A simulated photon-number detector in quantum information processing

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The fundamental limitations to classical communication in optical channels are due to the quantum nature of the signals being transmitted. The limitations can be characterized by the channel capacity as it has been shown that any attempt at communication beyond this capacity necessarily fails due to unrecoverable errors [1, 2]. For a single-mode optical communication channel the optimal capacity, under a mean energy constraint, is achieved with a source alphabet of photon-number states and ideal photon-number detection [3, 4]. In this ideal case the orthogonality of the signals and hence their perfect distinguishability makes error correction unnecessary. Unfortunately, *such ideal operation is currently impractical*, since neither ideal photon-number state preparation nor ideal photon-number detection is achievable.

The detection of weak signals (few photons) is especially difficult at communications wavelengths (1.3 - 1.55 μm). At communication frequencies, photon counting has been achieved with InGaAs or Ge avalanche photodiodes operating in the so-called Geiger mode [5, 6]. Due to the high dark-count rate, performance of these detectors as photon counters is very low. The best efficiency reported is around 20% at 1.3 μm at its optimal temperature 77 K [5], and is around 10% at 1.5 μm at the optimal temperature 213 K [6].

In this paper we consider an alternate encoding and detection strategy which is suitable for truly weak signals and current technological limitations. The basic idea is to *simulate* direct detection via a dual-homodyne scheme. Because strong local oscillators are continuously producing a strong output photocurrent, even high dark-count-rate detectors like PIN photodiodes, which have the highest quantum efficiencies [7, 8], may be used. We determine the mutual information for inefficient direct detectors and compare it to that of efficient homodyne detectors for a source alphabet preferring direct detection strategies. In doing so we are able to compute an equivalent efficiency for our simulated photon-number detectors for communication purposes.

We model loss by introducing extra beam-splitters into the channel or in front of the detector, discarding photons in the unused port. The parameter η is the *amplitude* efficiency corresponding to the amplitude-transmission coefficient of the beam-splitter, so η^2 represents the quantum efficiency of the overall detector. The mutual information $I^{\eta}(\bar{n})$ for a non-ideal photon detector is characterized by the efficiency η^2 and the mean-photon number $\bar{n}$. This function shows that a small amount of loss away from perfect detection results in a significant decrease in the mutual information.

Simulating photon-number detection via homodyne detectors, we have a communication penalty to pay. However it might be possible to exploit very high efficiencies of homodyne detectors for this purpose. Hence as a first approximation we will treat the homodyne measurements as ideal. Our detection strategy is based on dual homodyne detection, which can simultaneously detect both quadrature-phase amplitudes. We note here that we could have replaced the dual homodyne detection by hetrodyne measurement [9, 10]. We evaluate the simulated photon-number detector based on the mutual information given by the finite efficiency photon-number detector. We will equate the mutual informations achievable in each of these two schemes with an alphabet chosen to prefer direct detection. The choice of finite efficiency ${\eta^*}^2$ in the direct detection scheme for which this equivalence holds is dubbed by us the *equivalent efficiency* of the dual homodyne detection scheme (at least for the purposes of classical communication as analyzed here). Thus, the equivalent efficiency may be determined by inverting the relation

$$I^{\eta^*}(\bar{n}) = \mathcal{I}^{hd}(\bar{n}) \;, \tag{1}$$

for the efficiency ${\eta^*}^2$. This efficiency is obviously dependent on the mean photon number $\bar{n}$ for the source alphabet. For instance, the equivalent efficiency of the dual homodyne measurement for a mean-photon number $\bar{n} = 1$ is just ${\eta^*}^2 \simeq 0.327$. More general cases are shown on the graph of equivalent efficiency versus mean photon number in Fig. 1.

For an input alphabet of photon-number states, it is clear that schemes based upon homodyne measurement cannot be expected to perform as well as ideal direct photon detection. Nonetheless, such ideal direct photodetectors are not currently technologically realistic, especially at communications wavelengths. By contrast, since homodyne detectors may be operated without regard to dark current a significantly higher quantum efficiency is readily available for them. We have found that replacing inefficient direct detectors with homodyne-based simulated direct detectors can yield reasonable improvements, even near the single-photon level of operation.

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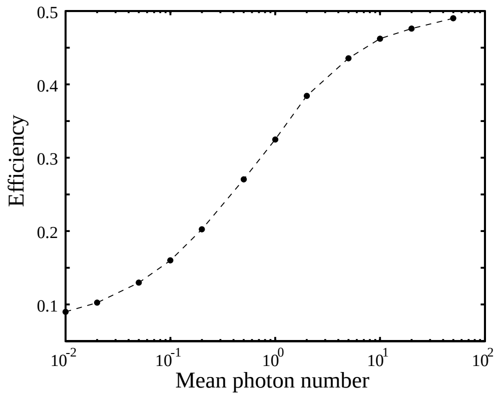

FIG. 1: The equivalent efficiency ${\eta^*}^2$ determined by inverting Eq. (1); shown with a semi-log scale in the mean photon number.

In this paper, we have shown that this improvement is theoretically possible for the purposes of classical communication through a single-mode bosonic channel. However it still remains to be considered whether the approach studied here really has any applicability to either quantum communication or computation. The maximum 50% equivalent efficiency of the simulated photon detection here might rule out these possibilities for detecting quantum information represented within discrete photonic Hilbert spaces. Thus, if we hope to use these ideas beyond classical communication this low efficiency implies that discrete Hilbert spaces will need to be abandoned. This suggests using homodyne detectors within some kind of continuous quantum variable scenario. For such variables a generalized Gottesman-Knill (GK) theorem can be derived [11]. The GK theorem provides a valuable tool for assessing the classical complexity of a given process.

The GK theorem for continuous variables shows that any circuit built up of components described by quadratic Hamiltonians (such as the set of gates SUM, Fourier transform, squeezing, and single oscillator Pauli operator [11]), that begins with finitely squeezed states and involves only measurements of canonical variables, such as homodyne measurements, may be efficiently classically simulated. Further, including direct photodetection is sufficient to provide these circuits with the capability to perform universal quantum computations [12, 13]. This observation suggests that our particular simulation strategy cannot be improved arbitrarily. For if homodyne-based measurements (and linear optics) could come arbitrarily close to simulating photocounting with only a polynomial number of components then a universal quantum computer could be simulated classically in polynomial time by this above theorem, which would imply BQP = BPP. (BPP is the class of problems that can be solved using randomized algorithms in polynomial time, while BQP is the class of all computational problems which can be solved efficiently on a quantum computer.) This outrageously unlikely outcome suggests that simulating direct detection using homodyne detectors must have limited efficiency, probably is not much different than the dual-homodyne scheme studied here.

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