Optimal realization of arbitrary gates in HQC

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We report the first attempt to implement holonomic quantum computing (HQC) numerically. We have developed a method for finding holonomy loops corresponding to any desired unitary one-qubit or two-qubit quantum operations in a realization-independent three-state model. In holonomic quantum computing unitary operations are achieved by selecting a 2^N times degenerate qubit system and allowing for adiabatic time-development that does not change the degeneracy structure. Even though the Hamiltonian of this subspace is completely trivial, it turns out that a non-Abelian and irreducible gauge potential appears, using which any unitary evolution can be carried out. As the word 'holonomy' suggests, we drive the system around loops in the control parameter space (or manifold) and after each loop there is a nontrivial change in the state of the system.

Let us consider a three-state model of a holonomic quantum computer with a degenerate eigenvalue spectrum described by the reference Hamiltonian

$$H_{\lambda_0} = \sum_j^N \delta E_j |2_j\rangle\langle 2_j| \tag{1}$$

where $|2_j\rangle$ is an auxiliary state of the j^{th} qubit and the corresponding logical states are $|0_j\rangle$ and $|1_j\rangle$. Here λ_0 refers to some set of system parameters. Using the above reference Hamiltonian we define a time-dependent degenerate Hamiltonian $H_{\gamma(t)}$ which can be taken without any loss of generality to be

$$H_{\gamma(t)} = W_{\gamma(t)} H_{\lambda_0} W_{\gamma(t)}^\dagger \tag{2}$$

where $W_{\gamma(t)} \in U(2^N)$. If the degenerate quantum system described by this Hamiltonian evolves adiabatically and cyclically around a loop γ in the parameter manifold, its unitary development is dictated by the formula

$$U_\gamma = \mathcal{P} \exp\left(-\oint_\gamma \mathcal{A}\right) \tag{3}$$

where $\mathcal{A} = \sum_i A_{\gamma_i} d\gamma_i$ and $(A_{\gamma_i})_{ab} = \langle \eta_a | W_\gamma^\dagger \frac{\partial}{\partial \gamma_i} W_\gamma | \eta_b \rangle$. Here $|\eta_a\rangle$ stands for the eigenket of the Hamiltonian H_γ labeled by the quantum number a. All the loops start at λ_0 and $W(\lambda_0) = I_{2^N}$. We only consider rotations that act nontrivially on at most two qubits at a time. We factorize the rotations W_γ acting on the qubits a and b by writing

$$W_\gamma = W_\xi \left(W_\gamma^{\text{a}} \otimes W_\gamma^{\text{b}}\right). \tag{4}$$

Here $W_\xi = e^{i\xi|11\rangle\langle 11|}$ is associated with nontrivial two-qubit operations while W_γ^{a} and W_γ^{b} are associated with single-qubit operations. The single-qubit rotations are further factored using Givens rotations, which results in a six-dimensional parameterization of single-qubit loops whereas the two-qubit loop parameterization results in a 13-dimensional space.

Arbitrary single- and double-qubit loops can be found via solving the optimization problem of minimizing

$$f(\gamma) = \|\hat{U} - U_\gamma\|_F \tag{5}$$

where F stands for the Frobenius trace norm. Here $\hat{U}$ is the desired quantum gate. In order to solve this task we have calculated the connection components and then limited ourselves to polygonal loops. The coordinates of the vertices

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form the optimization space. We have found several different implementations for some example quantum-gates using the prescribed method and two different derivative-free optimization algorithms. The optimization landscape seems to contain many minima. In particular, we have discovered implementations for the Hadamard gate, the CNOT gate, the SWAP gate, and the two-qubit Fourier transform. The numerical errors in the resulting loops were negligible. We found out that the Fourier transform can be constructed using a loop with only three vertices. However, adding more vertices resulted generally in shorter loops. The length used in the study was Euclidean. This must be seen as only a preliminary step towards a more realistic scheme employing a curved metric. For instance, the single-qubit control manifold is $S^5 \simeq SU(3)/SU(2)$. We attempted adding a penalty term for the length of the loop. In this manner we managed to find shorter implementations for the holonomic quantum-logic gates. It must be emphasized, though, that it is certainly desirable for the loop to have the shortest length possible to achieve fast operation speed without sacrificing the adiabaticity.

We have also developed a technique for improving an initially known quantum gate by minimizing with respect to the length and adding a penalty term for excess error given by the Frobenius norm. This method may be used for combining loops in an effective manner. For instance, given a factorization of a certain gate, i.e., a sequence of loops, our method enables one to construct a single shorter loop the yields the very same gate. Our calculations prove the presented three-state model capable of universal HQC.

Because HQC is adiabatic, and hence time-consuming, it is important to optimize the construction of quantum gates. Our optimization method could also be extended to dynamical quantum computing. Superconducting nanostructures could provide a platform for testing our results experimentally.