Quantum Cloning Machines

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Abstract

We study and review some recent works on quantum cloning machines, in particular, the universal quantum cloning machines and phase-covariant quantum cloning machines. The unitary cloning transformation for universal cloning machine is presented. And the physical implementation of this cloning machine is studied. We present the optimal phase-covariant quantum cloning machine for qudit. For qubit case, we study the general N to M case of phase-covariant cloning machine.

1 Universal quantum cloning machine for qudits

We present the result of UQCM formulated by unitary quantum cloning transformation[6]. And we show that the fidelities, both single qudit fidelity and multiqudit fidelity, of this unitary quantum cloning transformation achieve the upper bound obtained by Werner and Keyl and Werner[1].

We propose the N to M quantum cloning transformation for d-level quantum system as follows[6],

$$U_{NM}|\mathbf{n}\rangle \otimes R = \sum_{\mathbf{j}=0}^{M-N} \alpha_{\mathbf{nj}} |\mathbf{n}+\mathbf{j}\rangle \otimes R_{\mathbf{j}}, \tag{1}$$

where $\mathbf{n}+\mathbf{j}=\mathbf{m}, i.e., \sum_{k=1}^{d} j_k = M-N$, $R_{\mathbf{j}}$ denotes the orthogonal normalized internal states of QCM, and

$$\alpha_{\mathbf{nj}} = \sqrt{\frac{(M-N)!(N+d-1)!}{(M+d-1)!}} \sqrt{\prod_{k=1}^{d} \frac{(n_k+j_k)!}{n_k! j_k!}}. \tag{2}$$

Because all kinds of symmetric states $|\mathbf{n}\rangle$ can be allowed as input states in this quantum cloning transformation, this quantum cloning machine actually not only can copy identical pure states but also arbitrary quantum states restricted to symmetric subspace. And we can show that the shrinking factor which corresponds to single qudit fidelity achieves its upper bound.

For pure input states $|\Psi\rangle^{\otimes N}$, we can have two kinds of fidelities to quantify the quality of copies, $F = \langle\Psi|\rho_{red.}^{out}|\Psi\rangle$, $\mathcal{F} = ^{\otimes M}\langle\Psi|\rho^{out}|\Psi\rangle^{\otimes M}$. After tedious and straightforward calculations, we can find the fidelities of the quantum cloning transformation (1,2) are as follows

$$F = \frac{MN+M+N}{M(N+d)}; \quad \mathcal{F} = \frac{M!(N+d-1)!}{N!(M+d-1)!}. \tag{3}$$

These two fidelities achieve the optimal fidelities obtained by Werner and Keyl and Werner[1]. Thus the cloning transformation (1,2) is the optimal UQCM for qudits.

Recently, the experiment of UQCM for qubit was reported in [3] by photon stimulated emission. For qudit, a similar scheme can be proposed[6]. The Hamiltonian of the cloning system in terms of harmonic-oscillator operators is written as [2]

$$\mathcal{H}_d = \gamma(a_1 b_1 + \cdots + a_d b_d) + H.c. \tag{4}$$

We consider general initial states in the symmetric subspace (Bose subspace)

$$|\Psi_{in},\vec{j}\rangle=\prod_{i=1}^{d}\frac{(a_i^{\dagger})^{j_i}}{\sqrt{j_i!}}\frac{(c^{\dagger})^L}{\sqrt{L!}}|0\rangle\equiv|\vec{j}\rangle_a|\vec{0}\rangle_b|N\rangle_c, \tag{5}$$

where $\vec{j}=(j_1,j_2,\cdots,j_d)$. There are L excited states in the initial state, so the number of additional copies is restricted by N. We remark that the initial states (5) to be cloned span arbitrary states in the Bose subspace and constitute an orthonormal basis.

2 Phase-covariant quantum cloning machine for qudit

If we want to optimal copy the BB84 [4] state in QKD, the UQCM can achieve the fidelity $5/6\approx 0.83$, while the phase-covariant cloning machine can achieve $1/2+1/\sqrt{8}\approx 0.85$ [5, 6]. So, phase-covariant quantum cloning machine is better than the UQCM in the coping of BB84 states. Here, we study the quantum cloning of d-level states in the form $|\Psi\rangle^{(in)}=\frac{1}{\sqrt{d}}\sum_{j=0}^{d-1}e^{i\phi_j}|j\rangle$. We propose the following phase-covariant quantum cloning transformation

$$U|j\rangle|Q\rangle=\alpha|jj\rangle|R_j\rangle+\frac{\beta}{\sqrt{2(d-1)}}\sum_{l\neq j}^{d-1}(|jl\rangle+|lj\rangle)|R_l\rangle, \tag{6}$$

The optimal fidelity can be found to be $F_{optimal}=\frac{1}{d}+\frac{1}{4d}(d-2+\sqrt{d^2+4d-4})$. In case $d=2,3$, this results agree with some known results. As expeceted, this optimal fidelity of phase-covariant quantum cloning machine is higher than the corresponding optimal fidelity of UQCM $F_{optimal}>F_{universal}=(d+3)/2(d+1)$. The optimal fidelity can be achieved when α,β take the following values,

$$\alpha=\left(\frac{1}{2}-\frac{d-2}{2\sqrt{d^2+4d-4}}\right)^{\frac{1}{2}};\quad \beta=\left(\frac{1}{2}+\frac{d-2}{2\sqrt{d^2+4d-4}}\right)^{\frac{1}{2}}. \tag{7}$$

In case $d=2$, the cloning transformation (6) recovers the previous known result [5, 6]. The gap between the fidelities of phase-covariant cloning machine and UQCM decreases when d becomes larger. When d is large enough, this gap becomes negligible.

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