Parallel Quantum Fetching and its implementation in a 7-qubit NMR quantum computer*

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The perspective enormous computing power of quantum computers(QCs) has sparked intensive efforts worldwide. Various schemes have been put forward. Liquid NMR, quantum dot, cavity quantum electrodynamics(QED), linear ion trap, superconducting qubit, solid-state NMR are just a few examples in this long list. By the way they measure, they can be put into two main categories: single qubit-measurement quantum computer (SQC), and ensemble qubit-measurement quantum computer(EQC). Quantum computers compute using quantum superposition of states and hence has quantum parallelism. Notably, it can do prime-factorization and simulate quantum systems exponentially fast. However, for some problems, the speedup is not exponential. For instance, in the unsorted database search problem, Grover's algorithm achieves square-root speedup, and it has been shown to be optimal for quantum computer. For some problem, quantum computer can not achieve any speedup. If we can make quantum computers to work in parallel, then we may gain additional computing power.

In this talk, we will first introduce the concept of parallel quantum computation(PQC). Then we give one specific example of parallel quantum computing algorithm: a quantum fetching algorithm [1]. In the parallel quantum fetching(PQF) algorithm, unsorted database search can be completed with the ultimate maximum: a single query is sufficient to find out all the marked states. Here we report the experimental realization of the PQC fetching algorithm in a 7-qubit NMR system.

The capability of doing quantum computing in parallel lies in the unitarity nature of QC operations. A function f in a QC is carried out by a unitary transformation U_f. To calculate the function value of basis state $|i\rangle$, we apply the unitary operation U_f on the basis state

$$U_f(|0\rangle|i\rangle) = |f(i)\rangle|i\rangle, \tag{1}$$

where the second register records the function value, and the first register records the argument. This unitary transformation is realized in quantum computer by a series of basic one- ad two- qubit gates. Most importantly, this unitary transformation contains *all* the rules for computing all the basis states. This unitary operator evaluates the function value for $|00...0\rangle$, and the same pulse sequence also evaluates the function value of $|11...1\rangle$ and any other inputs! When applied to a superposition of basis states it evaluates the function values of all the basis states!

$$U(\sum_{i=0}^{N-1}|0\rangle|i\rangle) = \sum_{i=0}^{N-1}|f(i)\rangle|i\rangle. \tag{2}$$

The capability of computing using quantum superposed state, especially entangled states provides quantum parallelism which is an important source for quantum computer. When applied to an ensemble in mixed state, the same unitary operator transforms each constituent sub-ensemble of basis states into their corresponding function values:

$$\rho \to \rho' = U_f \rho U_f^{-1} = p_0|f(0),0\rangle\langle f(0),0| + p_1|f(1),1\rangle\langle f(1),1| + ... + p_{N-1}|f(N-1),N-1\rangle\langle f(N-1),N-1|. \tag{3}$$

This property enables us to combine the ideas of both quantum parallelism and classical parallelism to gain greater computing powers. The unitary operator is ubiquitous to all molecules in the ensemble and acts as a super-commander to all the basis states. Each basis state goes through the transformation required by the specific function irrespective of its surroundings: alone, or lined up hand by hand with other basis states in a superposed pure state, or scattered here and there in different sub-ensembles in a mixed state.

Liouville space NMR quantum computation(L-QC) [2] is a special case of parallel quantum computation. Using L-QC, we have achieved the absolute optimum for unsorted database search problem [1]. The eigen-state of the nuclear

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spin system $|\phi_m\rangle$ of a weakly-coupled system can be expressed as a direct product of the single-spin eigen-functions $|\alpha_i\rangle$, $|\beta_i\rangle$ of the spin projection operator of the i-th nuclear spin $I_{iz} = 1/2\sigma_z$, for instance,

$$|\phi_m\rangle = |\alpha_1\rangle|\alpha_2\rangle|\beta_3\rangle...|\alpha_{n-1}\rangle|\beta_n\rangle = |\alpha\alpha\beta...\alpha\beta\rangle = |001...01\rangle \Longleftrightarrow \rho_m = I_1^\alpha I_2^\alpha I_3^\beta ... I_{n-1}^\alpha I_n^\beta, \quad (m = 1, ..., 2^n), \tag{4}$$

where $|\alpha\rangle = |0\rangle$(up) and $|\beta\rangle = |1\rangle$(down) where $I^\alpha = |\alpha\rangle\langle\alpha| = (\mathbf{1} + 2I_z)/2$, $I^\beta = |\beta\rangle\langle\beta| = (\mathbf{1} - 2I_z)/2$.

