ph/0201042, 2002.

[5] J. Preskill. quant-ph/9712048, 1997.

[6] P. W. Shor. In Proc. of the FOCS, 1996.

[7] A. Steane. Royal Society of London A, 452, 1996.