A peak in the spectrum of an ancilla qubit 0, corresponds to the transition between two levels $|i_0 = 0i_1...i_n\rangle$ and $|i_0 = 1i_1...i_n\rangle$, and its frequency is $\omega_{0i_1i_2...i_n \leftrightarrow 1i_1i_2...i_n} = \omega_0 + \sum_{k=1}^{n} \pi J_{0k}(-1)^{i_k}$. Each of the 2^n peak in the ancilla bit's spectrum corresponds to a transition between $|0i_1i_2...i_n\rangle$ and $|1i_1i_2...i_n\rangle$. The phase of the peak can tell the direction of the transition of the ancilla bit. If the state of the ancilla bit is in 0, then the transition is from $|0i_1i_2...i_n\rangle$ to $|1i_1i_2...i_n\rangle$, and the peak is upward in the spectrum. If the state of ancilla bit is at 1, then the transition is from $|1i_1i_2...i_n\rangle$ to $|0i_1i_2...i_n\rangle$, and the peak is downward. Thus the 2^n peaks in the ancilla bit spectrum correspond to 2^n numbers of the n qubit, $|i_1i_2...i_n\rangle$. The upward or downward nature of the peak indicates the ancilla bit's state. This technique has been used in experiment to realize the Bruschweiler's algorithm [3], and it has been shown effective [4]

Our algorithm consists of three steps: 1) First prepare the $n+1$ qubit system in state I_0^α state, i.e., the ancilla bit is in state $|0\rangle$ and the remaining n qubit are in complete mixed state. This state actually represents 2^n items of the database, since apart from a scaling factor proportional to $\frac{\delta}{2^n} = \frac{\gamma_0 B}{kT 2^n}$,

$$\rho_{in} = |000...0\rangle\langle 000...0| + |000...1\rangle\langle 000...1| + ... + |011...1\rangle\langle 011...1|. \tag{5}$$

2) Apply the query function f to the system and store the result of the query in the ancilla bit. After the operation of the query, the state of the NMR system becomes

$$\rho_{out} = |f(0)000...0\rangle\langle f(0)000...0| + |f(1)000...1\rangle\langle f(1)000...1| + ... + |f(N-1)011...1\rangle\langle f(N-1)011...1|. \tag{6}$$

3) Measure the ancilla bit's spectrum and read out the results directly from the spectrum. Marked items will be those states with peaks downwards in the spectrum.

One qubit is used as ancilla as required by L-QC [2], the other six qubits represent a database with $2^6 = 64$ items, namely $|0\rangle = |000000\rangle$, $|1\rangle = |000001\rangle$, ...,$|63\rangle = |111111\rangle$. We have implemented the fetching algorithm in a 7-qubit NMR quantum computer [5]. Our experiment is carried out on a VARIANT INOVA-600MHz spectrometer. The liquid sample is ^{13}C labeled crotonic acid with formula $C^1H_3^3C^2H^1 = C^3H^2C^4O_2H$, which is manufactured by Cambridge Isotope Laboratories Inc. The seven qubits are the four carbon nuclear spins, two proton spins in the middle connected with the two carbon in the middle. The 3 methyl proton spins(denoted M) have the same chemical shift, and they are treated as a single qubit. The sample is firstly dissolved in a 0.5 ml acetone and then injected into a 5 mm tube, deaired and sealed. The ancilla qubit is chosen as C^2. Nuclear spins of H^1, C^3, C^1, H^3, C^4 and H^2 are assigned as qubit 1, 2, 3, 4, 5 and 6 respectively, in decreasing order by the J magnitude of respective nuclear spin with the ancilla bit. The query checks whether an items's first 3 bits values are 100. The whole pulse sequence involves the coherent control of all the 7 qubits. It consists of 47 spin-selective radio frequency pulses, together with several free evolution intervals. The decoupling is implemented using the compound decoupling pulse sequence WALTZ16. The total duration of the pulse sequence is about 90 ms which is well within the system's $2s$ decoherence time (T_2). We actually implemented the algorithm directly on the thermal equilibrium state without preparing I_0^α because we have used spin-selective pulses. The fetching of the marked states is well demonstrated.

The algorithm can be scaled up to systems with arbitrary n qubit system. They might be used in other ensemble quantum computers.

We have put forward the basic ideas of parallel quantum computing. We gave a specific example of the PQC by giving a fetching algorithm that finds marked items with only a single query. We also implement the algorithm in a 7 qubit NMR ensemble quantum computer. This work is supported in part by China National Science Foundation, the National Fundamental Research Program, Contract No. 001CB309308 and the Hang-Tian Science foundation.

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